Robust Optimal Decision Policies for Adaptive, Time-Varying Interventions Using Model Predictive Control

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Presentation Outline

- Fundamentals of adaptive time-varying interventions
- Formulation of a hypothetical time-varying adaptive intervention (inspired by the *Fast Track* program) using "IF-THEN" rules.
- Model Predictive Control (MPC) problem formulation
- MPC application to the hypothetical intervention
  - Comparison to the "IF-THEN" rules.
  - Constrained resource with augmentation scenario.
- Summary and conclusions

Basic Components of Adaptive Interventions

(Collins, Murphy, and Bierman, *Prevention Science*, **5**, No. 3, 2004)

- The assignment of a particular dosage and/or type of treatment is based on the individual's values on variables that are expected to moderate the effect of the treatment component; these are known as *tailoring variables*.
- In a *time-varying* adaptive intervention, the tailoring variable is assessed periodically, so the intervention is adjusted on an on-going basis.
- *Decision rules* translate current and previous values of tailoring variables into choice(s) of treatment and their appropriate dosage.

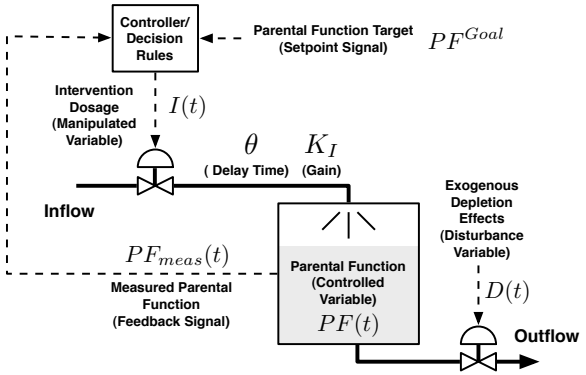
Adaptive Intervention Simulation

(inspired by the *Fast Track* Program, Conduct Problems Prevention Research Group)

- A multi-year program designed to prevent conduct disorder in at-risk children.
- Frequency of home-based counseling visits assigned quarterly to families over a three-year period, based on an assessed level of parental functioning.
- Parental function (the tailoring variable) is used to determine the frequency of home visits (the intervention dosage) according to the following decision rules:
  - If parental function is "very poor" then the intervention dosage should correspond to weekly home visits,
  - If parental function is "poor" then the intervention dosage should correspond to bi-weekly home visits,
  - If parental function is "below threshold" then the intervention dosage should correspond to monthly home visits,
  - If parental function is "at threshold" then the intervention dosage should correspond to no home visits.

Parental Function - Home Visits Adaptive Intervention as an Inventory Management Problem

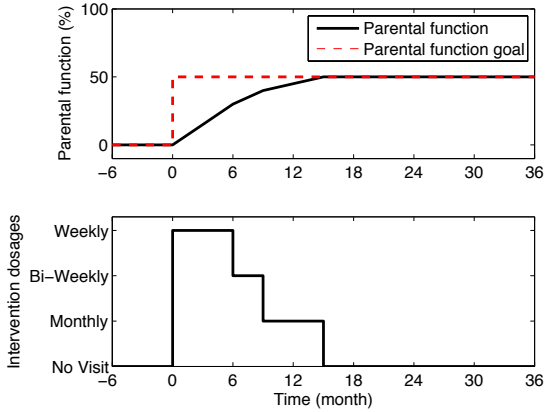
(Rivera, Pew, and Collins, "Using engineering control principles to inform the design of adaptive interventions," *Drug and Alcohol Dependence*, Vol. 88, Suppl. 2, May 2007, Pages S31-S40)



$$PF(t+1) = PF(t) + K_I\, I(t-\theta) - D(t)$$

Parental function *PF(t)* is built up by providing an intervention *I(t)* (frequency of home visits), that is potentially subject to delay, and is depleted by potentially multiple disturbances (adding up to *D(t)*).

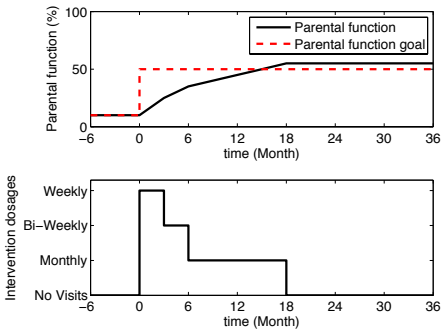
Parental Function - Counselor Home Visits Adaptive Intervention Single Participant Family Illustration



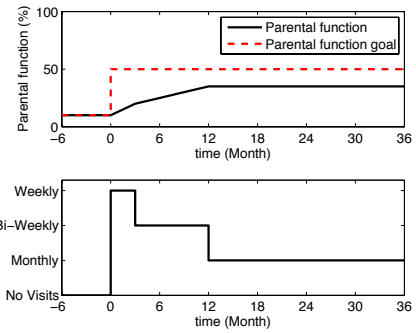
- The assigned dosage (frequency of counseling visits) decreases as the tailoring variable (parental function) increases, as prescribed by the decision rules.

"IF-THEN" Rules May Not Be Optimal

No Depletion (*D(t) = 0*)



High Depletion (*D(t) = 5*)

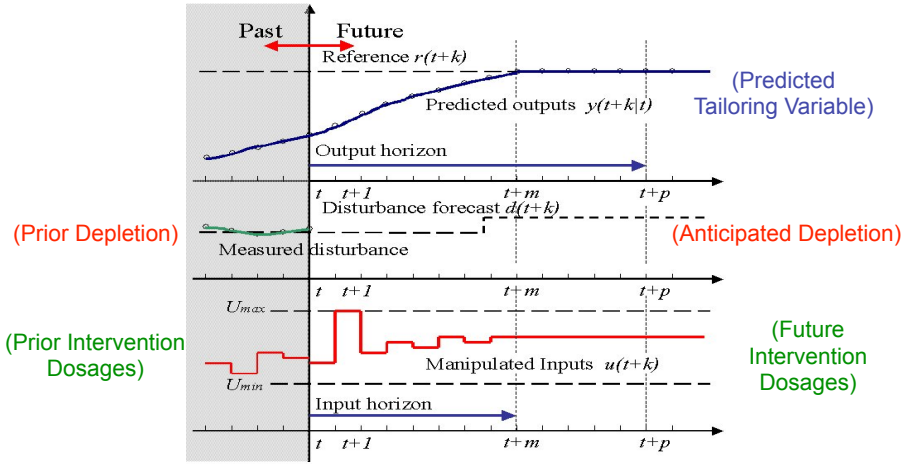


Single participant family scenario. *Offset* (where parental function fails to reach a desired goal at the end of the intervention) occurs when high depletion (representing large magnitude disturbances) is present.

Model Predictive Control (MPC)

- Control engineering technology widely used in many industrial applications (from chemical mfg to automotive and aerospace)
- As an *optimizer*, MPC can minimize or maximize an objective function that represents a suitable metric of intervention performance.
- As a *controller*, MPC can be tuned to achieve stability, robustness, and performance in the presence of model error, measurement unreliability, and disturbances that may affect the intervention.
- Three major steps in MPC:
  - *Prediction* of intervention outcomes at time instants in the future (i.e., the prediction horizon) based on a model,
  - *Optimization* of a sequence of future dosage decisions (i.e., move horizon) through minimizing an objective function,
  - Receding horizon strategy.

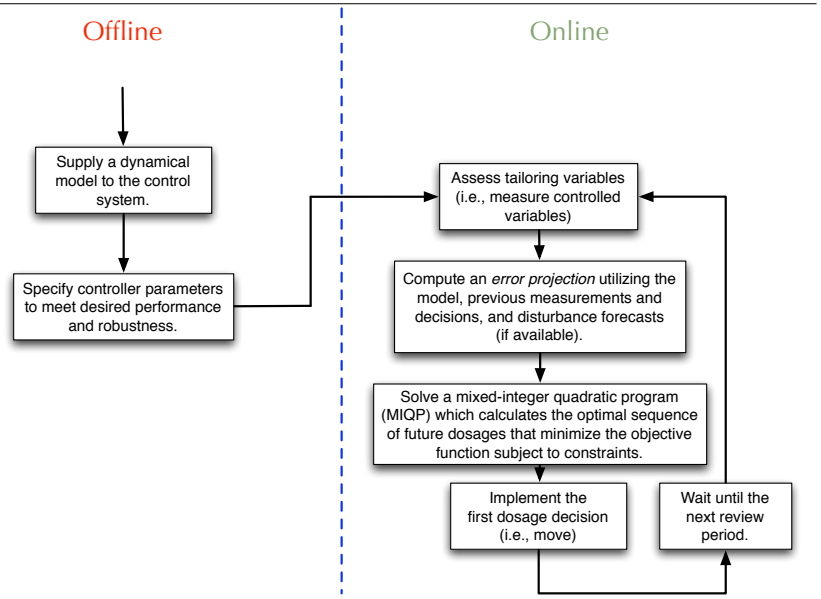
Model Predictive Control
Receding Horizon Formulation



Take Controlled Variables to Goal Penalize Changes in the Manipulated Variables

$$J = \overbrace{\sum_{\ell=1}^{p} Q_e(\ell)(\hat{y}(t+\ell|t) - r(t+\ell))^2}^{} + \overbrace{\sum_{\ell=1}^{m} Q_{\Delta u}(\ell)(\Delta u(t+\ell-1|t))^2}^{}$$

Model Predictive Control
Flowchart



Simulation Cases

- Scenario 1: Comparison with the "IF-THEN" rules.
- Scenario 2: Constrained, augmented operation.

  Total number of counseling visits constrained to an upper limit; group counseling may be offered when in-home visit limit is reached.

  – Scenario 2(a):
    » Unlimited counseling visits, no group meeting available.

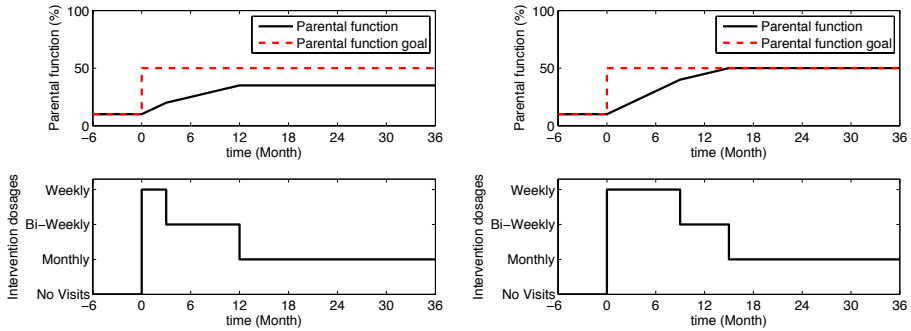
  – Scenario 2(b) :
    » Overall dosage limited to a maximum of 48 in-home counseling visits;
    » no group meeting available.

  – Scenario 2(c):
    » Overall dosage limited to a maximum of 48 in-home counseling visits;
    » Group counseling available when in-home visit limitation is reached.

Controller/Decision Rule Comparison, Scenario 1
High Depletion Rate ($D(t) = 5$)

"IF-THEN" rules

Model Predictive Control
($Q_e = 1, Q_{du} = 0.05, p = 30, m = 10$)

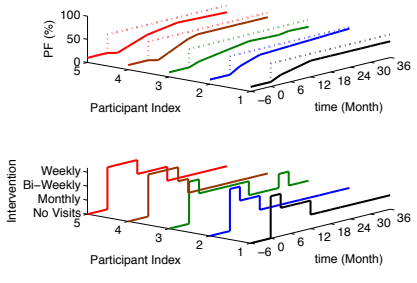


36 month intervention reviewed at quarterly intervals. Offset problem is eliminated in the MPC controller through judicious assignment of intervention dosages during the course of the intervention.

IF-THEN vs. MPC Comparison (Robustness Analysis)

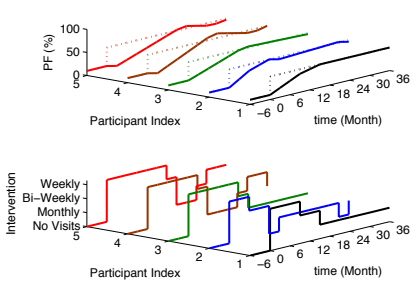
- The system response of five participant families, each characterized by its own dynamical model, is evaluated using a controller tuned on the basis of an average ("nominal") effect.

"IF-THEN" Decision Rules



MPC Control

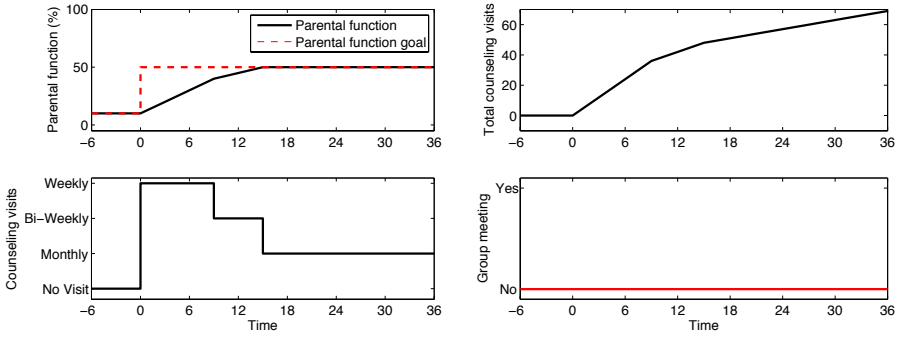
(Q_e = 1, Q_{du} = 0.05, p = 30, m = 10)



- A single MPC controller individually assigns appropriate intervention dosages to each participant family, leading to no offset and more consistent outcomes. This is achieved at the expense of greater variability in the intervention dosages.

Scenario 2(a)

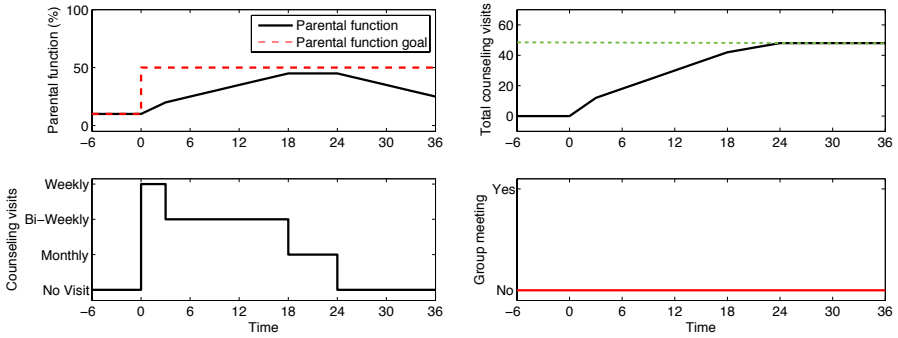
(No limit on number of counseling visits; no group meeting offered)



- 69 total counseling visits required in this unconstrained scenario to achieve the parental function goal w/o offset.

Scenario 2(b)

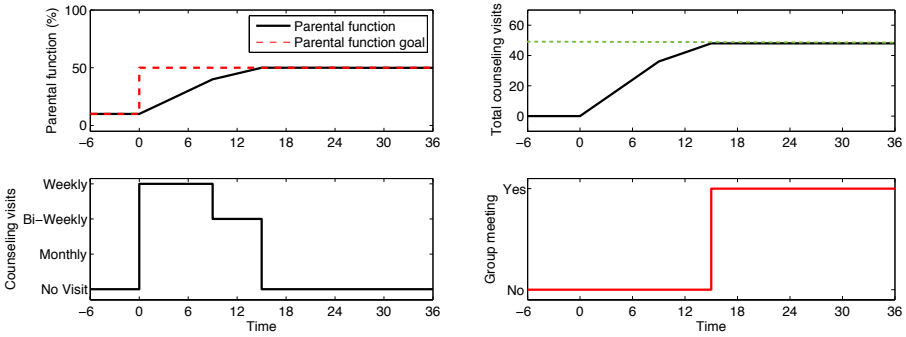
(Counseling visits limited to 48 *(total)*; no group meeting offered)



- The dosage constraint places a fundamental limit on the effect of the intervention. The controller does the best it can (for the given objective function parameters), within these restrictions.

Scenario 2(c)

(Counseling visits limited to 48 *(total)*; group meeting available)



- The MPC adapts the intervention to meet the outcome goal, properly sequencing the additional intervention component now available (once the counseling visits limit is reached).

MPC Controller/Decision Rules Summary

- PROS:
  - model-based decision-making results in improved outcomes,
  - works with either continuous or categorical measurements of tailoring variables; categorical dosage assignment is naturally part of the formulation,
  - can enforce constraints on tailoring variables, dosage assignments
  - application to multivariable problems (i.e., interventions involving multiple components and/or multiple outcomes) is straightforward.
- CONS:
  - Model parameters, adjustable parameters, need to be estimated and specified systematically,
  - Solution procedure involves using Mixed Integer Quadratic Programming (*miqp*); more difficult to explain to non-experts than both the "IF-THEN" and IMC-PID (from pre-workshop tutorial)

References

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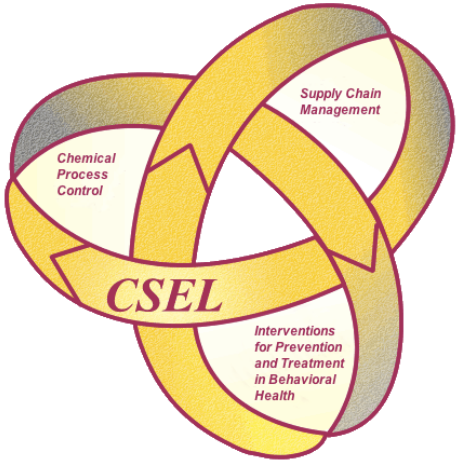
For more information...



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