A Novel Approach to Plant-Friendly Multivariable Identification of Highly Interactive Systems

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Presentation Outline

- **Multivariable System Identification using Multisine Signals**
  - Extension to highly interactive systems using modified "zippered" spectra
  - Optimization-based formulations for minimum crest factor signals, conducive to "plant-friendliness"
- **Case Study: High-Purity Distillation Column (Weischedel-McAvoy)**
  - Optimization-based design using an *a priori* ARX model
  - Closed-loop evaluation of data effectiveness with MPC
  - Extension to input signal design for nonlinear identification using NARX models
- **Latest Efforts:**
  - Input signal design for data-centric estimation (such as MoD)

System Identification Challenges Associated with Highly Interactive Processes:

- Need to capture both low and high gain directions under noisy conditions
- Plant-friendliness must be achieved during identification testing

Some Solutions to the Highly Interactive Identification Problem

- ✓ Chien and Ogunnaike (*1992 AIChE Mtg*.) and Ogunnaike *et al.* (*1993 ECC*) use "high frequency" linear models and nonlinear empirical models, respectively.
- ✓ Li and Lee (*1996 Comp. Chem. Eng*) and Varga and Jorgensen (*1994 AIChE)* examine the problem using both open and closed-loop identification tests.
- ✓ Koung and MacGregor (*1993 I&EC Res*.) use correlated input signals based on a priori knowledge of high/low gain directions.
- ✓ Stec and Zhu (*2001 ACC*) and Butoyi and Zhu (*2002 CEP*) apply a sequential combination of correlated and uncorrelated signals of varying magnitudes to enhance the low gain information in the data.

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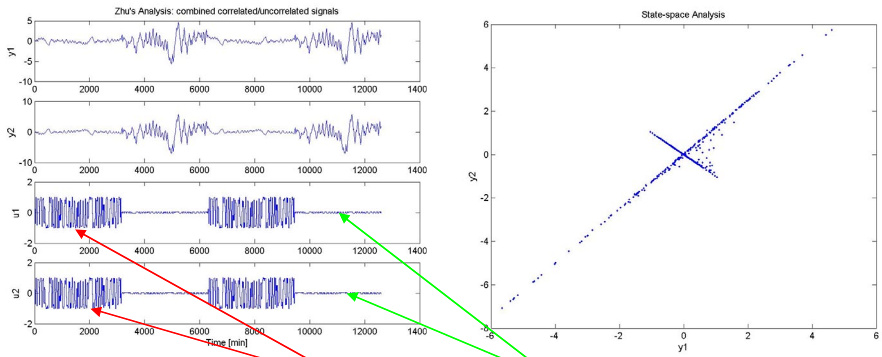
Identifying Highly Interactive Systems –

(Stec and Zhu, *2001 ACC)*



The sequential cycles of **correlated** and **uncorrelated** signals provide a mechanism for generating a data set with good information content in both high and low gain directions (e.g., tested with a simple model)

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Multisine Input Signals

A multisine input is a deterministic, periodic signal composed of a harmonically related sum of sinusoids,

$$u_j(k) = \sum_{i=1}^{m\delta} \hat{\delta}_{ji} \cos(\omega_i kT + \phi_{ji}^{\delta}) + \sum_{i=m\delta+1}^{m(\delta+n_s)} \alpha_{ji} \cos(\omega_i kT + \phi_{ji})$$

$$+ \sum_{i=m(\delta+n_s)+1}^{m(\delta+n_s+n_a)} \hat{a}_{ji} \cos(\omega_i kT + \phi_{ji}^{a}), \quad j = 1, \cdots, m$$

where T is sampling time, N_s is the sequence length, m is the number of channels, δ, n_s, n_a are the numbers of sinusoids per channel ($m(\delta+n_s+n_a) = N_s/2$), $\phi_{ji}^{\delta}, \phi_{ji}, \phi_{ji}^{a}$ are the phase angles, α_{ji} represents the Fourier coefficients defined by the user, $\hat{\delta}_{ji}, \hat{a}_{ji}$ are the "snow effect" Fourier coefficients

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Multisine Signal Design Guideline

(H.Lee, D.Rivera, H. Mittelmann, SYSID 2003)

For signal bandwidth denoted by (ω_*, ω^*), n_s, N_s, and T must satisfy the inequalities:

$$(1+\delta)\,\frac{\omega^*}{\omega_*} \le n_s \le N_s/2m$$

$$T \le \min\{\frac{\pi}{\omega^*}, \frac{\pi}{\omega^* - \omega_*}(1 - \frac{1+\delta}{n_s})\}$$

$$\max\{2mn_s, \frac{2\pi m(1+\delta)}{\omega_* T}\} \le N_s \le \frac{2\pi m n_s}{\omega^* T}$$

Finally, design values (denoted by the superscript "d") should satisfy:

$$\frac{(\omega^* - \omega_*)}{2\pi m} N_s^d T^d + (1+\delta) \le n_s^d \le \frac{N_s^d}{2m}$$

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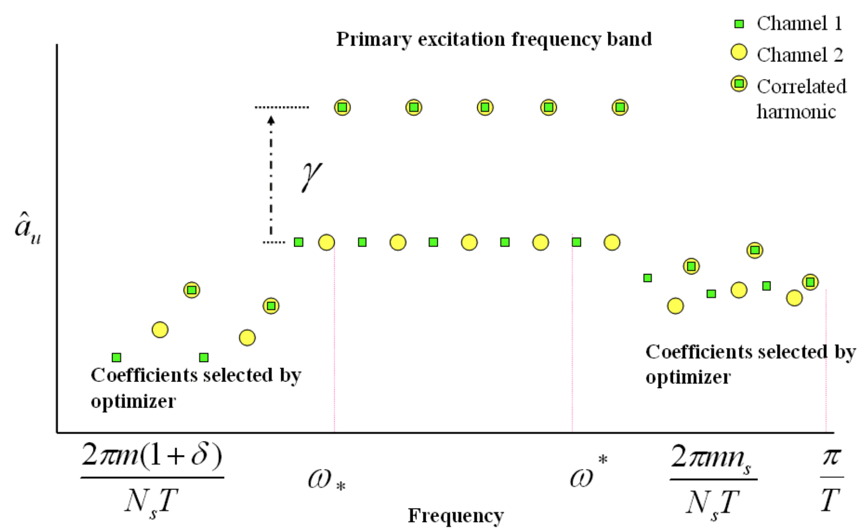
Modified Zippered Spectrum



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Modified Zippered Spectrum Design

Utilize the steady-state gain matrix from *a priori* model:

$$K = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}$$

$$\frac{\min(k_{11}^2, k_{12}^2, k_{21}^2, k_{22}^2)}{\max\{(k_{11}+k_{12})^2,\ (k_{21}+k_{22})^2\}} \leq \gamma^2 \leq \frac{\max(k_{11}^2, k_{12}^2, k_{21}^2, k_{22}^2)}{\min\{(k_{11}+k_{12})^2,\ (k_{21}+k_{22})^2\}}$$

For Weischedel-McAvoy Case Study: $\{\gamma\} = \{10.32 \ \leq \gamma_i \leq \ 15.67\}$

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Plant-Friendly Identification Testing

- A plant-friendly input signal should:
  - be as short as possible
  - not take actuators to limits, or exceed move size restrictions
  - cause minimum disruption to the controlled variables (i.e., low variance, small deviations from setpoints)

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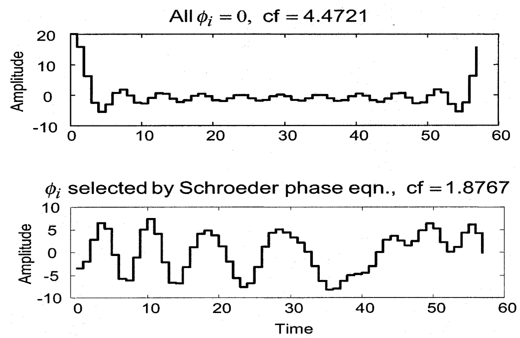
The Crest Factor (CF) is defined as the ratio of ℓ_∞ (or Chebyshev) norm and the ℓ_2 norm

$$CF(x) = \frac{\ell_\infty(x)}{\ell_2(x)}$$

A low crest factor indicates that most elements in the input sequence are located near the min. and max. values of the sequence.



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Problem Statement #1

$$\min_{\{\phi^a_{ji}\},\{\phi^\delta_{ji}\},\{\phi_{ji}\},\{\hat{a}_{ji}\},\{\hat{\delta}_{ji}\}} \max_j \text{CF}(u_j) \quad j = 1,\cdots,m$$

subject to maximum move size constraints on $\{u_j(k)\}$

$$|\Delta u_j(k)| \leq \Delta u_j^{max} \quad \forall\, k, j$$

and high/low limits on $\{u_j(k)\}$

$$u_j^{min} \leq u_j(k) \leq u_j^{max} \quad \forall\, k, j$$

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Problem Statement #2

$$\min_{\{\phi^a_{ji}\},\{\phi^b_{ji}\},\{\phi_{ji}\},\{\hat{a}_{ji}\},\{\hat{b}_{ji}\}} \max_{z} \mathrm{CF}(y_z)$$

$$j = 1, \cdots, m \qquad z = 1, \cdots, N_{outs}$$

subject to constraints in input

$$|\Delta u_j(k)| \leq \Delta u_j^{max} \quad \forall\, k, j$$

$$u_j^{min} \leq u_j(k) \leq u_j^{max} \quad \forall\, k, j$$

and output

$$|\Delta y_z(k)| \leq \Delta y_z^{max} \quad \forall\, k, z$$

$$y_z^{min} \leq y_z(k) \leq y_z^{max} \quad \forall\, k, z$$

This problem statement requires an *a priori* model to generate output predictions

Constrained Solution Approach

Some aspects of our numerical solution approach:

- ✓ The problem is formulated in the modeling language AMPL, which provides exact, automatic differentiation up to second derivatives.
- ✓ A direct min-max solution is used where the nonsmoothness in the problem is transferred to the constraints.
- ✓ The trust region, interior point method developed by Nocedal and co-workers (Byrd, R., M.E. Hribar, and J. Nocedal. "An interior point method for large scale nonlinear programming." *SIAM J. Optim., 1999*) is applied.

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Case Study: High-Purity Distillation

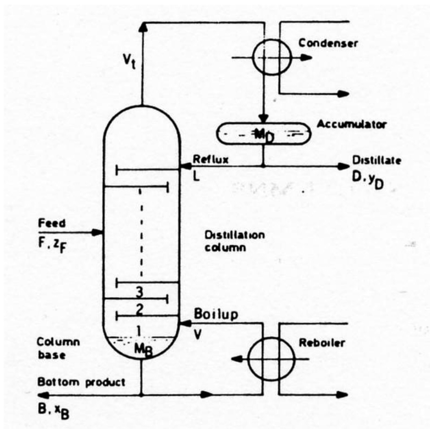
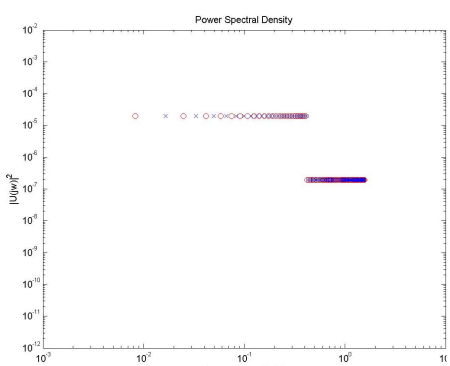


Fig. 2. Two-product distillation column.

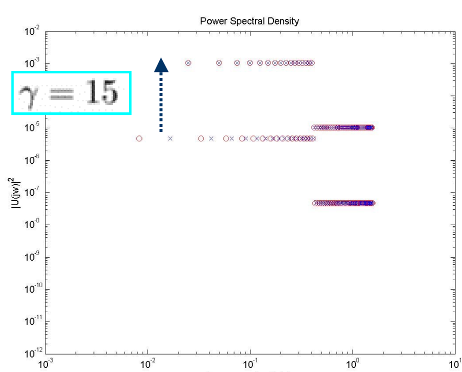
High-Purity Distillation Column per Weischedel and McAvoy (1980) : a classical example of a highly interactive process system, and a challenging problem for control system design

Standard & Modified Zippered Spectrum Design

Standard Zippered Spectrum



Modified Zippered Spectrum

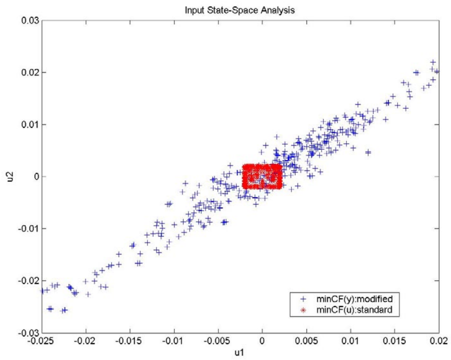


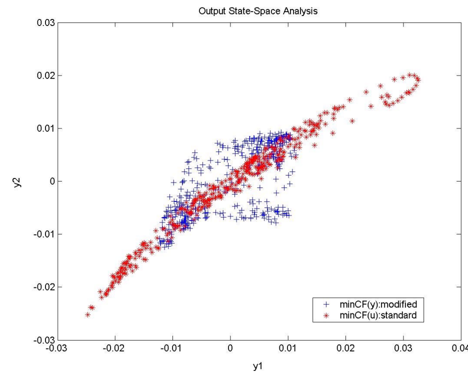
For $\tau^L_{dom} = 5, \tau^H_{dom} = 20$ min, $\delta = 0$, $\alpha_s = 2$, and $\beta_s = 3$, feasible design choices are $T = 2$ min, $n_s = 25$, $N_s = 378$, and $\gamma = 15$.

State-space Analysis

Input State-Space

Output State-Space





+(blue): min CF(y) signal with a modified zippered spectrum and *a priori* ARX model

*(red) : min CF(u) signal with a standard zippered spectrum

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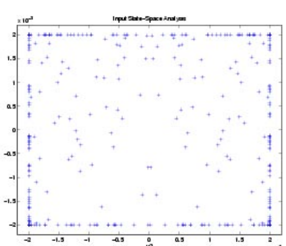
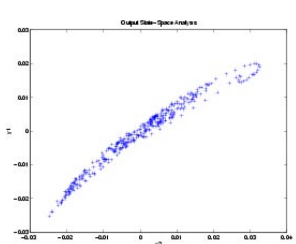
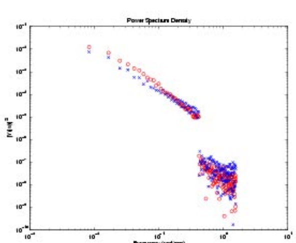
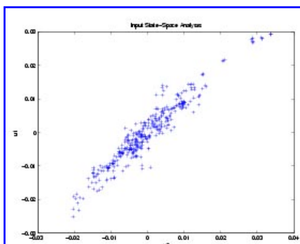
Standard & Modified Zippered Spectrum Design

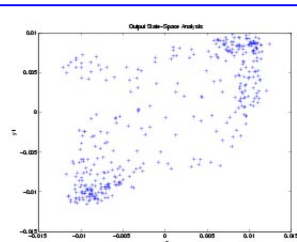


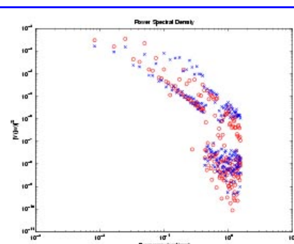




Input State-Space **Output State-Space** **Output Power Density**

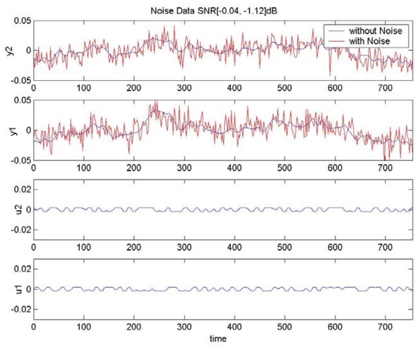


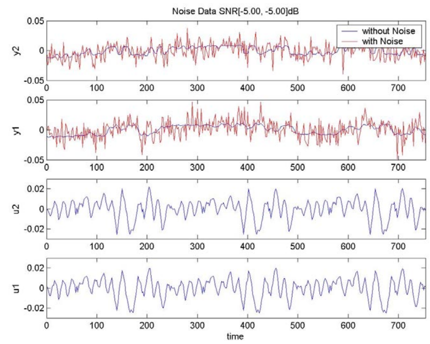




min CF signal design: time-domain

min CF(u) signal with Standard Zippered Spectrum	min CF(y) signal with ARX model and Modified Zippered Spectrum





Noise SNR [-0.04, -1.12]dB

Noise SNR [-5.0, -5.0]dB

min CF Signal Design :Test signals statistics

Type	Signal (x)	CF(x)	PIPS(%)	max Δx	max x	min x
min CF (u) design; standard zippered spectrum	u_1	1.227516	81.465337	0.002737	0.002000	-0.002000
	u_2	1.227516	81.465337	0.002227	0.002000	-0.002000
	y_1	2.524688	44.969514	0.003417	0.032565	-0.024796
	y_2	2.531872	43.839119	0.003436	0.020027	-0.025283
min CF (y) design; modified zippered spectrum by ARX model $\lvert y, \Delta y\rvert \leq 0.0075$ and $\lvert\Delta x\rvert \leq 0.01$	u_1	2.590540	43.027738	0.009997	0.019783	-0.024907
	u_2	2.683625	40.314130	0.009999	0.021897	-0.025803
	y_1	1.687338	61.615468	0.004484	0.011356	-0.012298
	y_2	1.945124	58.748109	0.007131	0.009289	-0.012429
min CF (y) design; modified zippered spectrum by NARX model $\lvert y, \Delta y\rvert \leq 0.0075$ and $\lvert\Delta x\rvert \leq 0.01$	u_1	2.676489	37.607322	0.009999	0.025399	-0.025734
	u_2	2.852480	35.288221	0.010000	0.027069	-0.027428
	y_1	1.348449	74.850385	0.005174	0.008878	-0.008709
	y_2	1.341205	75.176406	0.007500	0.008769	-0.008606
min CF(y) design; modified zippered spectrum with even harmonic suppression by ARX model $\lvert y\rvert \leq 0.007$, $\lvert\Delta y\rvert \leq 0.0075$ and $\lvert\Delta x\rvert \leq 0.01$	u_1	2.902927	34.447985	0.009600	0.019552	-0.019552
	u_2	2.557524	39.100325	0.010000	0.017227	-0.017227
	y_1	1.607000	64.257126	0.003982	0.009401	-0.008804
	y_2	1.673482	62.556392	0.005353	0.007687	-0.008455

Closed-loop Performance Comparison using MPC Setpoint Tracking: models obtained from noise-free data

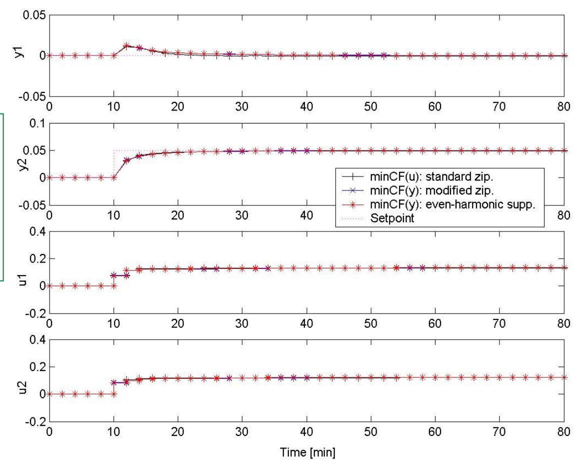
MPC Tuning Parameters:

Prediction Horizon PHOR : 100

Move Horizon : 25

Output Weighting: [1 1]

Input Weighting : [0.2 0.2]



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Closed-loop Performance Comparison using MPC Setpoint Tracking: models obtained from noisy data conditions

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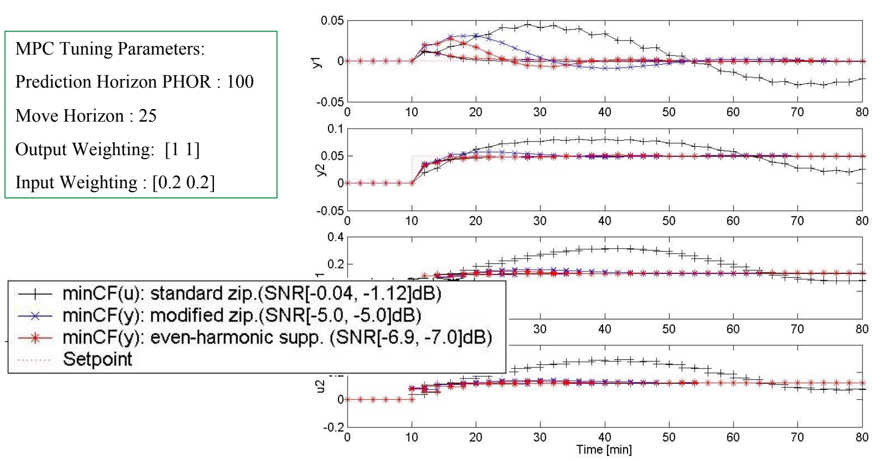

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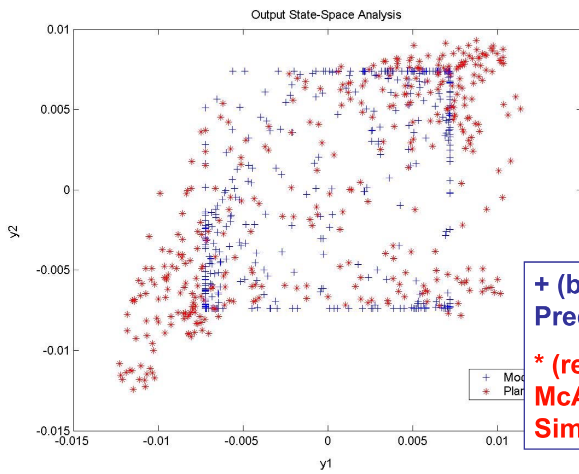
ARX Model Prediction vs. Plant Data



+ (blue) : Model Prediction

*** (red) : Weischedel-McAvoy Distillation Simulation**

NARX Model Estimation

Rely on a NARX model equation to predict the system outputs during optimization:

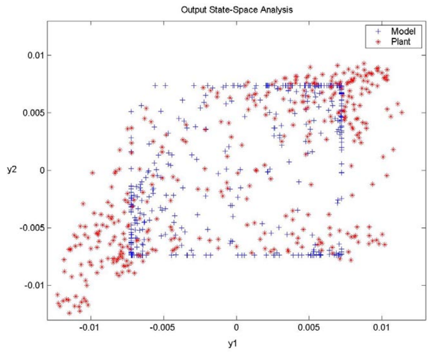
$$\begin{aligned} y(k) \;&=\; \theta^{(0)} + \sum_{i=1}^{n_y} \theta_i^{(1)} y(k-i) + \sum_{i=\rho}^{n_u} \theta_i^{(2)} u(k-i) + \sum_{i=1}^{n_y} \sum_{j=1}^{i} \theta_{(i,j)}^{(3)} y(k-i) y(k-j) \\ &+ \sum_{i=\rho}^{n_u} \sum_{j=\rho}^{i} \theta_{(i,j)}^{(4)} u(k-i) u(k-j) + \sum_{i=1}^{n_y} \sum_{j=\rho}^{n_u} \theta_{(i,j)}^{(5)} y(k-i) u(k-j) + \ldots \end{aligned}$$

Evaluation criterion (Sriniwas *et al.*, 1995):

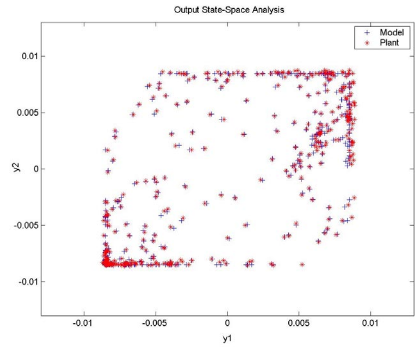
$$I \;=\; \frac{\sum_{k=1}^{N} [y(k) - \hat{y}(k)]^2}{\sum_{k=1}^{N} [y(k) - \bar{y}(k)]^2} \times 100\%$$

ARX vs. NARX Model Predictions

ARX Model



NARX Model



+ (blue) : Model Prediction

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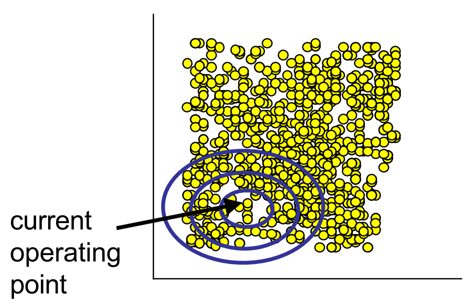
Model-on-Demand Estimation

(Stenman, 1999)

- A modern data-centric approach developed at Linkoping University
- Identification signals geared for MoD estimation should consider the geometrical distribution of data over the state-space.



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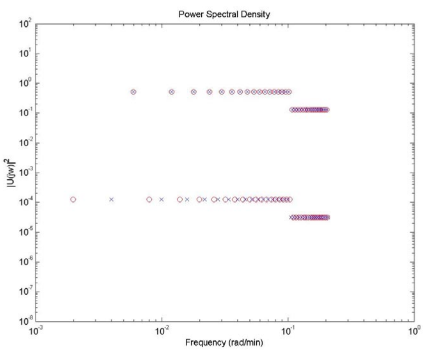
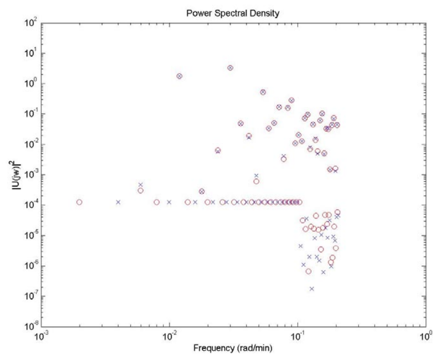
Data-Centric Output Distribution Approach

Modified Zippered, min CF (y) Signal



Modified Zippered, Data Centric Signal



Optimize the magnitudes of the correlated harmonics to obtain evenly distributed outputs signals in the state-space

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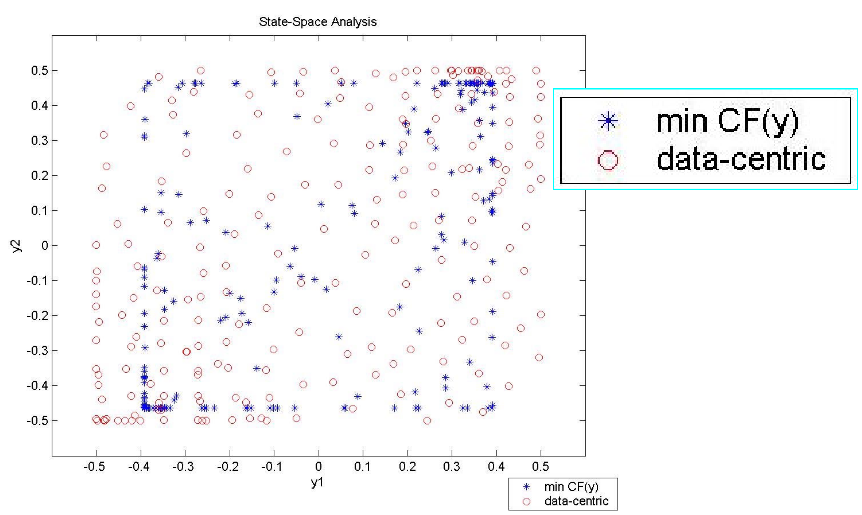
Data-Centric Output Distribution Approach



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Summary and Conclusions

- A comprehensive multisine signal design applicable to the identification of highly interactive systems has been presented
- A modified zippered spectrum design is proposed for highly interactive systems, which combined with constrained optimization leading to informative data under "plant-friendly" operation.
- Models estimated from modified zippered signals are more effective under noisy conditions compared to standard designs.
- NARX model estimation leads to less distortion in the model output predictions for the nonlinear Weischedel-McAvoy column
- The effective use of *a priori* knowledge is critical in the solution of this (or any other) control-relevant, plant-friendly identification problem.

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