

The Human Factor: Accounting For “Off Book” Flight Speeds Within Implied Safety Margins

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To ensure that flight safety is not compromised by engine failure, the FAA promulgates many Federal Regulations to govern aircraft dispatch. Two-engine commercial aircraft, for example, must attain at least 2.4% second segment climb gradient at V_2 with one engine inoperative. These Weight-Altitude-Temperature (WAT) limits ensure that aircraft always dispatch where they have some climb reserve. When planning for obstacle clearance, the FAA requires dispatch to employ data containing a 0.8% “margin-of-safety” on all published climb performance. While these measures ensure that aircraft will typically fly better than predicted—giving an allotment to either pilot error or engine failure—are they necessarily enough to account for multiple problem sources? This manuscript was inspired from an observation of two commercially certified pilots flying in a CRJ-200 high fidelity simulator; in a flight with an engine failure on the runway, pilots flew the airplane in a manner that found them trapped in a poorly performing second segment climb flown well below the V_2 speed. This has serious real world repercussions in the event that the flight path contains tall obstacles or terrain. We demonstrate the consequences of flying below the suggested cue speed, using a point-mass-simulation code. We found that the 0.8% safety margin could be completely consumed if pilots fly second segment climb at $V_2 - 15$. This implies that pilots can easily find themselves in circumstances that leave no further room for error. Future risk-based dispatch systems need to consider the possibilities, and consequences, of flight at “off book” speeds, especially with respect to deriving obstacle clearance limited take off weights.

Nomenclature

a	=	speed of sound (nM/hr)
AGL	=	altitude above ground level (ft)
ALT	=	altitude (above sea level) (ft)
AR	=	wing aspect ratio
CFR	=	Code of Federal Regulations
C_D	=	coefficient of drag
C_{D_i}	=	coefficient of induced drag

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C_{D_0}	=	coefficient of zero-lift drag
C_L	=	coefficient of lift
d_{fan}	=	engine fan diameter (ft)
$\Delta C_{D_{RE}}$	=	coefficient of drag increment due to Reynolds number (altitude & Mach) effects
$\Delta C_{D_{ENG}}$	=	coefficient of drag increment due to jammed engine
$\Delta C_{D_{FLAPS}}$	=	coefficient of drag increment due to flap deflection
$\Delta C_{D_{TRIM}}$	=	coefficient of drag increment due to trim
e	=	Oswald's efficiency factor
n_{eng}	=	number of engines
q	=	dynamic pressure (lbf/ft ²)
ROC	=	rate of climb (ft/min)
SET	=	specific excess thrust (climb gradient)
S_{ref}	=	wing planform reference area
T	=	thrust (lbf)
V_{FTO}	=	final takeoff speed (nM/hr)
V_{LOF}	=	lift-off speed (nM/hr)
V_{MC}	=	minimum control speed (nM/hr)
V_{SR}	=	reference stall speed (nM/hr)
V_2	=	second segment speed (nM/hr)
V_4	=	fourth segment speed (nM/hr)
W	=	aircraft weight (lbm)
WAT	=	Weight, Altitude, Temperature

I. Introduction

PILOTS do not intend to crash their aircraft; yet “pilot error” is the most common explanation behind aircraft mishaps that lead to loss of property or life. The Federal Aviation Administration (FAA) regulates the design and operation of all aircraft flown commercially within the United States. All these regulations set forth by the FAA are in the interest of safety for all stakeholders: the general public, passengers, pilots, operators and manufacturers. Aircraft designers must provide evidence that their design can both withstand expected flight loads and provide a minimum amount of expected performance for various flight scenarios. Commercial pilots must be rigorously trained before they can obtain a Federal license to fly. Once such a license has been obtained, pilots must continue training to be familiar with the intricacies of the aircraft that they fly during normal as well as emergency situations.

Safe operation requires engineers and pilots to comply with many different criteria, including the incorporation of factors of safety and required redundancy. Commercial aircraft designers must engineer within the bounds called for in Title 14 of the Code of Federal Regulations.¹ Pilots must abide by strict operational limits; engineers include “factors of safety” when preparing the certified flight manual upon which pilots depend. To ensure aircraft are operated in a safe manner, the FAA requires pilots to use an “FAA Approved” evidence-based Aircraft Flight Manual (AFM) developed by the manufacturer.

The AFM ensures safe operation when pilots fly according to published procedure. For example, to avoid accidental stall during aircraft operation, regulation 14 CFR § 25.107(g)² requires engineers to select a final takeoff speed, V_{FTO} , so that it produces at least the gradient of climb prescribed by 14 CFR § 25.121(c)³ (for two-engine commercial aircraft, the steady gross climb gradient must be no less than 1.2% at V_{FTO} with a critical engine inoperative). At the same time, V_{FTO} cannot be less than 1.18 times the reference stall speed, V_{SR} , which provides a margin of safety against accidental stall. Through regulations such as these, the FAA expects that multi-engine aircraft are easily controllable by trained pilots even in emergency situations. Pilots may safely climb, turn, descend or fly level even with a critical engine inoperative.

Pilots and dispatch operators use the AFM for mission planning purposes. Not only does it provide important cue speeds, the “V speeds” (such as V_I , V_R , V_2 and V_{FTO}), for safe operation, but it provides maximum dispatch weight guidance charts—“WAT limit” charts. These charts, which are functions of airfield altitude and temperature, ensure that the pilots can achieve minimum-required climb gradient performance in the critical engine-inoperative condition, so long as the aircraft is flown to the prescribed cue speeds.

But what happens when “pilot error” results in the aircraft departing from the prescribed speed? Are foreseeable deviations from the “book” cue-speeds properly accounted for by the “factors of safety” embedded in Federal Regulation? This paper, a companion to others prepared by our research group,^{4,5,6,7,8,9} outlines the impacts of a particular scenario witnessed during an observed flight; we explore the impacts of the observed “pilot error” in detail. Future papers will address aircraft performance shortfalls resulting from “pilot error” in a broader, stochastic framework.

II. Observed Scenario

The research team has embarked on an observational experiment utilizing a Canadair Regional Jet CRJ-200 level six simulator located at the Arizona State University Polytechnic campus. Aircrews ranging in experience from high time to student pilots are observed operating the simulator as a 14 CFR § 121 carrier, following all of the required operational procedures therein. These observations are in an effort to understand the typical pilot timings used during takeoff and landing flight regimes, with examinations in both normal and emergency operations. A more detailed description of the experimental plan can be found in Wood, Takahashi and Bays’ conference paper covering this topic.⁷

Piloting techniques and timings used during near field operations can have profound effects on runway usage. Dispatch utilizes a prediction tool that is often proprietary, but based on the values presented in the flight manuals in order to calculate critical field length requirements. This determines the V-speeds the aircrew will fly to, as well as identifying if the runway is long enough to accommodate accel-go and accel-stop distances. We aim to determine if these field length calculations are accounting for variability experienced from human factors. These variables can be introduced where the pilot is commanding the aircraft in a transient phase of flight such as rotation rates on takeoff, and pitching the aircraft to achieve the appropriate climb out speed. Lack of accounting for this variability leads to large deviations in the runway lengths required for operations that may not be accounted for, which invites disastrous outcomes.

We selected a flight scenario designed to maximize pilot anxiety during a continued take-off with engine failure after V_1 . We chose operations out of a long runway at high gross weights but with a low scheduled V_1 speed, just above VMCG. This would ensure that the “wheels up” distance would be near the end of the runway.

For pilot observation in the Arizona State University Canadair Regional Jet CRJ-200 simulator, we followed the Canadair CRJ Airplane Flight Manual (CSP A-012-013) to determine the following scenario.¹⁰

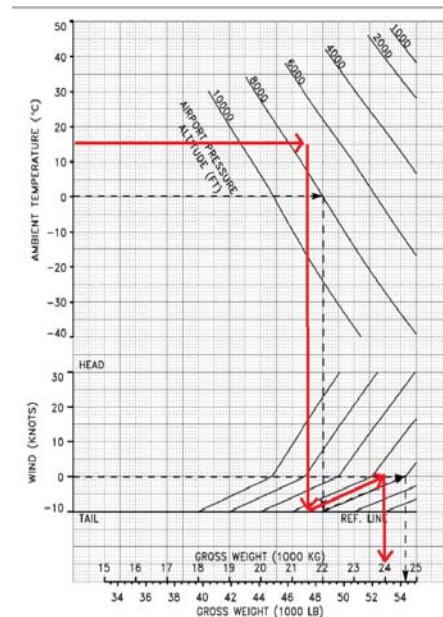


Figure 1. CRJ-200 DISPATCH. WAT LIMIT CHART (worked example from CRJ200 AFM p. 06-03-02).

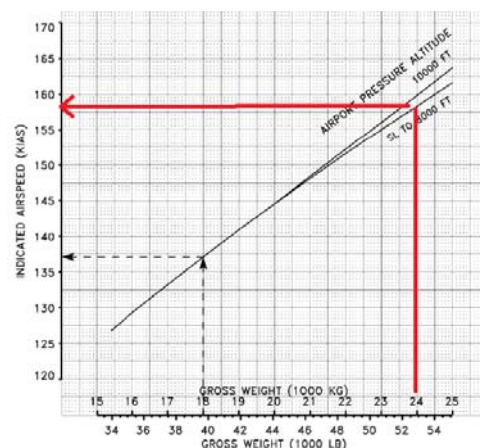


Figure 2. CRJ-200 DISPATCH. V2 CHART (worked example from CRJ200 AFM p. 06-04-13).

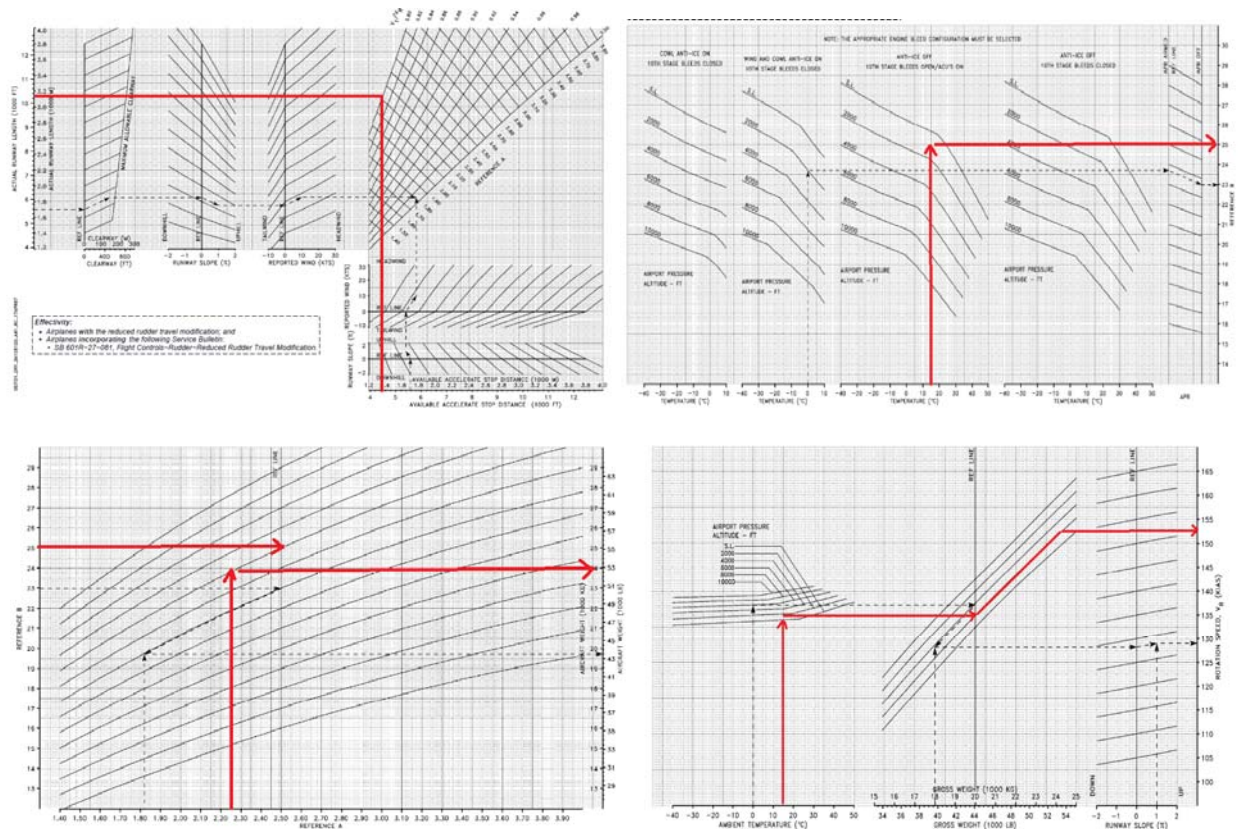


Figure 3. CRJ-200 DISPATCH. Runway Utilization Charts (worked example from CRJ200 AFM p. 06-03-05,06 and 07).

To begin, our pilots flew out of a familiar airport (KPHX). In Figure 1, the reader can see that dispatch at $W=53,000$ -lbm is well within the WAT limit chart for this airport ($PA=1130$ -ft, $OAT=13^{\circ}C$). Indeed, the WAT limit for 2nd and 4th segment climb is achieved at ~ 7000 -ft pressure altitude. Thus, dispatch at $PA\sim 1130$ -ft and $OAT=13^{\circ}C$ guarantees that 2nd and 4th segment climb performance exceed FAA minimums. Moving on to Figure 2, we chase through the obstacle clearance speed chart to determine a scheduled $V_2\sim 157$ KIAS at $W=53,000$ -lbm. Turning to Figure 3, we may determine that the aircraft may safely dispatch at weights up to 53,000-lbm (Reference A ~ 2.25 ; Reference B ~ 25) with a low decision speed ($V_I/V_R \ll 1$) and a scheduled rotation speed of $V_R\sim 152$ KIAS.

The scenario has the pilots experience an engine failure during ground roll, followed by an extended single-engine ground acceleration phase, rotation and initial climb-out at an obstacle clearance speed V_2 approximately 5-KIAS faster than rotation.

Following Figure 4 (overleaf), we see that the manual predicts the second segment “net” climb gradient, flown at V_2 , to be $\sim 3.8\%$. Regulation 14 CFR § 25.115(b) dictates that “the net takeoff path flight data must be determined so that they represent the actual [gross] takeoff flight paths ... reduced at each point by a gradient of climb equal to – (1) 0.8 percent for two-engine airplanes”.¹¹ In other words, the manual infers that the CRJ200 should actually climb at $\sim 4.6\%$ gradient. But for obstacle clearance planning purposes, dispatch includes a prescribed “margin of safety.” Thus, when flown at a scheduled $V_2=157$ -KIAS, the flight manual predicts an initial climb rate of ~ 600 -ft/min (“net”) or ~ 725 -ft/min (“gross”).

We believe that this scenario is realistic. Because operators routinely dispatch under a “FLEX TEMP” reduced thrust operating standard, aircraft often operate from runways close to their WAT limited weight and with a scheduled critical field length nearing the actual available runway length. Pilots train to experience the unexpected (but foreseeable) problem, such as a compressor-stall or major bird strike that leaves one engine inoperative at an inopportune moment.

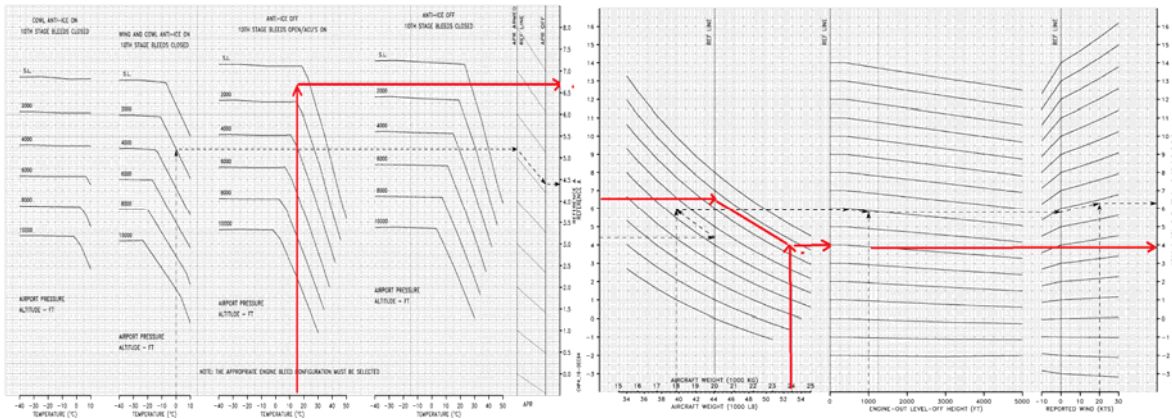


Figure 4. CRJ-200 DISPATCH. *Obstacle Clearance Charts (worked example from CRJ200 AFM p. 06-04-19 and 20).*

We will see that on one occasion, on a simulated flight with experienced pilots at the helm, things did not go according to plan.

The CRJ-200 simulator was configured for a max gross takeoff (53,000lbs) with all of the fuel tanks full and a significant payload. Center of Gravity calculations determined the aircraft would be nose heavy, but just within the forward limit. The aircraft was lined up for takeoff on runway 7L at Sky Harbor International airport (KPHX) with a TORA/TODA/ASDA distance of 10,300-ft. Weather at the field presented with calm winds and a temperature of 13° C with the altimeter reading 29.92-in Hg. We determined the minimum control speed on the ground for the CRJ-200 aircraft was approximately 101 KIAS, so the engine failure was set at this airspeed in the form of a throttle cut.

The pilots are both certified CRJ-200 aircrew with the Pilot Flying (PF) an experienced, high time aviator. The Pilot Observing (PO) is a recent graduate of the CRJ-200 training program at Arizona State University. Both Pilots were briefed that the aircraft would be at max gross and to expect an engine failure significantly below V_R . The PO would retard the throttle of their choosing to idle the moment V_I was reached. We have the capability to fail an engine on the simulator control interface; however, we were interested in the aircrew's reaction to a known failed engine without setting off caution and warning horns. This scenario would not accurately duplicate a failed engine, as the idle engine will still be producing some residual thrust. The pilots completed their takeoff checklist and the aircraft was configured for normal takeoff with a flap setting of 8° selected. The aircraft would not be performing a "FLEX" thrust takeoff, so full thrust would be selected.

Takeoff was initiated and the throttle levers were moved to full thrust. After 28 seconds and traveling 2200-ft, the aircraft reached $V_I = 101$ KIAS and the right engine was commanded to idle. At 49 seconds after takeoff initiation and consuming 7,350-ft of runway, the aircraft reached $V_R = 150$ KIAS and the nose was pitched up. Four seconds later, the aircraft achieved flight $V_{LOF} = 154$ KIAS) as the main wheels left the runway, resulting in a Takeoff Ground Run (TOGR) of 8,400-ft. The astute reader will notice that the aircraft consumed approximately 1,000-ft of runway after rotation initiation before the main wheels left the ground. This exposes the purpose of our investigation to determine if the dispatch distances align with the actual distance used. Even though this is an exceptional case as to daily operation, it is not outside the scope of possible events encountered in reality. The aircraft reached 35-ft above the runway at 57 seconds after takeoff initiation and traveled another 1,000-ft after liftoff resulting in a Takeoff Distance of 9,400-ft; nearly the entire usable runway available. Runway heading was maintained through the entire climb out portion of the flight.

After reaching V_{LOF} and becoming airborne, the airspeed momentarily stabilized at 154 KIAS, approximately 4 knots below V_2 . Within 12 seconds of rotation initiation, the aircraft was pitched through 15° nose attitude as the pilot followed the flight director; this appears to be a standard training practice for the CRJ200. This resulted in a vertical speed of over 2,000-ft/min, which the now single engine CRJ-200 could not sustain. At this point, 59 seconds after takeoff initiation, landing gear retraction was initiated and the airspeed began to sag. The PF noticed the slowing airspeed and responded by lowering the nose to a pitch angle of 9°. At 75 seconds after takeoff

initiation, the aircraft stabilized at 380-ft AGL and 144 KIAS, more than 10 knots below V_2 . After the PF pitched over, the vertical speed was reduced to 200-ft/min and the aircraft began to accelerate. V_2 was finally achieved 94 seconds after takeoff initiation, and about 40 seconds in the air, at an altitude of 460ft AGL, now several miles off the end of the runway. At this point, the aircraft was stabilized and began to climb away.

This high fidelity flight simulator allows us to evaluate the many ways human factors can affect the performance on aircraft without endangering lives or equipment. By subjecting pilots to these emergency scenarios, we can see how the reactions cause the aircraft to respond negatively when flown off design. In this case the pilot over rotated the aircraft after achieving flight, which resulted in the airspeed dropping well below the best climb speed. The aircraft was unable to sustain an appropriate climb gradient and languished along at approximately 400-ft for several miles as the aircrew attempted to recover. At the given airport, there were no real obstacles that would have threatened the aircraft, however, if a similar situation arose at an airfield with terrain or structures, the situation would have become even more dire. If the reader recalls in this scenario, the aircraft did in fact have all the engines operating, with the “failed” engine still generating idle thrust. Given a more realistic simulation with a truly failed engine and the in-cockpit warning light and horns that follow, the outcome may not have been as positive.

Aircraft manufacturers publish flight manuals in an effort to inform operators of the performance parameters that aircraft can achieve in normal and emergency situations. This information is used to dispatch the aircraft to appropriate airfields and weights to ensure a safe takeoff and landing. When pilots depart from the manufacturer recommended airspeeds, the aircraft will react negatively, potentially to the point where recovery becomes impossible. Unfortunately, many aircraft manufacturers only publish performance figures for when an aircraft is operating at the design speed. There need not be any accounting for any off-design performance and how the aircraft will react to being flown “off speed.” Some manufacturers, like Boeing, include limited information for climb at an elevated V speed (for example climb out at $V_2+10\text{KIAS}$). In response to this, we decided to evaluate aircraft performance off design utilizing a numerical code simulation.

III. Overview of General Simulation

This study uses the same collection of aerodynamic prediction and performance tools used in our previous manuscripts.^{4,5,6,7,8,9} For the sake of understanding, we will repeat the equations mentioned there, but in a briefer form. The performance equations herein come from Takahashi’s text, *Aircraft Performance and Sizing*.¹²

For this study, we consider the effects of various aerodynamic, propulsive and mass-properties uncertainties upon the takeoff distances of a notional narrow-body twin-engine commercial airliner, reminiscent of an Airbus A320. While our team does not have access to Airbus proprietary data, we have reverse engineered our basis data to match published performance data for standard day conditions.

To get a good grip on the A320’s calibration, four tools were necessary: (A) *EDET*, (B) *NPSS*, (C) *SKYMAPS*, and (D) *MISSION*, all of which are discussed briefly below.

A. Enhanced Drag Estimation Technique (EDET) & Further Aerodynamic Modelling

A drag estimation technique developed by Feagin and Morrison, the “Delta Method ... [estimates] the clean wing drag polar for cruise and maneuver conditions up to buffet onset.”¹³ This method was codified into a program—later enhanced by Takahashi using more complex form factor equations for drag estimation found in Takahashi, German, et al.—which accepts an input file which details the physical parameters of the aircraft and outputs estimated drag polars for a variety of operating conditions.¹⁴ As all real aircraft have “imperfections” such as fasteners, ridges, and flap tracks which are collectively difficult to account for, EDET includes a “crud drag” correction, which adds drag on top of the nominal count to obtain a better match to reality. For the curious reader, our A320 model incorporates a 35% crud drag correction.

An aircraft flying with an inoperative engine is penalized with additional drag counts. In our study, we assume that when an engine fails, it is jammed, creating a sort of “barn door” drag. While we would assume the expression of the additional drag penalty due to an engine inoperative is of the following form,

$$\Delta C_{D_{ENG}} = n_{eng_{inop}} \frac{\pi \left(\frac{d_{fan}}{2} \right)^2}{S_{ref}} \quad (1)$$

We have found that this estimate is slightly too high, creating a model which is too draggy and would not match the climb gradient values implied by the AFM. So while Eqn. (1) suggests approximately 167 drag counts, we desire a model which better matches published data; through multiple trade studies, we identified that approximately 135 drag counts best fits our model – implying that the AFM must have used some form of windmilling assumption for their inoperative engine model.

Additionally, aircraft flying with an inoperative engine experiences a yawing moment that is induced by the asymmetric thrust. This yawing moment causes the aircraft to turn away from the centerline; this is countered by use of rudder and aileron deflection, but this induces a small drag penalty. This drag penalty associated with rudder and aileron deflection is estimated as,

$$\Delta C_{D_{TRIM}} \cong 0.001 \quad (2)$$

As all commercial aircraft typically take-off with deployed flaps, we need to consider the drag penalty associated with flaps. For our A320 model, we estimated the drag addition associated with leading-edge and limited fowler flap extension and deflection as,

$$\Delta C_{D_{FLAPS}} \cong 0.021 \quad (3)$$

This is our drag estimation for the “Flaps 2” setting on the A320 model that enabled it to best-match estimated climb gradients at the prescribed book speeds in its AFM.

As EDET only provides induced drag corrections for the “clean wing” setting, we used the following classic equation for drag-due-to-lift estimations with “Flaps 2” engaged,

$$C_{D_i} = \frac{C_L^2}{\pi AR e} \quad (4)$$

As the span efficiency factor, e , is a free variable that is not defined in the AFM, we had to run trade studies to get a good estimate on it. Parameter e was picked so that the model’s climb gradients at V_2 and V_4 came close to those suggested by our resource. The value which was found to give the model best fidelity was $e \cong 82\%$.

B. Numerical Propulsion System Simulation (NPSS)

While possession of an aerodynamics model on-hand is nice, a propulsion model is also required. NPSS,¹⁵ a physics-based engineering tool, provides us usable five-column data through feeding it key engine parameters such as maximum turbine inlet temperature, reference bypass ratio, reference fan pressure ratio, etc. Using this program, we generated a propulsion model of the V2527-A5 engine used by the A320.

C. SKYMAPS

SKYMAPS produces contour plots of key “point-performance” parameters such as rate-of-climb, climb-gradient and specific-excess-thrust as a function of flight speed and altitude. This tool uses the aerodynamics profile provided by EDET and the propulsion model from NPSS for calculations, as well as all corrective drag terms produced in subsection A of this discussion. This point performance code truncates unrealistic points of data, such as where the required lift coefficient exceeds maximum lift coefficient, or where total drag exceeds maximum thrust.¹⁶

As climb gradient is largely regulated within the Code of Federal Regulations and by the takeoff minimums in departure procedures, this is a variable that we need to track carefully to ensure an accurate model calibration.

To begin, we compute the specific excess thrust:

$$SET(M, ALT) = \frac{n_{eng} T(M, ALT, PLA_{max}) - D(M, ALT)}{W} \quad (5)$$

This term represents the linear acceleration capability of an airframe.

The specific excess power, P_s , is a term closely related to the specific thrust of the aircraft. Recalling from basic physics, the units of power are that of force times length over distance; we can develop a power metric based on the product of the specific excess thrust and the aircraft velocity in knots true airspeed normalized by the weight of the aircraft:

$$P_s(M, ALT) = SET(M, ALT) \cdot V_{KTAS}(M, ALT) \quad (6)$$

P_s indicates the ability of an aircraft to change its energy state; a surplus of energy can be used to accelerate or climb. The reason that engineers like to use P_s is that it forms the basis of a simplified method to directly estimate the rate of climb at constant true airspeed. We will see through example, later in this paper, that this method is not as accurate as we would hope for when computed engine-inoperative flight path trajectories.

If we think about the work-energy theorem from college physics, and we make a series of simplifying assumptions regarding flight at small angles of attack (i.e. the thrust vector is aligned with drag) and climb at small flight path angles (i.e. lift is aligned to oppose weight) at a constant velocity, we can write the following expression:

$$P_s = \frac{T-D}{W} V = \frac{dALT}{dt} \approx R.O.C._{unaccelerated} \quad (7)$$

where T is thrust in lbf, D is drag in lbf, V is velocity in ft/sec, ALT is the geopotential altitude of the aircraft in ft, and W is aircraft mass in lbf. In the world of small angles, the excess power of the aircraft translates into a change in potential energy (change in altitude).

We can also express the rate of climb, for obstacle clearance purposes, in terms of the climb gradient. The gradient is the rate-of-climb per distance travelled, expressed in terms of a percentage. When the climb gradient is zero, the airplane neither climbs nor descends. When the climb gradient is 100%, the airplane climbs one foot for every foot it travels down range.

Continuing to follow the small angle approximation, used so-far in this section, we may write the climb gradient as:

$$\text{Gradient} \approx \frac{R.O.C.}{V} \approx \frac{T(M, ALT) - D(M, ALT)}{W} = SET(M, ALT) \quad (8)$$

This, miraculously, turns out to be the specific excess thrust of the aircraft. Eqn. (8) is best expressed as a percentage as 14 CFR § 25.121 expresses minimum climb gradients as rise-over-run in percent.¹³

Because regulatory compliance calls out minimum climb performance in terms of small gradients, engineering tradition holds that these small angle approximations are appropriate. Thus, a CFR requirement that mandates that a two engine aircraft demonstrate 2.4% percent gradient with one engine inoperative really means that its specific excess thrust level must be greater than 0.0024.

We will see later in this paper that these assumptions render predictions that are less accurate than engineers would commonly expect.

D. MISSION

Although *SKYMAPS* provides performance data at a particular weight setting, this tells us very little about how the aircraft performs when performing flight. Another tool is required to understand how the A320 performs *over time and distance*, especially in response to decisions in cue speeds and takeoff profile.

MISSION is an explicit point-mass simulation tool in which the power lever angle, aircraft flight speed, and altitude are principal state variables. This tool is equipped with multiple “modes” which are used to shape the flight trajectory; only three of them are of interest to us: (1) ground run, (2) constant KIAS climb, and (3) level acceleration. The ground run mode simulates takeoff from rest up until the aircraft reaches V_2 (used in absence of V_{LOF}). Upon reaching V_2 , the code switches to constant KIAS climb mode to simulate second segment climb. Third segment acceleration is commanded by the level acceleration mode in *MISSION*. Third segment acceleration terminates at V_{FTO} and is followed by fourth segment climb which is commanded by the constant KIAS climb mode.

MISSION employs a large-flight-path-angle physics model that incorporates thrust effects upon climb performance that are neglected in the traditional *SKYMAPS* point-performance model. This simulation moves beyond the simple work-energy theorem; it estimates climb performance with a force-balance approach derived from Newton’s laws.

In Figure 5, we can see the force balance for an aircraft flying along a trajectory with a flight path angle called out by γ and an aerodynamic angle-of-attack relative to the flight path angle called out by α . In this case, lift does not precisely oppose weight; thrust does not precisely oppose drag.

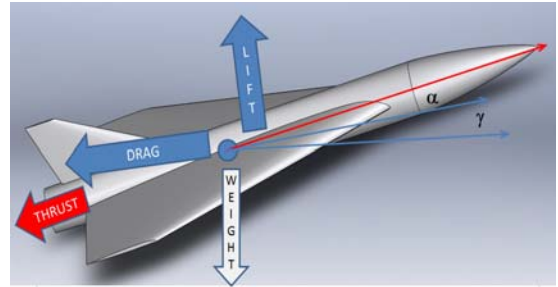


Figure 5. Geometry - Aerodynamic Force Balance.

If we write the force balance for unaccelerated flight in an earth fixed coordinate frame, remembering that an object in motion tends to stay in motion unless acted upon by an unbalanced force:

$$\sum F_x = 0 \rightarrow D \cdot \cos(\gamma) = L \cdot \sin(\gamma) + T \cdot \cos(\gamma + \alpha) \quad (9)$$

$$\sum F_y = 0 \rightarrow W + D \cdot \sin(\gamma) = L \cdot \cos(\gamma) + T \cdot \sin(\gamma + \alpha) \quad (10)$$

Thus, the rate of climb is determined simply as:

$$R.O.C._{\text{unaccelerated}}(M, ALT) = V \cdot \sin(\gamma) \quad (11)$$

This simulation iterates to converge on the exact *CL* to maintain the force balance during climb and descent. Consequently, the results produced by *MISSION* may differ somewhat from that estimated in *SKYMAPS* from a simple work-energy relationship.

In prior work,¹⁷ predicting the climb performance of very high performance aircraft, Takahashi noted:

While Rutowski’s (*work-energy*) methods are useful to provide insight, they do not define truly “optimal” trajectories for high performance aircraft. It appears that the simplifications inherent in the work-energy theorem derived basic performance equations (flight at small angles of attack, thrust vector is aligned with drag, climb at small flight path angles (i.e. lift is aligned to oppose weight) ... fundamentally detract from the veracity of the general method.

In the latter parts of this paper, we will demonstrate that the work-energy approach to flight performance, so common in industry,¹⁸ is not particularly accurate even for engine-inoperative climb at shallow gradients. Consider an initial climbout with takeoff flaps: the aircraft may be pitched to 10° angle of attack although it flies at ~2.4%

climb gradient (a 1.3° flight path angle), thus the aircraft attains a 11.3° deck angle. Because weight acts in the ground reference frame, the thrust from the remaining engines acts upon a vector displaced 11.3° from normal. Since lift and drag act in a reference frame aligned to the flight path angle, not to the aircraft body, a fraction of lift will offset weight. In the case of a notional A320 operating near its WAT limit, $\sim 19.5\%$ of the thrust ($\sin(11.3^\circ)$) offsets weight ($\sim 4,300$ -lbf lift relief from the 168,000-lbm flight weight) while 98% of the thrust ($\cos(10^\circ)$) opposes drag. This 2.5% relief in lift results in a $\sim 5.25\%$ reduction in induced drag. The aircraft will climb better than predicted through work-energy because the $\sim 5.25\%$ induced drag reduction more than offsets the 2% loss in thrust. While these errors seem small, they compound. Their effects become evident when computing the climb profile of actual aircraft flown on typical missions particularly in fourth segment engine-inoperative climb.

IV. Simulator Scenario and Modelling Results

For purposes of illustration of the effect of “pilot error” that leads to initial climbout below scheduled V_2 , we will utilize this A320 model. We will plan dispatch at around maximum weight permitted under the WAT limit given by the Airbus A320 Aircraft Flight Manual.¹⁷

Consider dispatch of a notional A320 at $W \sim 168,000$ -lbm at sea-level and standard day with the high-lift system set to “Flaps 2.” Here, the A320 AFM calls out a target OEI second segment obstacle clearance climb speed of: $V_2 = 144$ -KIAS. The final segment, en-route climb speed (the “green dot speed”) in the AFM is $V_{FTO} = 225$ -KIAS.

Figure 6 represents one-engine-inoperative climb gradient performance at a variety of weights. Figure 6a indicates second segment climb performance with one engine inoperative; Figure 6b provides final segment climb performance with one engine inoperative. Interrogating the two charts together, we can see that the critical-engine inoperative A320 with “Flaps 2” may attain a gross climb gradient greater than 2.4% at weights less than $\sim 168,000$ -lbm whereas the regulatory minimum 1.2% gross climb gradient in 4th segment can be flown at any legal dispatch weight. Thus, the WAT limit of the A320 is found for $W \sim 168,000$ -lbm.

Turning to Figure 7, we see that as the aircraft increases in altitude, the climb gradient diminishes. Our predictions indicate that the best speed for climb gradient is for the pilot to target takeoff rotation speeds to attain $V_2 = 169$ -KIAS; this provides the best possible second segment climb gradient of $\sim 2.7\%$. We can see in this figure that any deviation reduces the aircraft’s climb potential. Since the minimum regulatory gross second segment climb gradient is 2.4%, Airbus could conceivably schedule V_2 as slow as ~ 144 -KIAS. Indeed, we note that for a “Flaps 2” departure at $W = 168,000$ -lbm, the A320 AFM suggests precisely this value: $V_2 = 144$ -KIAS.¹⁹ We believe that Airbus chooses this low V_2 speed in order to minimize take-off ground roll distances to improve their aircraft’s compatibility with short runways at the expense of diminished engine-inoperative obstacle clearance capability so long as V_2 complies with regulation 14 CFR § 25.107(b)

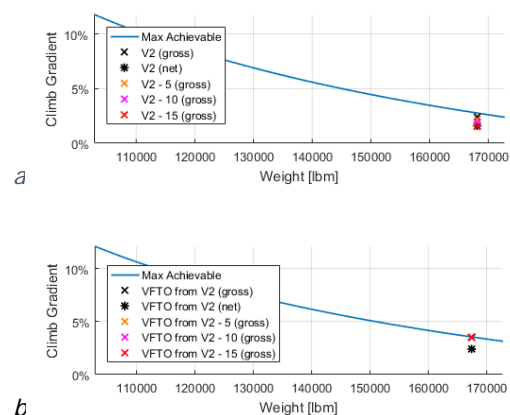


Figure 6. Reverse Engineered A320, climb gradient estimates. a) One engine inoperative climb in second segment; b) One engine inoperative climb in fourth segment / en-route configuration.

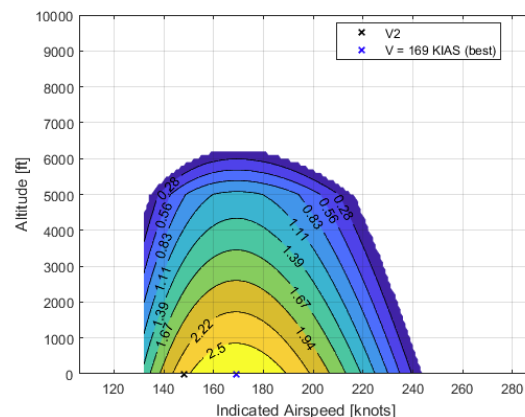


Figure 7. Reverse Engineered A320, $W = 168,000$ -lbm OEI “Flaps 2” gross climb gradient data.

which limits V_2 to be no slower than $1.13 V_S$ or $1.1 V_{MCA}$.²

Following 14 CFR § 25.107(b),² we may infer from the scheduled V_2 that the A320 stall speed at $W=168,000$ -lbm is $V_S=131$ -KIAS. Thus, it is entirely possible for the pilot to “hand fly” the aircraft during initial climbout to trade kinetic energy for altitude; this results in a stabilized V_2 beneath the scheduled speed. Similar to what we observed in the CRJ simulation, the pilot could end up flying a one-engine-inoperative second segment climb at speeds up to 17-KIAS below the scheduled V_2 speed without triggering stall warnings.

Consider the implications of “pilot error” upon obstacle clearance capability. Recall that regulation 14 CFR § 25.115 imposes a “margin-of-safety” of 0.8% in the take-off climb gradient for a two engine aircraft.¹¹ Returning to Figure 5, we see that the 2.4% climb gradient at the scheduled V_2 speed declines to 2.2% when flown at V_2-5 KIAS, 1.9% at V_2-10 KIAS and 1.5% at V_2-15 KIAS. Thus, the “margin-of-safety” has been fully consumed if pilot error leads to a stabilized second segment climb at V_2-15 KIAS. The reader should note that the rate-of-climb capability implied by the gradients is ~ 355 -ft/min at V_2 ; ~ 315 -ft/min at V_2-5 KIAS, ~ 265 -ft/min at V_2-10 KIAS and only ~ 200 -ft/min at V_2-15 KIAS.

Continuing to examine Figure 7, we also see that the climb gradient degradation is a function of aircraft altitude. At a pressure altitude of 1000-ft, the density lapse in engine thrust may take a tenth of a percent off of the climb gradient flown at the nominal V_2 . As the aircraft flies slower than scheduled, the climb degradation grows stronger.

To further illustrate the impacts of an engine-inoperative departure flown “off” book speeds consider the following cases:

- Obstacle clearance flyout with flap retract @ 400-ft AGL – nominal AEO, nominal OEI, “net flight path” OEI, actual flown off speed (V_2-5 KIAS, V_2-10 KIAS and V_2-15 KIAS).
- Obstacle clearance flyout with flap retract @ 1000-ft AGL – nominal AEO, nominal OEI, “net flight path” OEI, actual flown off speed (V_2-5 KIAS, V_2-10 KIAS and V_2-15 KIAS).

In Section III, we mentioned the potential difference between a climb gradient computed using the simple, work-energy theorem approach (with small angle approximations) and that obtained from a full model. Refer to Figure 8, the reader will see the difference in observed climb gradients and rates-of-climb between these methods. The aircraft departs at the work-energy theorem WAT limit weight $W=168,000$ -lbm at $V_2=148$ KIAS; a 2.4% gradient results in a 358-ft/min rate-of-climb. After the gross-to-net 0.8% safety factor is applied, the “official” second segment climb gradient is 1.6% (238-ft/min). While the correction factor for thrust/lift offset is small, as the aircraft slows down the thrust axis ($\alpha+\psi$) grows in magnitude to reach a peak value of 14.2° for flight at V_2-15 KIAS. Under these conditions, 24.5% of the thrust offsets the weight of the aircraft. At 13.0° angle of attack, only 97.4% of the thrust is used to accelerate the aircraft along its flight path. Changing the zero-lift angle-of-attack for second segment climb can bias the solution. We see that the complete physics model predicts a climb gradient of 2.6% (or 385-ft/min climb rate) for OEI climb at V_2 (a 7.5% increase in performance over the prediction). As the aircraft falls below the scheduled V_2 speed, the climb performance declines. Returning to Figure 8, we see that OEI climb at V_2 declines to 352-ft/min when flown 5-kts under V_2 and then down to 268-ft/sec when flown 15 knots under V_2 .

Config	Climb Speed Deviation	Climb Speed (KIAS)	Gradient (%)	R.O.C (ft/min)	γ ($^\circ$)	α ($^\circ$)	$\alpha+\gamma$ ($^\circ$)
AEO	V_2+10	158	16.7%	2674	9.7	6.1	15.8
OEI	V_2	148	2.6%	385	1.5	8.7	10.2
OEI	V_2-5	143	2.4%	352	1.4	10.0	11.4
OEI	V_2-10	138	2.2%	313	1.3	11.4	12.7
OEI	V_2-15	133	2.0%	268	1.2	13.0	14.2
OEI (small angle)	V_2	148	2.4%	358			
OEI - NET GRADIENT	V_2	148	1.6%	238			

Figure 8. Comparison of Climb Gradient and Rate-of-Climb. Differing simulations and climb speeds.

Instead of the “point performance” data found in Figures 6 and 7, Figure 9 (overleaf) plots an altitude-vs-distance flight trace. We may examine the climb profile of the simulated aircraft flown on and off “book speeds” as compared to the dispatch planning altitude-vs-distance profile derived from “net” climb gradients that were computed in a dynamic fashion.

Figure 9 bases the “net” OEI flight path on an altitude dependent work-energy based climb model as opposed to the large-flight-path angle used to compute the other “gross” gradients. The pessimism inherent in the work-energy based climb performance plus the “gross-to-net” decrement produces a flight profile (purple line) with a considerably shallower slope than found with the higher fidelity physics model.

We can see that the FAA mandated “gross-to-net” margin-of-safety protects the pilot from “far out” obstacle avoidance problems (see Figure 9a and 9b). However, there are some regions in flight where the “gross-to-net” margin produces an optimistic flight profile. Second segment climb at $V2-15KIAS$ nearly consumes the “gross-to-net” margin. In addition, the third segment acceleration distance grows longer because the aircraft must accelerate an additional 15 knots before flap retract may commence. Thus, there is an additional low altitude region shortly after flap retraction where the aircraft could be flying a flight profile beneath that which is used to schedule departure. The additional level acceleration distance may add several nautical miles to the flight profile before final segment climb may commence. If an obstacle exists near the flap retraction altitude, the optimism of the “pro-forma” obstacle clearance check may lead to a situation where the pilot will fly a controlled flight into terrain.

In addition, the reader should be cognizant of the sheer distances and times involved in these WAT limited departures. For an aircraft similar to an A320, flown at its WAT limit, the aircraft will fly ~40-nM downrange of the airport before reaching 5,000-ft AGL. The third segment flap retraction itself will consume a nerve-wracking 10-nM (3.5 minutes); a period of time where pilot anxiety regarding controlled flight into terrain might lead the crew to attempt a final segment climb beneath the scheduled speed. In our CRJ-200 simulation observation discussed in the introduction to this paper, the pilots inadvertently lost several hundred feet of altitude during the prolonged third segment. While they flew the final segment at the appropriate reference speed, they began their climb at a lower altitude than was otherwise scheduled; this sort of “real-world” experience falls outside the scope of FAA “pro-forma” obstacle clearance plans.

Moreover, the reader should be aware that these profiles were computed for an idealized still wind condition. Dispatch is allowed to take into account prevailing winds and give climb gradient credit for flight into a prevailing headwind. We speculate that it is a rare day when headwinds remain constant from the departure runway through an altitude of 5,000-ft, ~40-nM downrange.

Finally, the reader should be aware that these profiles were computed for an idealized straight-out flight departure. Very few, if any, standard instrument departures (SIDs) comprise a 40-nM climb out without significant heading change. If, as so common, an aircraft dispatched near the WAT limit proves incapable of flying the SID, pilots must depart from the planned departure and circle back to attempt a landing within the valley from which they just departed. The degradation in climb turn performance for a 90° or 180° heading change is not accounted for here. Previous work by the team (Takahashi & Bays)⁹ has shown that even a gentle 15° bank made during second segment can easily reduce the climb gradient by nearly 1%, and thus consume the entire “margin-of-safety” used by dispatch.

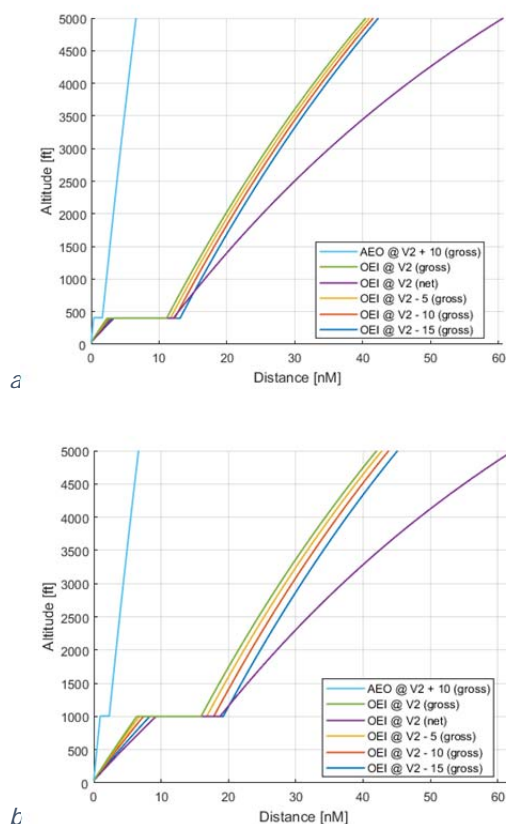


Figure 9. Reverse Engineered A320, “Flaps 2” OEI takeoff rate-of-climb history $W = 168,000\text{-lbm}$. a) Flap retract at 400-ft AGL; b) Flap retract at 1000-ft AGL. The OEI net gradient is computed using the simple work-energy method.

In a dispatch where the obstacles are even taller than implied here; the pilot would need to ensure steeper gradients. Therefore, the aircraft would need to be limited to dispatch at a lighter weight. At these lighter weights (which imply greater gradients and steeper flight path angles), the small-angle approximation will be even more pessimistic.

V. Conclusions

This numerical simulation, exploring the consequences of an event witnessed during an emergency training situation flown by certified pilots, highlights the current state-of-the-art for commercial aircraft dispatch. We believe that the current dispatch rules, with obstacle clearance predicated on a flight profile that derives from a 0.8% decrement applied to “gross” predicted climb rate of an ideal aircraft, provides a reasonable margin-of-safety for any one issue: that is 1) a poorly performing aircraft with worn engines, 2) an aircraft that needs to turn when climbing to avoid an obstacle, 3) an aircraft where “pilot error” leads to an initial climb flown at a stabilized speed below scheduled V_2 , or 4) an aircraft flown into winds that differ from that used to estimate its obstacle clearance capability. At the same time, current margins-of-safety do not account for foreseeable situations where multiple issues compound.

To conclude, we believe that future risk-based dispatch systems should consider the possibilities and consequences of flight stabilized at “off book” speeds to derive obstacle clearance limited take-off weights especially when considering close-in obstacles.

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