PERTURBATION SIGNAL DESIGN

H.A. Barker[1], D.E. Rivera[2], A.H. Tan[3] and K.R. Godfrey[4]

1: *School of Engineering, University of Wales Swansea, Swansea, SA2 8PP, U.K.*
2: *Department of Chemical and Materials Engineering, Arizona State University, Tempe, Arizona, 85287-60006, U.S.A.*
3: *Faculty of Engineering, Multimedia University, 63100 Cyberjaya, Malaysia.*
4: *School of Engineering, University of Warwick, Coventry, CV4 7AL, U.K.*

Abstract: This tutorial paper focuses on a number of designs for perturbation (input) signals for system identification; all of the signals can be designed using software readily available on the World Wide Web. Pseudorandom signals have fixed spectra, and both binary and multilevel signals based on maximum-length sequences are discussed. Other classes of pseudorandom binary signals that greatly increase the number of available signal periods are described. Computer-optimized signals have spectra that can be specified by the user, and the paper deals with three types – multisine (sum of harmonics) signals, and binary and multilevel multiharmonic signals. Perturbation signal quality measures are also considered in the paper.

Keywords: Binary signals; Frequency responses; Input signals; Multilevel codes; Pseudorandom signals; System identification; Time-domain responses.

1. INTRODUCTION

The origins of perturbation signal design for linear system identification lie in the concept that an impulse response model of a system can be obtained from the cross-correlation function of its input and output signals. The procedure is particularly simple when the input signal is a white noise, with an autocorrelation function that is an impulse, in which case the cross-correlation function is the required impulse response model. Despite two important disadvantages, first that a white noise signal is not physically realisable and second that an impulse response model is not the most convenient form for use in control systems, this approach was followed until the 1960's using random signals that adequately approximated white noise for the purposes of the identification.

During the 1960's, the first disadvantage was overcome as the focus of perturbation signal design changed from random white noise approximants to deterministic white noise approximants, now commonly termed pseudorandom signals. Although the most successful of these were based on the maximum-length sequences described by Zierler (1959), other types of importance do exist, as shown in surveys by Everett (1966), and Barker and Raeside (1968). Two obvious advantages of this approach are that a deterministic input signal can be properly designed and implemented for each identification experiment, and that periodic averaging eliminates the indeterminacy associated with random signals. A third advantage, less obvious at the time, is that the concept lends itself to digital signal processing methods that have been used in system identification ever since.

During the 1970's, the second disadvantage was overcome when the fast Fourier transform (FFT) became available for digitally processing the perturbation signal and the sampled system response.

The radix-2 FFT of Cooley and Tukey (1965) had been of little interest for perturbation signal design because the signal periods were not powers of 2 (Lamb and Rees, 1973), but the mixed-radix FFT of Singleton (1969) proved to be much more useful because it could be applied to perturbation signals with any period. Barker and Davy (1975) described the applications of this transform, and showed that working in the frequency domain has many inherent advantages for perturbation signal design, including methods for suppressing perturbation signal harmonics to facilitate the use of harmonic separation in the system output, and methods for correcting input transducer errors by pre- or post-compensation.

The availability of this computationally efficient method for obtaining the discrete Fourier transform (DFT) of a signal also made possible the development of further perturbation signal design methods that involved optimization to obtain the desired characteristics. These include the discrete interval binary (DIB) signal design method of Van den Bos and Krol (1979) and the sum of harmonics (SOH) design method of Van der Ouderaa, *et al.* (1988). The SOH design is based on a time-frequency swapping algorithm that was later used by McCormack, *et al.* (1995), and subsequently developed by Tan and Godfrey (2004), for the design of multilevel multiharmonic (MLMH) perturbation signals.

2. PSEUDORANDOM SIGNALS

The largest class of pseudorandom signals are the maximum-length pseudorandom (MLPR) signals, which are based on maximum-length sequences in Galois fields. The software GALOIS for generating these signals, described by Barker (2001), is freely available at

http://www.eng.warwick.ac.uk/eed/dsm/galois

A Galois field GF(q) may be defined when q is a prime or a power of a prime. GF(q) has q field elements that may be represented by the integers $0\,1\,2\,3\ldots q-1$, or by the powers $0\,1\,g\,g^2\ldots g^{q-2}$, where g is a primitive field element. A sequence $s_{q,n}(i)$ of elements of GF(q) is defined by the linear recurrence relationship of order n

$$s_{q,n}(i)+c_1 s_{q,n}(i-1)+\ldots+c_n s_{q,n}(i-n)=0 \quad \text{for all } i \qquad (1)$$

As shown by Zierler (1959), if the characteristic polynomial in the delay, D, given by $f_n(\mathrm{D})=1+c_1\mathrm{D}+\ldots+c_n\mathrm{D}^n$ of $s_{q,n}(i)$ is a primitive polynomial, then $(-1)^n c_n$ is a primitive field element and $s_{q,n}(i)$ is a maximum-length sequence in GF(q) with the greatest possible period $N=q^n-1$. In the period q^n-1, each field element occurs q^{n-1} times, except for the element 0, which occurs $q^{n-1}-1$ times.

A MLPR signal $u_{q,n}(i)$ with period $N=q^n-1$ is generated from the maximum-length sequence $s_{q,n}(i)$ by converting the field elements $0\,1\,g\,g^2\ldots g^{q-2}$ into the signal levels $\mathrm{u}(0)\,\mathrm{u}(1)\,\mathrm{u}(g)\,\mathrm{u}(g^2)\ldots\mathrm{u}(g^{q-2})$, and the field element conversions define properties that are the same for all pseudorandom signals $u_{q,n}(i)$ generated from $s_{q,n}(i)$. Although the q field elements are by definition distinct, it is not necessary that the signal levels are also distinct, and if they are restricted to r distinct levels then an r-level pseudorandom signal is generated.

If $n=1$, then as shown by Barker (2004), a primitive maximum-length sequence $s_{q,1}(i)$ is generated, with period $N=q-1$ and characteristic polynomial $f_1(\mathrm{D})=1+c_1\mathrm{D}$. A period of $s_{q,1}(i)$ is therefore $1\,g\,g^2\ldots g^{q-2}$, where $g=-c_1$ is a primitive field element, so in the period $q-1$, each field element occurs once, except for the element 0, which does not occur. A primitive MLPR signal $u_{q,1}(i)$ with period $N=q-1$ is generated from the primitive maximum-length sequence $s_{q,1}(i)$ by the field element conversions defined above, so a period of $u_{q,1}(i)$ can be expressed as $u(1)\;u(2)\;u(3)\ldots u(q-1)$ or as $\mathrm{u}(1)\,\mathrm{u}(g)\,\mathrm{u}(g^2)\ldots\mathrm{u}(g^{q-2})$. The conversions of the nonzero field elements for which the MLPR signal $u_{q,n}(i)$ has the same properties as those of the primitive MLPR signal $u_{q,1}(i)$ are therefore

$$\mathrm{u}(g^{i-1})=u_{q,1}(i) \qquad \text{for } i=1,2,3,\ldots,q-1 \quad (2)$$

and the zero field element conversion u(0) is itself zero, that is $\mathrm{u}(0)=0$.

Equation (2) shows that, to design a multilevel MLPR perturbation signal to have a set of desirable properties for system identification, it is necessary only to design the corresponding primitive multilevel MLPR signal to have the same properties. As the primitive multilevel MLPR signal has only a short period, this is accomplished relatively easily. The properties may be in either the time domain or the frequency domain, which for a signal $u(i)$ with period N is defined through its DFT $U(k)$, where

$$U(k)=\sum_{i=0}^{N-1} u(i)\exp\left(-\frac{2\pi \mathrm{j} ik}{N}\right)$$
$$\text{for } k=0,1,2,\ldots,N-1 \tag{3}$$

Some properties may be interpreted equally well in either domain. For example, if it is required to eliminate even-order nonlinear distortion effects by suppressing the even harmonics of $u(i)$, so that $|U(k)|=0$ when k is even, then this is achieved by ensuring that $u(i)$ is an inverse-repeat signal with the property that

$$u(i)+u(i+N/2)=0 \qquad \text{for all } i \tag{4}$$

Other desirable properties might include the additional suppression of harmonic multiples of 3, or a uniform spectrum of nonzero harmonics, or the optimization of measures of signal quality, such as those described in Section 3.

Three methods have been used successfully to design primitive multilevel MLPR signals with specified desirable properties that are then conferred on the corresponding multilevel MLPR signals. The first is an analytical method, which has the advantage of giving symbolic results of great generality, as described by Barker, *et al*. (2004). The second is a direct search method, as described by Barker, *et al*. (2005), which has the advantage of giving accurate results. The third method uses the software *multilev_new*, described by Tan and Godfrey (2004), to generate the primitive multilevel MLPR signal as a MLMH signal, which has the advantage that it can be used when the first two methods become computationally inefficient, as described in more detail in Tan, *et al*. (2005).

When $q=2$, a maximum-length binary (MLB) sequence $s_{2,n}(i)$ is obtained from which a MLB signal $u_{2,n}(i)$ with period 2^n-1 may be generated using the field element conversions $\mathrm{u}(1)=1$ and $\mathrm{u}(0)=-1$. In common with other MLPR signals, MLB signals may be generated using the software GALOIS. Although MLB signals are the most important and popular class of pseudorandom binary (PRB) signals, other classes do exist. As shown by Everett (1966), and Barker and Raeside (1968), these include, *inter alia*, quadratic residue binary signals, with periods $4k-1$ when $4k-1$ is prime, Hall binary signals, with periods $4k^2+27$ when $4k^2+27$ is prime, and twin prime binary signals, with periods $k(k+2)$ when k and $k+2$ are prime. These signals complement MLB signals by increasing the number of periods available, and the software *prs* for their generation, described by Tan and Godfrey (2002), is freely available at

http://www.eng.warwick.ac.uk/eed/dsm/prs

All PRB signals have a full harmonic spectrum that is uniform except at multiples of the period, and although they can be used as perturbation signals in their own right it is preferable to use them to generate signals with even more desirable properties. This is accomplished by inverting alternate members of a PRB signal to generate an inverse-repeat PRB (IRPRB) signal, with a period N that is twice that of the signal from which it is generated. An IRPRB signal $u(i)$ satisfies (4), so its even harmonics $|U(2k)|$ are zero and even-order nonlinear distortion effects can be eliminated through input-output processing. In addition, the spectrum of its odd harmonics is uniform and the signal quality measure PIPS, defined in (5) has its maximum value of 100%.

3. SIGNAL QUALITY MEASURES

Four measures of quality are commonly applied to a perturbation signal $u(i)$ used for the identification of the underlying linear dynamics (Schoukens, *et al*., 2001) of a system in the presence of noise and nonlinear distortions. The first of these is PIPS, which is defined as

$$\mathrm{PIPS}=\frac{200}{N\left(u(i)_{\max}-u(i)_{\min}\right)}\left(\sum_{k=1}^{N-1}|U(k)|^2\right)^{1/2}\ \% \tag{5}$$

PIPS takes into account the necessary compromise between a high power content, to maximize the signal-to-noise ratio, and a low amplitude, to minimize the excitation of nonlinearities. It is closely related to an older measure of signal quality, crest factor, which is $100/(\sqrt{2}\mathrm{PIPS})$. For applications where not all harmonics are used, Godfrey, *et al*. (1999) proposed the signal quality measure Effective PIPS, or PIPSE, where

$$\mathrm{PIPSE}=\frac{200}{u(i)_{\max}-u(i)_{\min}}\left(\sum_{\text{specified}\,k}|C'_u(k)|^2\right)^{1/2}\ \% \tag{6}$$

and $|C'_u(k)|^2$ is the power of the k-th harmonic in a unilateral spectrum of positive frequencies.

While the use of PIPS and PIPSE provides information on the power content, they do not give any indication of the power at the individual specified harmonics. In order to quantify the normalised minimum ratio between actual and specified harmonic amplitude at any of the specified harmonics, Godfrey, *et al*. (1999) proposed that the signal quality measure EMINE should be used, where

$$\text{EMINE} = \min \frac{100 |C'_u(k)| \left(\frac{1}{R} \sum_{\text{specified } k} |C'_u(k)|^2_{\text{specified}} \right)^{1/2}}{|C'_u(k)|_{\text{specified}} \left(\frac{1}{R} \sum_{\text{specified } k} |C'_u(k)|^2 \right)^{1/2}} \% \tag{7}$$

where R is the number of specified harmonics. Both (6) and (7) can be simplified if the specified DFT spectrum is uniform (Tan, *et al.*, 2005). The quality measures PIPS, PIPSE and EMINE are all independent of the signal mean and amplitude scaling factor, and their values range from 0% for a signal with the worst performance to 100% for a signal with the best performance.

A fourth quality measure can be derived from PIPSE and EMINE. This is the Time Factor (TF), defined by Pintelon and Schoukens (2001). TF measures the minimum accuracy obtained in a fixed measurement time for a specified maximum peak value of excitation, and for signals where $u(i)_{\min} = -u(i)_{\max}$ it is given by

$$\text{TF} = 0.5 \left(\frac{100}{\text{PIPSE}} \right)^2 \left(\frac{100}{\text{EMINE}} \right)^2 \tag{8}$$

4. MULTISINE SIGNALS

A series of very informative survey and comparison papers appear in the literature which examine multisine inputs; this includes the articles by Schoukens, *et al.* (1988), Godfrey, *et al.* (1999), Godfrey, *et al.* (2005) as well as the edited book by Godfrey (1993) and the text by Pintelon and Schoukens (2001). A multisine SOH signal is a sum of harmonically-related sinusoids, represented in the single input case as

$$u(k) = \lambda \sum_{i=1}^{n_s} \sqrt{2\alpha_i} \cos(\omega_i kT + \phi_i) \quad \begin{array}{c} \omega_i = 2\pi i / N_s T \\ n_s \le N_s / 2 \end{array} \tag{9}$$

The amplitude of each sinusoid and hence the power spectrum in a multisine input can be directly specified through the selection of the scaling factor λ, the Fourier coefficients α_i, the number of harmonics n_s, and the signal length N_s. T is the sampling time. Theoretical requirements such as persistence of excitation, harmonic suppression, and control-relevance can be satisfied without loss of generality through the choice of the power spectrum. The choice of phases ϕ_i strongly influences the time-domain realization of a multisine signal, and hence making the proper selection of this variable in this class of signals has been a subject of research for a number of decades. Most of the approaches centre on minimizing the crest factor, which is equivalent to maximizing PIPS.

Early seminal work in the design of multisines leading to high PIPS comes from the work of Schroeder (1970), who presented a closed-form formula to select the phases in a multisine. The formula gives a reasonable result when the user-defined spectrum is flat and wideband over consecutive harmonics, or for consecutive odd harmonics. However, for band-limited signals, multisines with sparse spectra (where the frequency lines are few and far apart) or when the amplitude spectra are not flat, Schroeder-phasing does not yield better results than random phase selection (Pintelon and Schoukens, 2001). Despite these limitations, Schroeder-phased multisines have been reported as being effective in a variety of application environments, ranging from large space structures to industrial boilers.

The deficiencies of Schroeder-phasing justify the need for more rigorous approaches, such as those involving optimization. Two significant efforts in this vein have been developed at Vrije Universiteit in Brussels; these are the iterative "clipping" algorithm by Van der Ouderaa, *et al.* (1988) and the sequential p-norm optimization approach of Guillaume, *et al.* (1991). Solutions involving constrained optimization and nonlinear programming have also been proposed (Rivera, *et al.*, 2002; Lee, *et al.*, 2003). All of these algorithms enable generating signals featuring the "snow effect," in which additional power is added by the algorithm at frequencies other than the primary ones to further increase PIPS.

The algorithm *msinclip* in the Frequency Domain System Identification Toolbox (FDIDENT, Kollár, 2005) implements the iterative clipping approach. Two forms of zero-order hold (ZOH) multisine signals are available using *msinclip*, both of which result in a signal with some power in the non-specified harmonics. The first form is a band-limited signal, passed through a ZOH, which has a $(\sin^2 x)/x^2$ power spectrum envelope. The second form has pre-compensation for the shape of the ZOH spectrum. For this form, the value of EMINE is 100%, regardless of the specification, because the actual and harmonic amplitudes are the same at the specified harmonics, but this is at the expense of relatively low PIPS and PIPSE values.

The optimization problem associated with maximizing PIPS is not straightforward because the infinity norm is non-convex and non-differentiable. The work by Guillaume, *et al.* (1991) presents an approach which seeks to approximate the minimization of the Chebyshev norm by sequentially minimizing the p norm for p = 4, 8, 16, . . . Although a global solution cannot be guaranteed with this approach, most local minima are avoided and experimentally it performs very well. Extensions of the technique discussed in Guillaume, *et al.* (1991) include maximization of PIPS for linearly related multiple multisines, when PIPS for both input and output signals need to be maximized, and multivariable scenarios.

Despite the usefulness of the clipping and successive *p*-norm minimization algorithms, there are a number of considerations that motivate the use of constrained optimization approaches. One is the need to achieve plant-friendliness during identification testing, which will require that both input and output signals stay within time-domain constraints (Rivera, *et al.*, 2003). The other is the opportunity afforded by recent advances in optimization techniques to minimize the infinity norm directly. These ideas have been explored in a series of recent papers (Rivera, *et al.*, 2002; Lee, *et al.*, 2003). The methods readily generalize to the multivariable case.

For multivariable system identification, simultaneous multi-input testing can decrease the overall test time. Methods for reducing the inter-channel interactions include using delayed versions of a single multisine signal in different channels, and orthogonal power spectra (Lee, *et al.*, 2003).

5. BINARY MULTIHARMONIC SIGNALS

Another class of optimized frequency-domain excitation signals is the DIB signals. These are periodic binary signals, where the sign can change only at an equidistant discrete set of points in time (Van den Bos and Krol, 1979). The amplitude spectrum of the signal can be optimized by choosing a good switching sequence so that the energy is concentrated within the frequency band of interest. PIPSE is low because not all of the power is concentrated in the frequencies of interest; but even then, most of the energy can be confined to the frequency band required (Pintelon and Schoukens, 2001). Compared with PRB signals, DIB signals can be generated for any signal length.

The function *dibs* in the FDIDENT toolbox can be used to generate DIB signals, and the objective of the optimization in this case is to force as much power as possible into the specified harmonics. The function is based on an algorithm of Van den Bos and Krol (1979). The value of PIPS for such a signal is close to 100%, because the signal is binary and the mean of the signal is either zero or very close to it. However, the value of PIPSE is less than 100% for any harmonic specification. The value of EMINE can also be low for some harmonic specifications, but then the function *dibsimpr* in the FDIDENT toolbox can be used to try to increase EMINE. (The function *dits*, available in the graphical user interface version of the FDIDENT toolbox, can be used to generate discrete interval ternary (DIT) signals.)

6. MULTILEVEL MULTIHARMONIC SIGNALS

As shown in Section 1, a method for the design of MLMH perturbation signals was first described by McCormack, *et al.* (1995), and subsequently developed by Tan and Godfrey (2004). MLMH signals are a compromise between multisine SOH signals (Kollár, 2005) and DIB signals (Van den Bos and Krol, 1979), and combine the advantages of SOH signals, which normally have high power in the specified harmonics, and DIB signals, which have a small number of levels. The original software *multilev* used the time-frequency swapping algorithm (Van der Ouderaa, *et al.*, 1988) to optimize EMINE. The modified software *multilev_new* optimizes TF, and is freely available at

http://www.eng.warwick.ac.uk/eed/dsm/multilev_new

This software includes an option to pre-compensate the perturbation signal spectrum to take account of the input transducer dynamics, assumed to be in the form of a ZOH, when identifying a continuous dynamic system. The uncompensated option, only available in the modified routine, was used to generate the primitive multilevel MLPR signals with specified properties that are then conferred on the corresponding multilevel MLPR signals with longer periods, as described in Section 2. The software *multilev_new* has fewer constraints on the harmonic specification than most other methods, as it can deal not only with the broadband uniform spectrum that is most commonly used, but also with both band pass and logarithmic frequency scale specifications. These designs, for which the use of pseudorandom signals is clearly ruled out, were considered in Tan and Godfrey (2004).

MLMH signals, being a compromise between SOH signals and DIB signals, are useful in applications where there are constraints on both the number of signal levels and the harmonic specification, particularly for the identification of the underlying linear dynamics in the presence of noise and nonlinear distortions. For such an application, it is advantageous to have harmonic multiples of both two and three suppressed in order to be able to completely eliminate the effects of even order nonlinearities on the estimates of the linear dynamics, while reducing the effects of odd order ones (Godfrey, *et al.*, 2005). Such a specification is not possible with binary signals, but suitable ternary signal designs are possible with MLMH signals, DIT signals (Kollár, 2005) and MLPR signals (Barker, 1993; Barker, *et al.*, 2005).

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