Robust Optimal Decision Policies for Adaptive, Time-Varying Interventions Using Model Predictive Control

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Presentation Outline

- Brief descriptions of control systems engineering and time-varying adaptive interventions.
- Model Predictive Control (MPC) problem formulation
- MPC application to a hypothetical intervention (based on the *Fast Track* program) and comparison.
- Extensions: adaptive interventions for smoking cessation and fibromyalgia.
- Summary and conclusions

Similarities Between Behavioral Interventions and Control Engineering

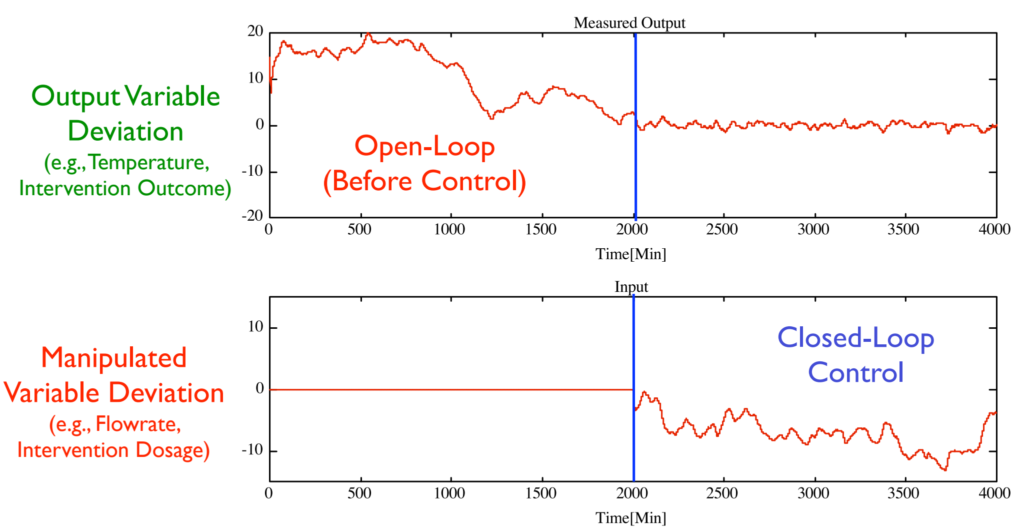
- Behavioral scientists are interested in developing and delivering effective interventions that:
  - demonstrate high levels of adherence.
  - display uniformity and reproducibility despite heterogeneity of the target population, and inherent variability associated with delivery of the intervention,
  - are cost-effective in nature.

All these goals are compatible with the objectives of control systems engineering...

Control Systems Engineering

- The field that relies on dynamical models to develop algorithms for adjusting system variables so that their behavior over time is transformed from *undesirable* to *desirable*.
- Control engineering plays an important part in many everyday life activities. Some examples of control systems engineering applications include:
  - Cruise control and climate control in automobiles,
  - The "sensor reheat" feature in microwave ovens,
  - Home heating and cooling,
  - The insulin pump for Type-I diabetics,
  - "Fly-by-wire" systems in high-performance aircraft,
  - Homeostasis
- A well-tuned control system will effectively *transfer variability* from the more "expensive" system resource to a less expensive one.

From Open-Loop Operation to Closed-Loop Control (Stochastic Viewpoint)



The transfer of variance from an expensive resource to a cheaper one is one of the major benefits of control systems engineering

Basic Components of Adaptive Interventions

(Collins, Murphy, and Bierman, *Prevention Science*, **5**, No. 3, 2004)

- The assignment of a particular dosage and/or type of treatment is based on the participant's values on variables that are expected to moderate the effect of the treatment component; these are known as *tailoring variables*.
- In a *time-varying* adaptive intervention, the tailoring variable is assessed periodically, so the intervention is adjusted on an on-going basis.
- *Decision rules* translate current and previous values of tailoring variables into choice(s) of treatment and their appropriate dosage.

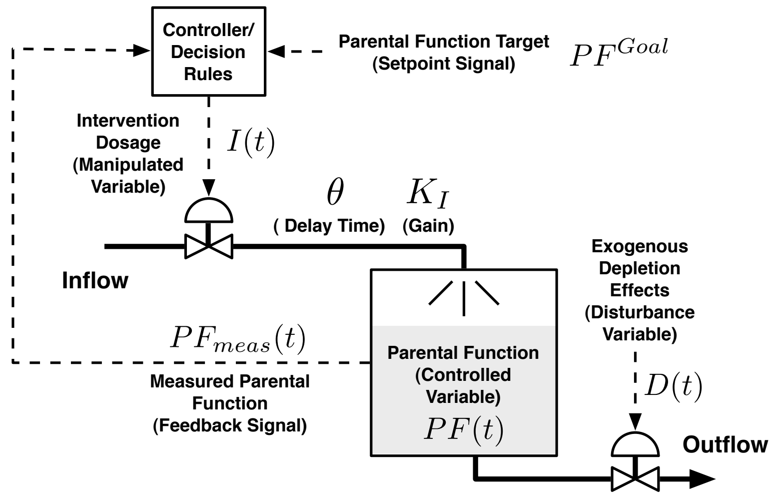
Adaptive Intervention Simulation

(inspired by the *Fast Track* Program, Conduct Problems Prevention Research Group)

- A multi-year program designed to prevent conduct disorder in at-risk children.
- Frequency of home-based counseling visits assigned quarterly to families over a three-year period, based on an assessed level of parental functioning.
- Parental function (the tailoring variable) is used to determine the frequency of home visits (the intervention dosage) according to the following decision rules:
  - If parental function is "very poor" then the intervention dosage should correspond to weekly home visits,
  - If parental function is "poor" then the intervention dosage should correspond to bi-weekly home visits,
  - If parental function is "below threshold" then the intervention dosage should correspond to monthly home visits,
  - If parental function is "at threshold" then the intervention dosage should correspond to no home visits.

Parental Function - Home Visits Adaptive Intervention as an Inventory Management Problem

(Rivera, Pew, and Collins, "Using engineering control principles to inform the design of adaptive interventions," *Drug and Alcohol Dependence*, Vol. 88, Suppl. 2, May 2007, Pages S31-S40)

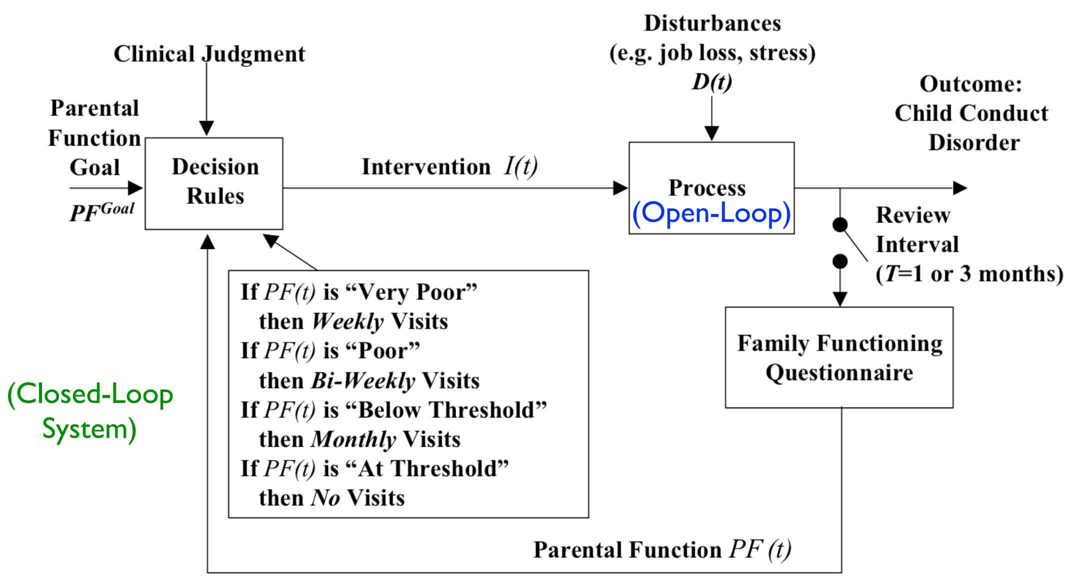


$$PF(t+1) = PF(t) + K_I\ I(t-\theta) - D(t)$$

Parental function *PF(t)* is built up by providing an intervention *I(t)* (frequency of home visits), that is potentially subject to delay, and is depleted by potentially multiple disturbances (adding up to *D(t)*).

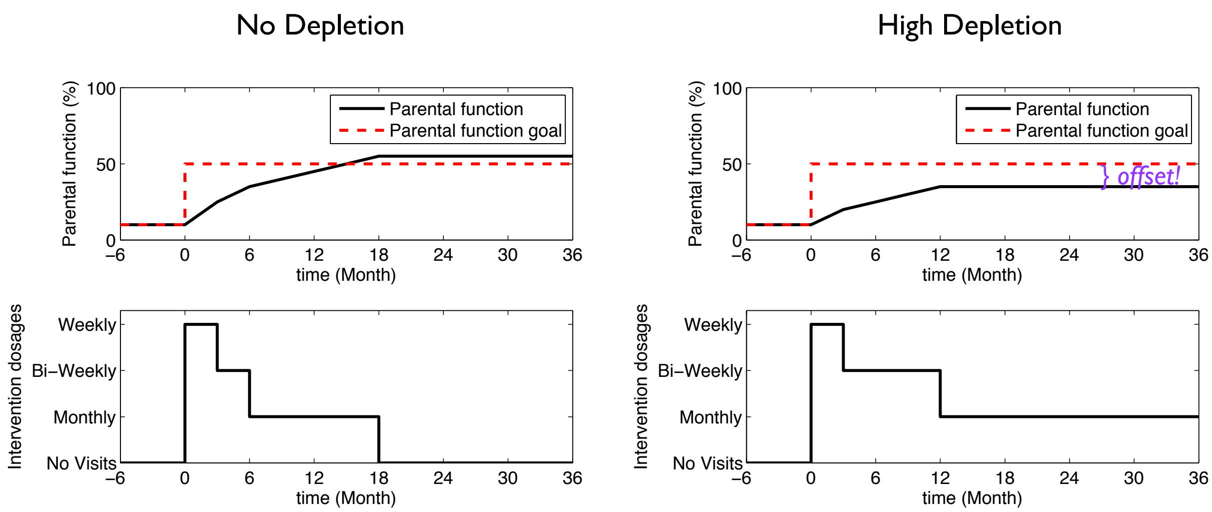
Parental Function Feedback Loop *Block Diagram**

(to decide on home visits for families with at-risk children)



From Rivera, D.E., M.D. Pew, and L.M. Collins, "Using engineering control principles to inform the design of adaptive interventions: a conceptual introduction," *Drug and Alcohol Dependence*, Special Issue on Adaptive Treatment Strategies, Vol. 88, Supplement 2, May 2007, Pages S31-S40.

"IF-THEN" Rules May Not Be Optimal

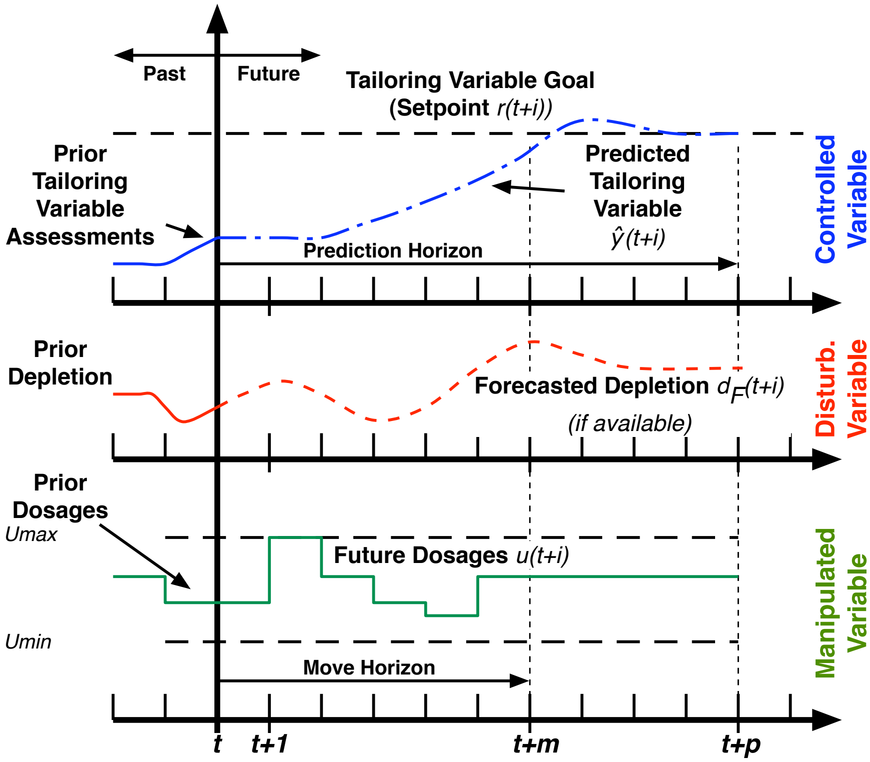


Offset (where parental function fails to achieve a desired goal at the end of the intervention) occurs when high depletion is present.

Model Predictive Control (MPC)

- Control engineering technology widely used in many industrial applications (from chemical manufacturing to automotive and aerospace)
- As an *optimizer*, MPC can minimize (or maximize) an objective function that represents a suitable metric of intervention performance.
- As a *controller*, MPC can be tuned to achieve stability, robustness, and performance in the presence of model error, measurement unreliability, and disturbances that may affect the intervention.
- MPC is *optimal* in the sense that an explicit mathematical criterion related to intervention performance is minimized (or maximized),
- MPC is *robust* in the sense that it is meant to operate under real-world conditions (e.g., noisy signals, inaccurate models, participant variability).

Model Predictive Control Conceptual Representation



Model Predictive Control Optimization Problem

$$\min_{\Delta u(t)\ldots\Delta u(t+m-1)} J = \overbrace{\sum_{i=1}^{p} Q_e(i)(\hat{y}(t+i) - r(t+i))^2}^{\text{Take Tailoring Variables to Goal}} + \overbrace{\sum_{i=1}^{m} Q_{\Delta u}(i)(\Delta u(t+i-1))^2}^{\text{Penalize Changes in the Intervention Dosages}}$$

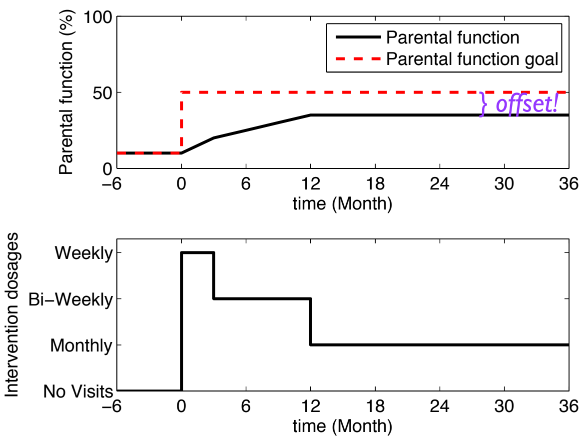
subject to constraints on:

- manipulated variables (i.e., intervention dosages)
- the rate of change of manipulated variables (i.e., dosage changes)
- controlled variables (i.e., tailoring variables),
- associated variables (i.e., secondary outcomes)

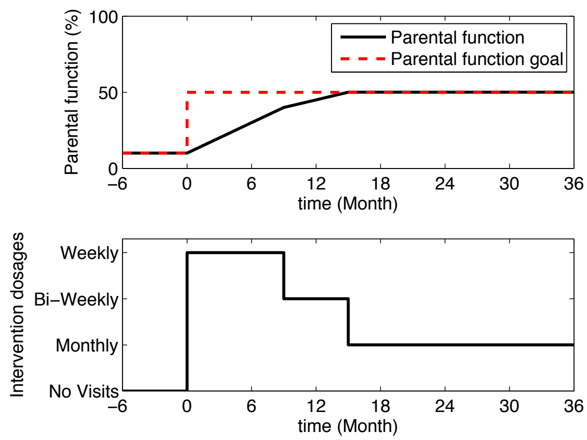
Any operating restriction that can be represented using propositional calculus can be translated into constraint equations for the Model Predictive Control optimization problem.

Controller/Decision Rule Comparison, Scenario 1
High Depletion Rate (*D(t)* = 5)

"IF-THEN" rules



Model Predictive Control
(Q_e = 1, Q_{du} = 0.05, p = 30, m = 10)

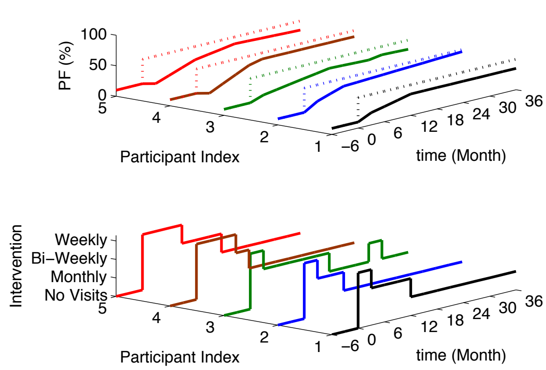


36 month intervention reviewed at quarterly intervals. Offset problem is eliminated in the MPC controller through improved assignment of intervention dosages during the course of the intervention.

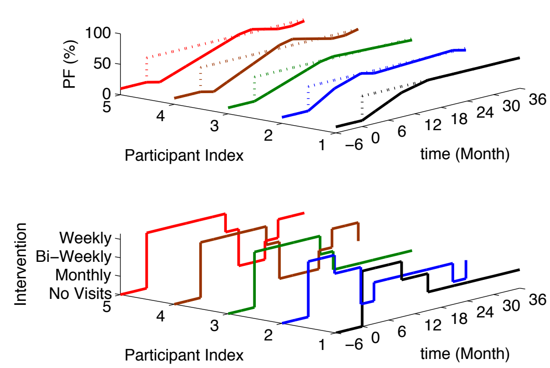
IF-THEN vs. MPC Comparison (Robustness Analysis)

- The system response of five participant families, each characterized by its own dynamical model, is evaluated using a controller tuned on the basis of an average ("nominal") effect.

"IF-THEN" Decision Rules



MPC Control
(Q_e = 1, Q_{du} = 0.05, p = 30, m = 10)



- A single MPC controller individually assigns appropriate intervention dosages to each participant family, leading to no offset and more consistent outcomes. This is achieved at the expense of greater variability in the intervention dosages.

Scenario 2: Constrained Intervention Resources*

- Total number of counseling visits constrained to an upper limit; group counseling may be offered when in-home visit limit is reached.

  – Scenario 2(a) (*unconstrained scenario*):
  » Unlimited number of in-home counseling visits;
  » no group meeting available.

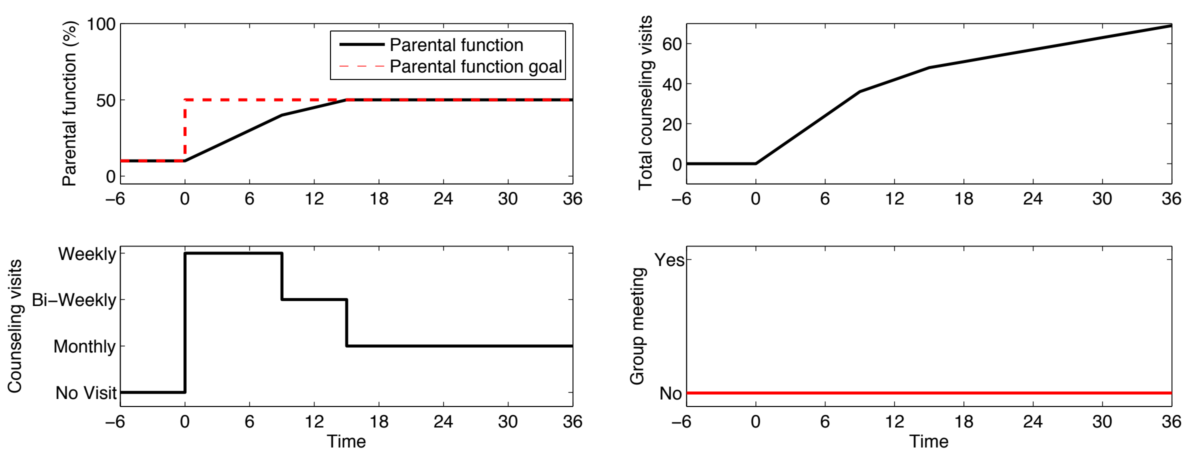
  – Scenario 2(b):
  » Overall dosage limited to a maximum of 48 in-home counseling visits;
  » no group meeting available.

  – Scenario 2(c):
  » Overall dosage limited to a maximum of 48 in-home counseling visits;
  » Group counseling available when in-home visit limit is reached.

*Nandola, N. and D.E. Rivera, "Hybrid Model Predictive Control Applied to Production-Inventory Systems," *18th IFAC World Congress*, Milan, Italy, August 28 - Sept. 2, 2011, in press.

Scenario 2(a)

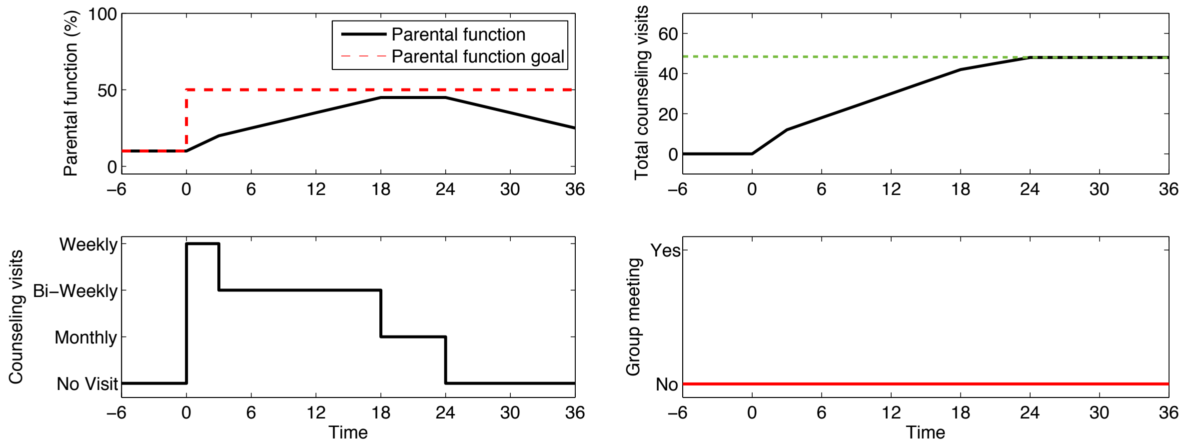
(No limit on number of counseling visits; no group meeting offered)



- 69 total counseling visits required in this unconstrained scenario to achieve the parental function goal w/o offset.

Scenario 2(b)

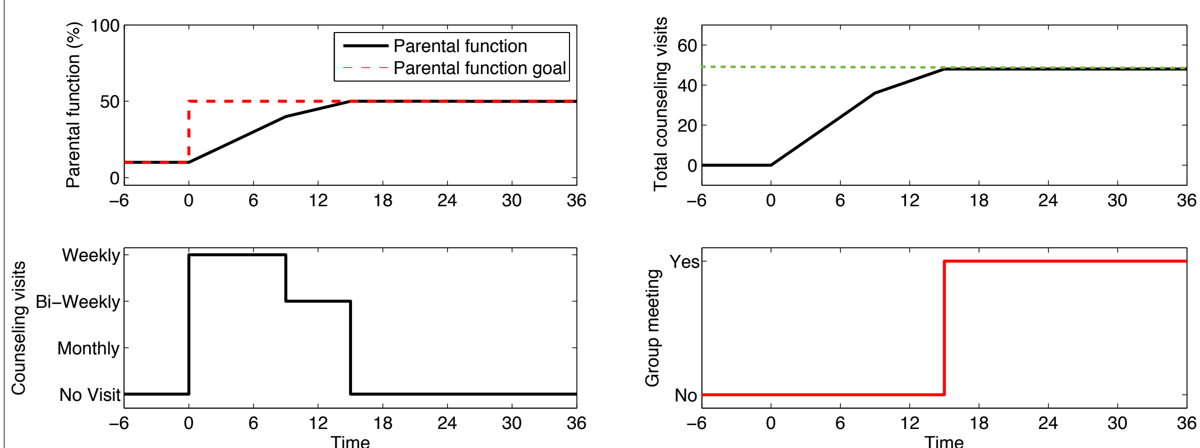
(Counseling visits limited to 48 *(total)*; no group meeting offered)



- The dosage constraint places a fundamental limit on the effect of the intervention. The controller does the best it can (for the given objective function parameters), within these restrictions.

Scenario 2(c)

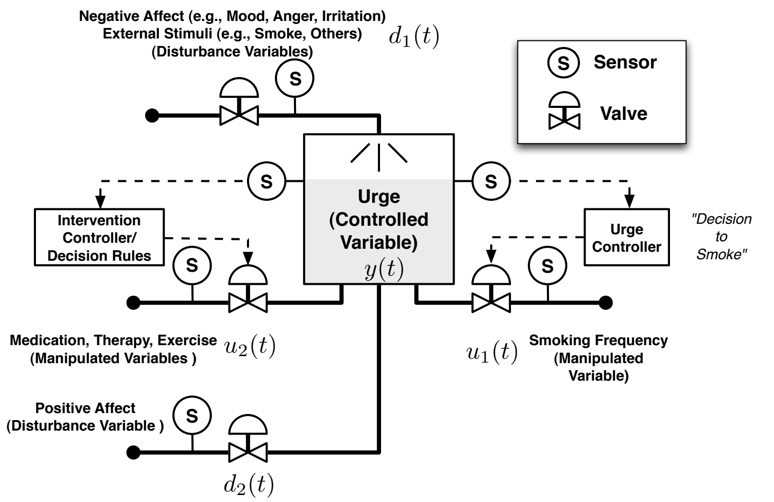
(Counseling visits limited to 48 *(total)*; group meeting available)



- The MPC adapts the intervention to meet the outcome goal, properly sequencing the additional intervention component now available (once the counseling visits limit is reached).

Adaptive Interventions for Smoking Cessation

Riley, W.T., D.E. Rivera, A.A. Autienza, W. Nilsen, S. Allison, and R. Mermelstein, "Health behavior models in the age of mobile interventions: are our theories up to the task?" *Translational Behavioral Medicine: Practice, Policy, Research*, Vol. 1, No. 1, pgs. 53 – 71, March 2011.

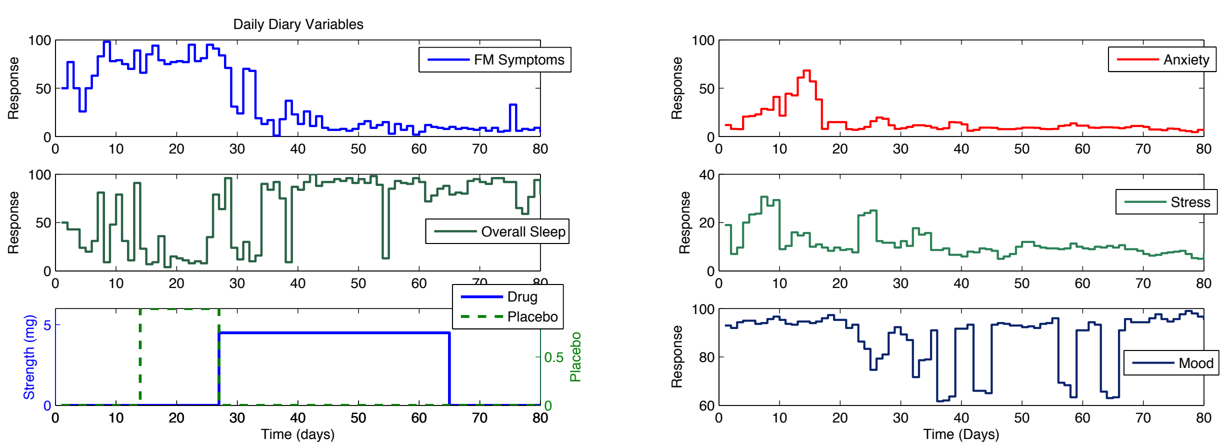


$$\frac{dy}{dt} = K_{d_1}\ d_1(t) - K_{u_1}\ u_1(t) - K_{u_2}\ u_2(t) - K_{d_2}\ d_2(t)$$

- An adaptive intervention for smoking cessation can be conceptualized as an additional feedback mechanism that ultimately replaces smoking as a means to reduce urge.

Dynamical systems: Enabling Better Use of Intensive Longitudinal Data

J. Younger and S. Mackey (Stanford School of Medicine) "Fibromyalgia symptoms are reduced by low-dose naltrexone: a pilot study, *Pain Medicine*, 10(4):665-672, 2009.



- Pilot study participant in fibromyalgia intervention using naltrexone; drug and placebo as manipulated variables; self-report of anxiety, stress, and mood as disturbances; FM symptoms and overall sleep quality as outputs. A dynamical systems model accounts for 73.9% of the output variance.

Dynamic Modeling, Fibromyalgia Intervention, Sample Participant

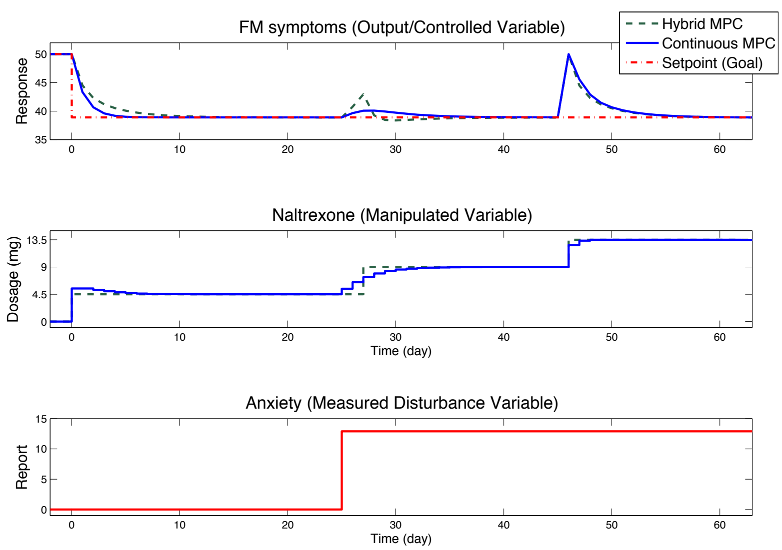
$$\tau^2 \frac{d^2y}{dt^2} + 2\zeta\tau \frac{dy}{dt} + y(t) = K_p \left(u(t) + \tau_a \frac{du}{dt} \right)$$

Model (Input-Output)	K_p, τ, ζ, τ_a	$T_{98\%}$(days)
Drug-FM symptoms	-2.47, 1.57, 1.26, 1.96	11.49
Placebo-FM symptoms	45.81, 1.57, 1.26, 1.15	13.06
Anxiety-FM symptoms	0.86, 1.57, 1.26, 0.24	14.24
Stress-FM symptoms	2.29,1.57, 1.26, 0.49	13.94
Mood-FM symptoms	-0.091, 1.57, 1.26, 4.67	11.93
Drug-Overall Sleep	4.98, 2.13, 1.04, -3.35	15.83

- *System identification* techniques are used for estimating the dynamical system parameters from daily diary data provided by participants.
- Deshpande, S., N. Nandola, D.E. Rivera, and J. Younger, "A control engineering approach for designing an optimized treatment plan for fibromyalgia," *Proc. of the 2011 American Control Conf.*, San Francisco, CA, June 29 - July 1, 2011, in press.

Closed-Loop Control, Naltrexone Intervention for Fibromyalgia

Deshpande, S., N. Nandola, D.E. Rivera, and J. Younger, "A control engineering approach for designing an optimized treatment plan for fibromyalgia," *Proc. of the 2011 American Control Conf.*, San Francisco, CA, June 29 - July 1, 2011, in press.



- Simulation result showing MPC system response to goal change (stpt. tracking), increase in anxiety report (measured disturbance rejection) and unpredicted change in pain report (unmeasured disturbance rejection).

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Fluid Analogy for the Theory of Planned Behavior

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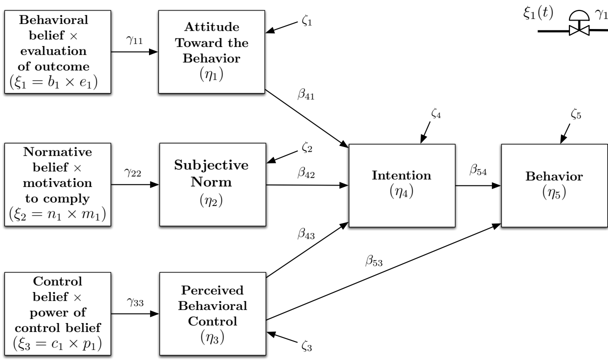
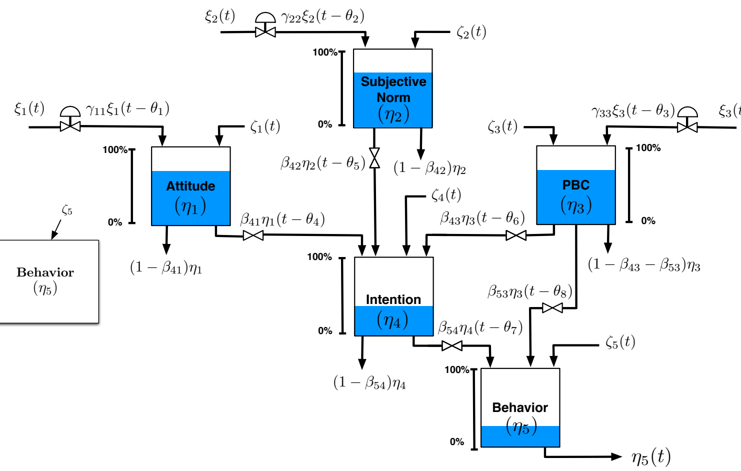
Navarro-Barrientos, J.E., D.E. Rivera, and L.M. Collins, "A dynamical model for describing behavioural interventions for weight loss and body composition change," *Mathematical and Computer Modelling of Dynamical Systems*, Volume 17, No. 2, Pages 183-203, 2011.





Any path diagram can be expressed into a corresponding fluid analogy described by a system of differential equations.

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MPC for Adaptive Interventions: Summary

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- Model Predictive Control (MPC) is a control engineering decision framework that can result in improved intervention outcomes,
  - dynamic and model based,
  - can enforce constraints on tailoring variables, dosage assignments,
  - can be tuned for robustness under conditions of uncertainty.
- Model parameters and adjustable parameters need to be estimated and specified systematically; requires relying on intensive longitudinal data.
- Solution procedure involves using Mixed Integer Quadratic Programming (*miqp*); more difficult to explain to non-experts than both the "IF-THEN" and IMC-PID (from pre-conference workshop talk); however, all this technology operates "under the hood" from the user's standpoint.
- Technological advancements in ecological momentary assessment, mobile devices, and cloud computing will make these ideas easier to implement in the future.

References

- Adaptive interventions and control systems engineering:
  - Collins, L.M., S.A. Murphy, and K.L. Bierman, "A conceptual framework for adaptive preventive interventions," *Prevention Science*, **5**, No. 3, pgs. 185-196, Sept., 2004.
  - Rivera, D.E., M.D. Pew, and L.M. Collins, "Using engineering control principles to inform the design of adaptive interventions: a conceptual introduction," *Drug and Alcohol Dependence*, Special Issue on Adaptive Treatment Strategies, Vol. 88, Suppl. 2, May 2007, Pages S31-S40.
  - Rivera, D.E., M.D. Pew, L.M. Collins, and S.A. Murphy, "Engineering control approaches for the design and analysis of adaptive, time-varying interventions," Technical Report 05-73, The Methodology Center, Penn State (also available from http://csel.asu.edu/AdaptiveIntervention)
  - Nandola, N. and D.E. Rivera, "A novel model predictive control formulation for hybrid systems with application to adaptive behavioral interventions," *Proc. of the 2010 American Control Conf.*, Baltimore, MD, June 30 - July 2, 2010, pgs. 6286-6292 (available through IEEE Xplore)
  - Zafra-Cabeza, A. , D.E. Rivera, L.M. Collins, M.A. Ridao, and E. F. Camacho, "A risk-based model predictive control approach to adaptive interventions in behavioral health," *IEEE Transactions on Control Systems Technology*, in press (available through early access *IEEE Xplore*).

References (continued)

- Behavioral interventions as dynamical systems; connections to path analysis and system identification; advanced formulations:
  - Navarro-Barrientos, J.E., D.E. Rivera, and L.M. Collins, "A dynamical model for describing behavioural interventions for weight loss and body composition change," *Mathematical and Computer Modelling of Dynamical Systems*, Volume 17, No. 2, Pages 183-203, 2011.
  - Riley, W.T., D.E. Rivera, A.A. Autienza, W. Nilsen, S. Allison, and R. Mermelstein, "Health behavior models in the age of mobile interventions: are our theories up to the task?" *Translational Behavioral Medicine: Practice, Policy, Research*, Vol. 1, No. 1, pgs. 53 – 71, March 2011.
  - Deshpande, S., N. Nandola, D.E. Rivera, and J. Younger, "A control engineering approach for designing an optimized treatment plan for fibromyalgia," *Proceedings of the 2011 American Control Conference*, San Francisco, CA, June 29 - July 1, 2011, in press.
  - Nandola, N. and D.E. Rivera, "Hybrid Model Predictive Control Applied to Production-Inventory Systems," *18th IFAC World Congress*, Milan, Italy, August 28 - Sept. 2, 2011, in press.

Additional References

- Some tutorial presentations that may be of interest:
  - Rivera, D.E., "Optimized behavioral interventions: what does dynamical systems and control engineering have to offer?," presentation at pre-conference workshop, 2011 SBM Meeting, can be downloaded from http://csel.asu.edu/adaptiveintervention (select item 19).
  - Rivera, D.E., "Engineering control theory: can it impact adaptive interventions?" tutorial presentation at 2010 Society for Prevention Research workshop, June 1, 2010. Can be downloaded from http://csel.asu.edu/adaptiveintervention (select item 9).
  - Rivera, D.E., "A brief introduction to system identification," Penn State Methodology Center Brown Bag presentation, March 20, 2008. Can be downloaded from http://csel.asu.edu/controleducation (select item 10).
  - Rivera, D.E., "An introduction to mechanistic models and control theory," tutorial presentation at the SAMSI Summer 2007 Program on Challenges in Dynamic Treatment Regimes and Multistage Decision-Making, June 18 - 29, 2007. Can be downloaded from http://csel.asu.edu/controleducation (select item 9).

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- J. Younger, Systems Neuroscience and Pain Laboratory, Stanford University Medical Center

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