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Climbing While Turning: Combat Energy Management Principles Applied to Civilian Obstacle Clearance

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Regulations that limit the maximum takeoff weight from a given airfield require safe clearance of all obstacles in the flight path in the event of an engine failure. At some airfields, a turning flight path allows safe lateral separation from tall obstacles. While potentially permitting much greater takeoff weights, an accurate estimate of the degradation in climb gradient arising from a turning-climb is still necessary to ensure sufficient vertical separation from any other obstacles in the flight path. Our review of the aircraft performance data provided by manufacturers reveals that there are no standards to specify gradient loss during turn-climbs. This paper takes a brief historical look at the regulations and formulations for turning and climbing flight. It proposes an adaptation of classical energy-maneuverability theory, as traditionally used for fighter aircraft, to address the problem of engine-out climb for transport aircraft. We present analytical formulations for gradient loss and discuss the potential strengths and limitations of such an approach.

Nomenclature

AEO	=	all engines operating
AFM	=	Airplane Flight Manual
CFR	=	United States Code of Federal Regulations
OEI	=	one engine inoperative
ALT	=	geopotential altitude (ft)
M	=	Mach number
W	=	aircraft gross weight (lbm)
KTAS	=	true airspeed (knots)
KIAS	=	indicated (equivalent) airspeed (knots)
q	=	dynamic pressure (lbf/ft^2)
V	=	true airspeed (ft/sec)
VMCA	=	minimum controllable airspeed in the air (KIAS)
VMCL	=	minimum controllable airspeed on landing (KIAS)
Vs	=	1-g Stall Speed (KIAS)
σ	=	atmospheric density (altitude) (slugs/ft^3)
AR	=	aspect ratio
S	=	reference area of the wing (ft^2)
C_L	=	lift coefficient
C_D	=	drag coefficient
L	=	lift (lbf)
D	=	drag (lbf)
T	=	aircraft net thrust (lbf)
η	=	net efficiency of the propeller
$t\,hp_a$	=	thrust horsepower available (hp)
$t\,hp_r$	=	thrust horsepower required for steady, level flight (hp)

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Nz	=	load factor normal to flight path
Nzmaxi	=	instantaneous load factor for turning
Gradient	=	climb gradient (%)
TurnRate	=	heading change rate (^{o}/sec)
SSR	=	stall speed ratio
ROC	=	rate-of-climb (ft/min)
Φ	=	bank angle (°)
γ	=	climb angle (°)
g	=	universal gravitational constant (32.2 ft/s^2).

I. Introduction

ENGINEERS design multi-engine aircraft to safely operate in the event that one engine fails. The rules for planning commercial and non-combat military aircraft operations require dispatch to limit aircraft operations in the event that a failure of a critical engine would jeopardize the safety of the flight. Aircraft operational procedures guarantee that under normal conditions the aircraft never flies at an airspeed or on a trajectory that would render, in the event that an engine suddenly fails, the aircraft to either lose control or suffer "controlled flight into terrain."[1]

This paper provides a brief history of aircraft performance computations as applied to turning while climbing. Initial interest in this capability came from combat pilots who wanted, in a time of war, to "dog-fight" against enemy aircraft.[2] Later on, this capability was required to ensure safety in "air commerce" for multi-engine civilian aircraft.[3] This paper also discusses the history of engine-inoperative climb regulations.

Although we present both the exact equations of motion as well as small-flight-path-angle approximations to compute these problems, the choice of end user dictates the appropriate equation. Because dogfights require highly maneuverable aircraft, military combat aircraft performance analysis must consider the exact equations of motion. Mission tactics and strategy require a precise understanding of how turning diminishes climb performance in circumstances where aircraft have significant inherent excess power. Alternatively, commercial obstacle clearance problems may rely on a small-flight-path-angle, so called "approximate" solution, because the rate-of-climb in an engine-inoperative scenario is quite modest.

This paper presents trade studies examining combined turn/climb capability for a moderate-thrust-to-weight subsonic aircraft reminiscent of an Airbus 320 narrow-body airliner (both all-engines-operating and critical-engine-inoperative). In this work, we seek to establish: 1) where and when the "approximate" small-flight-path-angle solution is appropriate and where the iterative large-fight-path-angle is required, 2) what sorts of limitations and performance tradeoffs are actually experienced when aircraft are expected to turn as they climb, and 3) what are the turn-climb performance implications inherent in the regulations that require pilots to fly engine-inoperative-takeoff at specific stall-speed ratios.

II. MODELLING AIRCRAFT CLIMB

In NACA TR-97, published in 1921, DeBothezat introduces the basic work-energy theorem model of aircraft performance.[4] He holds that the rate of climb is a function of the forward flight speed, *KTAS*, and the flight path angle, γ:

$$ROC \propto KTAS \cdot \gamma \tag{1}$$

Where, cast in terms of more modern nomenclature, he defines flight path angle as a function of thrust, *T*, aircraft flight weight, *W*, and the lift-to-drag ratio (*L/D*):

$$\gamma = \frac{T}{W} - \frac{1}{L/D} \tag{2}$$

For steady level flight, lift must equal weight (L=W), thus:

$$\gamma = \frac{T-D}{W} \tag{3}$$

And subsequently:

$$ROC = \frac{6080}{60} \cdot KTAS \cdot \frac{T-D}{W} \tag{4}$$

In 1933, Bailey Oswald published his famous treatise covering general formulas for aircraft performance as NACA TR-408.[5] Oswald, who mentored under Arthur Raymond who was then the assistant chief engineer at Douglas Aircraft as well as faculty at Cal Tech, derived an applied version of the work-energy theorem based not so much on the interplay of thrust and drag forces, but instead focused upon propulsive horsepower. In Oswald's formulation, the rate-of-climb is proportional to the difference between the thrust horsepower available, $t\ hp_a$, and the thrust horsepower required to maintain steady level flight, $t\ hp_r$:

$$ROC = \frac{dh}{dt} = \frac{550\,(t\ hp_a - t\ hp_r)}{W} \tag{5}$$

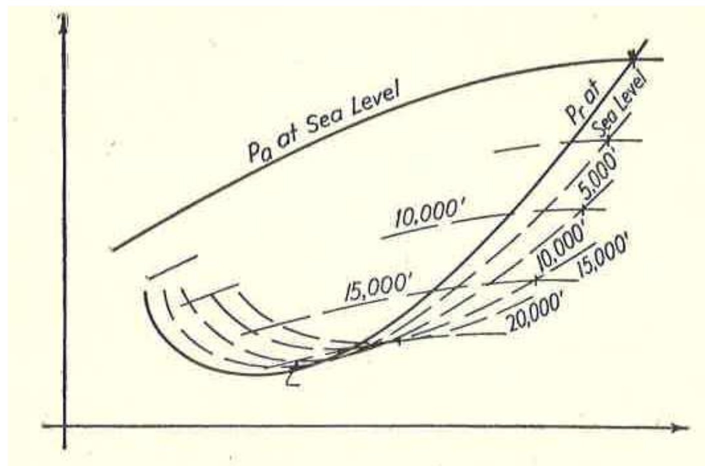

Figure 1 – Power Curve Approach to finding the speed associated with the best rate of climb. After Millikan [ref 6], p. 105.

Oswald's goal was to reduce the rate-of-climb problem from the three-variable form found in equation [5] into a succession of two two-variable functions suitable for graphical evaluation in design. Thus, Oswald immediately assumes that the thrust horsepower required derives from a quadratic form of the drag polar comprising a fixed "parasite drag" (D_p) term and an "effective induced drag" (D_i) term that is proportional to the square of the lift coefficient. (In this derivation, Oswald introduces his famous "span efficiency factor," e, in order to better adjust the theoretical value of induced drag to measured values.)

$$t\ hp_r = D \cdot V = \left(D_p + D_i\right) \cdot KTAS \tag{6}$$

Similarly, we compute the thrust horsepower available as the product of P, the power delivered by the engine to the propeller, and η, the net efficiency of the propeller.

$$t\ hp_a = P \cdot \eta = f(KTAS, \sigma) \tag{7}$$

Which are ultimately non-linear functions of flight speed, $KTAS$, and atmospheric density (altitude), σ.

Oswald's methods re-appear in slightly modified form within Clark Millikan's famous text "Aerodynamics of the Airplane." [6] This text formalizes the idea that the maximum rate of climb occurs at the flight speed that maximizes the excess power available (see Figure 1). Other pre-war authors such as B.M. Jones[7] and Dwinnel[8] express the same sentiment: a simple work-energy relationship captures the entire climb problem.

If we think more broadly about the work-energy theorem, we can write a more general expression that is valid at small angles of attack (i.e., the thrust vector is aligned with drag) and climb at small flight path angles (i.e., lift is aligned to oppose weight).

$$(T - D)\, V = W \frac{dALT}{dt} + \frac{W}{g} \frac{d}{dt} (\frac{V^2}{2}) \qquad (8)$$

Where T is thrust in lbf, D is drag in lbf, V is velocity in ft/sec, ALT is the geopotential altitude of the aircraft in ft, W is aircraft mass in lbm, and g is the universal gravitational constant. In other words, the excess power of the aircraft can translate into a combination of a change in potential energy (change in altitude) or a change in kinetic energy (change in airspeed).

If we divide Equation 8 through by the weight, W, we get the following expression for specific excess power, P_s:

$$P_s = \frac{(T-D)\, V}{W} = \frac{dALT}{dt} + \frac{d}{dt} (\frac{V^2}{2g}) \qquad (9)$$

We can then consider the case of constant kinetic energy climb. Equation 9 reduces to:

$$P_s = \frac{(T-D)\, V}{W} = \frac{dALT}{dt} \approx ROC_{unaccelerated} \qquad (10)$$

This equation infers that the rate-of-climb is linearly proportional to P_s.

Of course, the devil is in the details: real aircraft do not fly truly un-accelerated climb profiles. Due to air traffic control and piloting convention, aircraft fly either constant indicated airspeed (*KIAS*) or constant Mach number (*M*) climbs and descents. During a climb at constant indicated airspeed, the aircraft must increase its kinetic energy as it ascends. Conversely, during a climb at constant Mach number in a region of the atmosphere where there is a cooling temperature gradient with increasing altitude, the aircraft must slightly decrease its kinetic energy as it climbs.

Correction factors, based upon the lapse rate in outside air temperature with altitude, can account for this effect.[9] Thus, the small-angle approximation rate-of-climb at a given Mach number and altitude is:

$$ROC(M, ALT) \cong K_{accel} \cdot ROC_{\ unacellerated}(M, ALT) \qquad (11)$$

Where:

$$K_{accel} = \frac{1}{1 + \frac{KTAS(M, ALT)}{g} \cdot \frac{dKTAS}{dh}(M, ALT)}. \qquad (12)$$

Which when applied to the structure of the 1962 and 1976 Standard Atmosphere models results in:

$$K_{accel} = \frac{1}{1 + .566816 \cdot M^2} \qquad (13A)$$

for climb at constant indicated (calibrated) airspeed below the tropopause (ALT <36,089-ft), and

$$K_{accel} = \frac{1}{1 + .7 \cdot M^2} \qquad (13B)$$

for climb at constant equivalent indicated (calibrated) airspeed above the tropopause (ALT>36,089-ft).

Because climb at constant indicated airspeed requires the aircraft to physically accelerate and increase its kinetic energy as its climbs, less energy remains to influence changes in potential energy. Thus, an aircraft flying a constant indicated airspeed climb will gain altitude slightly more slowly than the basic, un-accelerated equations of motion would predict.

Similarly, the *k*-factors for climb at constant Mach number are:

$$K_{accel} = \frac{1}{1 - .133184 \cdot M^2} \tag{13C}$$

for climb below the tropopause (*ALT* <36,089-ft), and

$$K_{accel} = 1 \tag{13D}$$

for climb at constant Mach number above the tropopause (*ALT*>36,089-ft).

Because a climb at constant Mach number requires the aircraft to decelerate and lose kinetic energy as its climbs in the tropopause, more energy exists that can influence changes in potential energy. Thus, an aircraft flying a constant Mach number climb in the troposphere will gain altitude slightly more quickly than the basic, un-accelerated equations of motion would predict.

Moving beyond the simple work-energy theorem, we can also derive a more complex set of relationships to compute the un-accelerated rate-of-climb based upon a force-balance approach derived from Newton's laws.

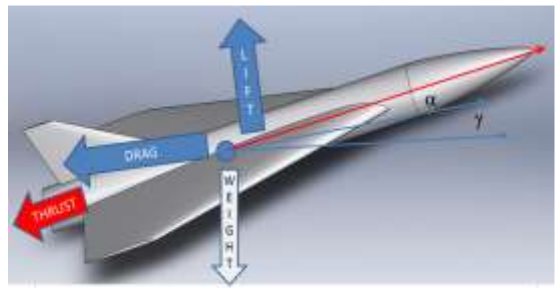

Figure 2 – Geometry - Aerodynamic Force Balance

Return to Figure 2, the force balance equation for the general conditions of an aircraft flying along a trajectory with a flight path angle, γ, and an aerodynamic angle-of-attack relative to the flight path angle, α. In this case, lift does not precisely oppose weight; thrust does not precisely oppose drag.

If we write the force balance for un-accelerated flight in an earth-fixed coordinate frame, remembering that an object in motion tends to stay in motion unless acted upon by an unbalanced force:

$$\sum F_x = 0 \Rightarrow D \cdot \cos(\gamma) = L \cdot sin(\gamma) + T \cdot \cos(\gamma + \alpha) \tag{14}$$

$$\sum F_y = 0 \Rightarrow W + D \cdot \sin(\gamma) = L \cdot cos(\gamma) + T \cdot \sin(\gamma + \alpha) \tag{15}$$

As before, we can determine the rate of climb as:

$$ROC_{unaccelerated}(M, ALT) = V \cdot \sin(\gamma) \tag{16}$$

This equation is valid for cases where $(T - D) / W <= 1$. Under circumstances where thrust is equal to the sum of weight and drag at zero lift, the flight path angle, γ, attains a peak value of 90^o. It should be intuitively obvious that this is the correct answer, the aircraft can fly straight up. Under small-angle approximations, we would under-predict performance because our value of drag would contain both zero-lift and drag-due-to-lift terms that represent flight where *Lift=Weight*.

Consider an aircraft with a dynamic thrust to weight ratio (*T/W*) of 0.25 climbing at an angle-of-attack, α, of 6^o and a flight path angle, γ, of 12^o. In this scenario, the vectored thrust manages to offset the total lift by 7.7%. Because the induced drag of the aircraft is a function of the square of the lift coefficient, flight under such a scenario results in a 14% reduction in induced drag compared to that of the small-angle approximation. The reduction in drag-due-to-lift can lead to a noticeable increase in specific excess thrust. These differences are recognizable to both pilot and simulation expert. The small-angle approximation will lead to a pessimistic estimate of aircraft climb performance. The actual aircraft will perform better than expected.

Alternatively, in an engine-inoperative situation, where an aircraft may have a dynamic thrust to weight ratio (T/W) of 0.10, climbing at an angle-of-attack, α, of 6^o and a flight path angle, γ, of 1.7^o (a 3% gradient). In this scenario, the vectored thrust manages to offset the total lift by only 1.3%. In this circumstance, the small angle approximation a scenario results in a 2.5% reduction in induced drag compared to the exact solution.

Thus, we can see that for engine-inoperative climb at shallow flight-path angles, the difference between the "approximate" small-flight-path-angle solution and the exact, iterative large-fight-path-angle solution is small. In Section VI, we will use numerical simulations to verify the implied pessimism behind the small angle equations.

III. MODELLING AIRCRAFT TURNING PERFORMANCE

Neither DeBothezat[4], Oswald[5], nor Millikan[6] mention aircraft turning or maneuvering performance in their respective aircraft performance treatises.

It was up to B.M. Jones[7] to present an early comprehensive discussion of aircraft turn performance.

Recall, in straight, level flight, the vertical, downward force of gravity is the dominant inertial force acting upon the aircraft; thrust and drag remain in equipoise. When flying in a steady turn, centrifugal forces develop in the lateral plane. Because lift must balance this resultant force, it must act "inward and upward" to counteract both the vertical and the lateral force components. Thus, it is customary for an aircraft to bank or "depress the inner wing" into the turn in order to properly align lift forces. Jones[7] introduces the basic correlation between bank angle, flight speed and turn radius.

$$tan(\Phi) = \frac{V^2}{g \cdot R} \quad (17)$$

Jones clearly noted that the bank angle is independent of weight, wing area and airfoil geometry.

Jones also observed that "if the angle-of-bank is correct … the magnitude of the lift force must equal the magnitude of the resultant force." Thus:

$$L = \frac{g \cdot R}{\sin(\Phi)} \quad (18)$$

In modern nomenclature, we can define the load factor, *Nz*, to represents the magnitude by which lift exceeds weight:

$$L = N_Z \cdot W \quad (19)$$

Thus, geometry implies a correlation between *Nz* and bank angle, Φ, for flight without loss of altitude (Figure 3):

$$Nz = 1/\cos(\Phi) \quad (20)$$

Turning radius in feet may be inferred from load factor, *Nz*, and flight speed in KTAS (Figure 4):

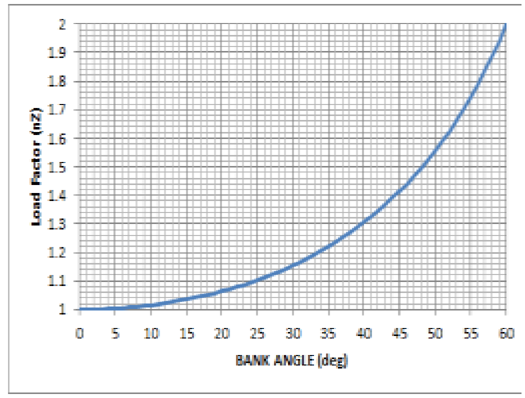

Figure 3 –Bank Angle / Load Factor Relationship

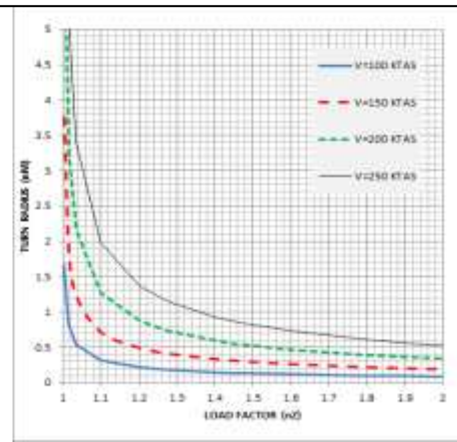
Figure 4 –Turn Radius as a function of Load Factor and Flight Speed

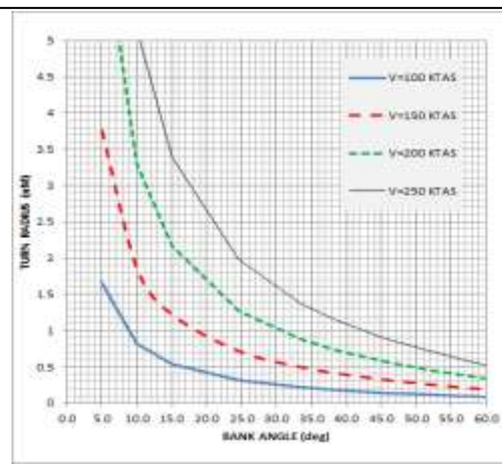

Figure 5 –Turn Radius as a function of Bank Angle

$$TURNRADIUS = \frac{(V)^2}{g\sqrt{Nz^2-1}} = \frac{(KTAS\cdot\frac{6080}{3600})^2}{32.2\cdot\sqrt{Nz^2-1}} \qquad (21)$$

Since the turn radius increases as a function of the true airspeed squared, the reader may examine the trade between maneuverability and agility. Beyond the scope of the chart, consider a supersonic fighter aircraft flying with a 5-g load factor at 1000 KTAS true airspeed, it will bank over 78^o to make a ~3 nM radius turn. Returning to Figure 4, examine the ~3 nM radius turn as executed by a subsonic transport aircraft. That 3 nM radius turn can be flown at 250 KTAS with a barely perceptible load factor of *Nz*~1.045. At 200 KTAS, the aircraft requires even less load factor to turn: *Nz* ~ 1.010!

Figure 5 depicts a combination of the turn radius equation (21) with the bank angle equation (20). Here we can see that a ~3 nM radius turn flown at 250 KTAS requires a bank angle of α~16^o. At 200 KTAS, about 10^o of bank angle is required while at 150 KTAS only ~5^o bank angle is required. Similarly, at a bank angle of 15^o, 250 KTAS implies a ~3.3 nM turn radius; 200 KTAS implies a ~2.2 nM turn radius; and 150 implies a ~1.2 nM turn radius. Thus, the agility of an aircraft is inversely proportional to the square of the speed and non-linearly proportional to the maximum allowable bank angle.

Although most aircraft structural design texts introduce the concept of the *V-n* diagram,[10] little attention is placed on understanding where in an aircraft's flight envelope it can generate the aerodynamic loads to fully stress its structure.

The Maximum Instantaneous Aerodynamic Load Factor, N_zMax_i, can be established as a function of the lift coefficient, C_L, necessary for steady level flight where lift equals weight, $L=W$:

$$N_zMax_i(M,ALT) = \frac{C_{L\max}(M)}{C_L(M,ALT)} \qquad (22)$$

Because maximum load factor and turn rate are coupled, we may compute additional parameters to describe a level turn. The turn rate, in degrees heading change per second is:

$$TurnRate = 360^\circ \cdot \frac{V_{KTAS}}{2\pi \cdot TurnRadius} / 3600\sec \qquad (23)$$

Since this relationship is independent of the specific aircraft geometry, we can develop generalized turn rate plots (see Figures 6 and 7) similar to those previously developed for turn radius.

The turn rate, turn radius, airspeed, and energy state are brought together in the "doghouse" plot, a variation of which is shown in Figure 8. The lines of constant bank angle (or load

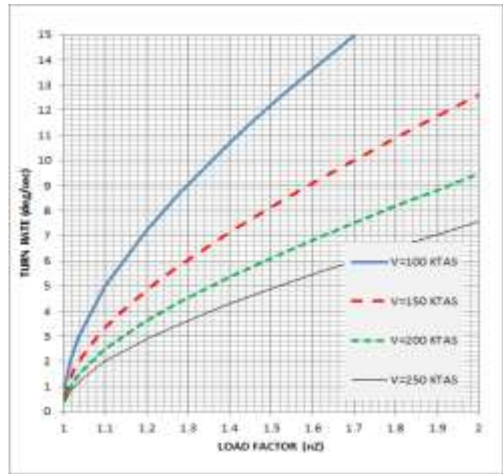

Figure 6 –Turn Rate as a function of Load Factor and Flight Speed

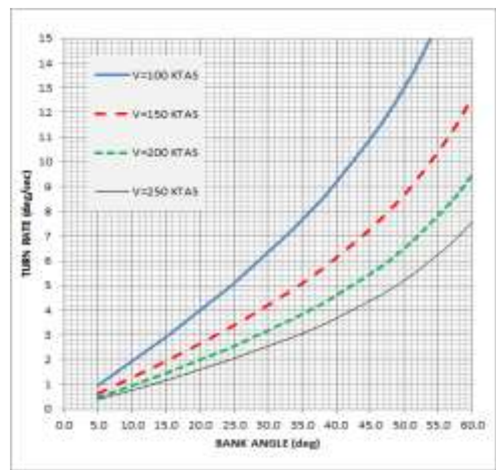

Figure 7 –Turn Rate as a function of Bank Angle

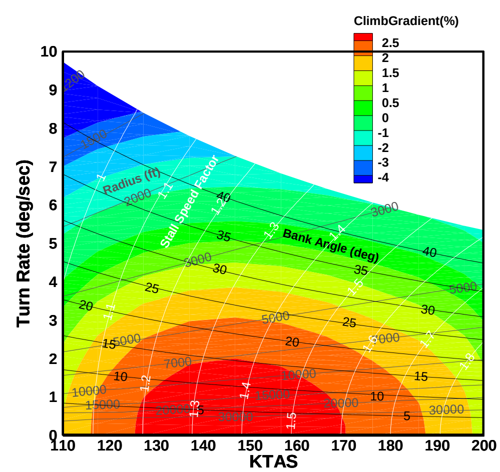

Figure 8– Classic "doghouse" plot

factor) and turn radius form the pitched "roof" of the doghouse, while the *Ps* contours form the "door." In terms of fighter aircraft performance, these plots express energy in the form of *Ps* contours, such that the peak of the curve for $P_S = 0$ identifies the condition for maximum turn rate – a speed of utmost importance in a dogfight. For purposes of evaluating climb performance of lower-thrust transport aircraft, the doghouse chart can be modified to show contours of climb gradient, as shown in Figure 8. Contours of constant stall margin have also been added to the chart, which indicate the available margin at a given bank and airspeed.

Similarly, N_{zmax} can be expressed in slightly different form as an expression of the flight stall speed ratio, *SSR*. When the aircraft flies at incipient stall, $C_L=C_{Lmax}$; therefore its maximum aerodynamic sustainable load factor is $N_{zMaxi}=1$, and $V=V_{stall}$. Additionally, note that when one flies at an airspeed $V=1.3\ V_{stall}$, the airframe can only utilize 59% of its maximum lift capacity (one flies with a reserve instantaneous aerodynamic load factor of 1.69).

Because lift scales proportionately with dynamic pressure, and dynamic pressure is a function of the square of the flight Mach number, the stall speed ratio is:

$$SSR(M, ALT) = \sqrt{\frac{C_{L\max}(M)}{C_L(M, ALT)}} \tag{24}$$

These two equations imply a Janus-like duality between maximum aerodynamic load factor attained at stall and formal stall speed ratio implied by flight at a scheduled airspeed. We can plot the relationship implied between equation (23) and (24) independent of any actual specification of the maximum lift coefficient; see Figure 9.

The FAA through Title 14 of the Code of Federal Regulations[11] requires the aircraft designer to call out minimum permissible flight speeds for operations to ensure that under all probable situations, the pilot never encounters stall or suffers controllability problems due to the malfunction of a critical engine. Today, newly certified aircraft must operate under the following guidelines (see Table 1, below)

V_{MCA} is the minimum controllable airspeed with the critical engine inoperative in the take-off or cruise configuration.[12]

V_{MCL} is the minimum controllable airspeed with the critical engine inoperative in the landing configuration.[12]

Regulation 14 CFR § 91.117 restricts commercial aircraft flight at speeds in excess of 250 KIAS at altitudes below 10,000-ft AGL. [13]

Table 1 – Minimum Permissible Flight Speeds for Jet Propelled Aircraft

Speed	Flight Condition	Flap Setting	Minimum Speed (KIAS)	Regulation
V2	Take-Off Safety Speed	T/O flaps deployed, gear retracted	Max(1.13 Vs TO flaps, 1.1 VMCA)	14 CFR § 25.107(b) [14]
Vref	Final Approach Speed (Balked Landing)	Landing flaps deployed, gear extended	Max(1.23 Vs LD flaps, VMCL)	14 CFR § 25.125 (b)(2) [15]
Vap	Initial Approach	Approach flaps deployed, gear retracted	Max(1.4 Vs AP flaps, VMCA)	14 CFR § 25.121 (d)(1) [16]
Vfuss	En-Route Climb, Flaps Up Safety Speed	Cruise, gear retracted	Max(1.18 Vs cruise, VMCA)	14 CFR § 25.123 (b)(2)[17]

Returning to Figure 9, we can see that flight at 1.13 times the stall speed (the take-off safety speed rule for second segment climb) permits an instantaneous aerodynamic load factor of 1.27 to be attained prior to stall. Flight at 1.18 times the stall speed (the minimum speed rule for fourth segment or en-route climb) permits a load factor of 1.39. Flight at 1.23 times the stall speed (recall the final approach speed limit, V_{ref}) permits an instantaneous aerodynamic load factor of 1.5 to be attained prior to stall. Conversely, a stall speed requirement for flight at 1.4 times the stall speed (for instance, initial approach speed) essentially permits a pilot to fly any maneuver up to 2-g's without risking stall.

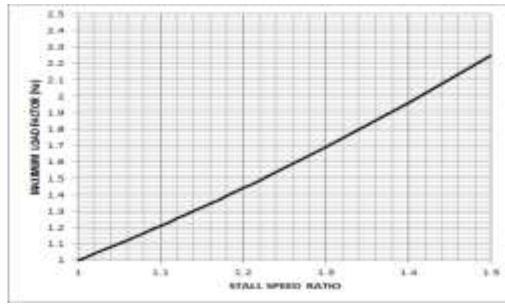

Figure 9 –Maximum Aerodynamic Load Factor as a Function of Stall Speed Ratio

Figure 10 stems from a combination of equations (20), (22) and (24). We can see here the limits of various stall-speed ratios upon the maximum level-flight bank angle of an aircraft. Flight at *1.13 Vs* restricts steady level turns so that incipient stall occurs at 39^{o}. Similarly, flight at *1.18 Vs* enables steady level turns so long as Φ<44^{o}. Flight at *1.23 Vs* limits level sustained turn bank angles to Φ<49^{o}.

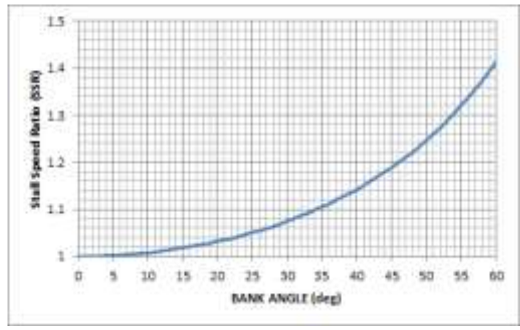

Figure 10 –Bank Angle as a Function of Stall Speed Ratio

Alternatively, Figure 11 derives from a combination of equations (22), (23) and (24). Here we can see the limits of various stall-speed ratios upon the minimum turn radius of an aircraft. For example, flight at *1.13 Vs* and 250 KTAS limits the turn radius to an arc no tighter than 1.15 nM. Slowing down to 200 KTAS, but holding the stall speed ratio constant (i.e. flying at a lighter weight), reduces the turn radius to 0.73 nM. Slowing further to 150 KTAS but still holding the stall speed ratio constant (i.e. even less weight), reduces the turn radius to only 0.41 nM.

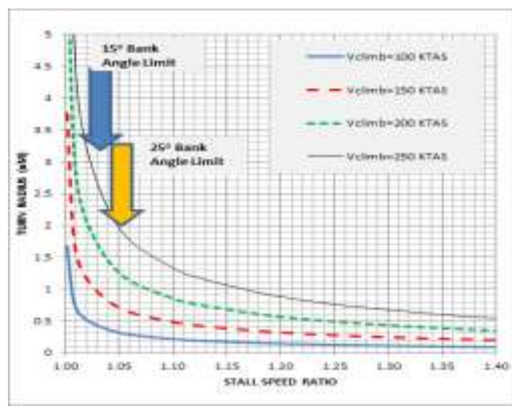

Figure 11 –Turn Radius as a Function of Stall Speed Ratio

Figure 12 (overleaf) is closely related to Figure 11, but combines equations (20), (22), (23) and (24). We can see here the limits of various stall-speed ratios upon the minimum turn rates of an aircraft. If we impose a 15^{o} bank angle, we greatly restrict the maximum turn rate of the aircraft. At 200 KTAS climb speed, the aircraft will change heading at a rate of 1.5 compass degrees per second. If we relax our analysis to impose a 25^{o} bank angle, we still restrict the maximum turn rate of the aircraft. At 200 KTAS climb speed, the aircraft will change heading at a rate of 2.5 compass degrees per section. If we fly *1.13 Vs* and 200 KTAS and bank to incipient stall, our turn rate quickens to a rate of ~4.5 compass degrees per second. These rates are quite slow, at a heading change rate of two degrees per second, a 90 degree heading change requires 45 seconds of elapsed time to fly through.

Clearly, during the initial take-off procedure, much can happen in terms of the aircraft altitude during a 45-second sustained turn maneuver.

Gradient loss in a turning climb can be approximated as the difference between the gradient for an airplane in non-turning flight and the gradient at the same conditions in turning flight:

$$\Delta\gamma = \gamma_{NO\ TURN} - \gamma_{TURN} \tag{25}$$

Substituting equation (3) into equation (25), and assuming that thrust is constant, gradient loss becomes an expression based on the different levels of drag:

$$\Delta\gamma = \frac{D_{NO\ TURN} - D_{TURN}}{W} \tag{26}$$

If drag is again expressed in the quadratic form proposed by Oswald[5] (i.e., $C_D = C_{D0} + k \cdot C_L^2$), then the gradient loss can be expressed as a function of the lift coefficient and load factor:

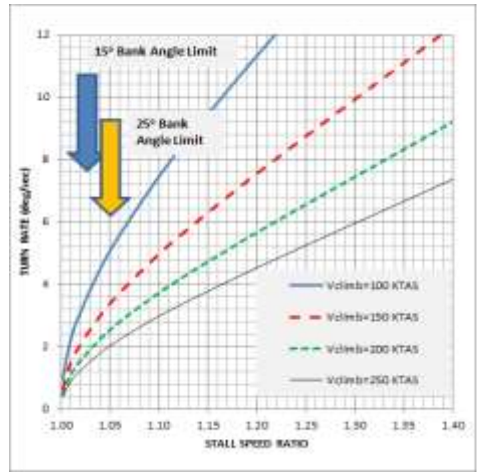

Figure 12 –Turn Rate as a Function of Stall Speed Ratio

$$\Delta\gamma = \frac{qS\left(C_{D_{NO\ TURN}} - C_{D_{TURN}}\right)}{W} = \frac{q \cdot S \cdot k \cdot \left({C_{L_{NO\ TURN}}}^2 - {C_{L_{TURN}}}^2\right)}{W} = \frac{q \cdot S \cdot k \cdot {C_{L_{NO\ TURN}}}^2 (1 - {n_z}^2)}{W} \tag{27}$$

Since $C_L = W / (q \cdot S)$, this further simplifies to:

$$\Delta\gamma = \frac{k(\frac{W}{S})(1 - n^2)}{q} = \frac{2k(\frac{W}{S})(1 - n^2)}{\rho \cdot V^2} \tag{28}$$

Since load factor n_z is equivalent to $1/\cos\Phi$, the term $(1 - n^2)$ is equivalent to $\tan^2\Phi$, leading to:

$$\Delta\gamma = \frac{k(\frac{W}{S})\tan^2\Phi}{q} = \frac{2k(\frac{W}{S})\tan^2\Phi}{\rho \cdot V^2} \tag{29}$$

Thus, the gradient loss can be expressed as a function of aircraft gross weight, bank angle, airspeed, and altitude (via the variation of air density with height).

An actual flight-test derived AEO (all engines operating) drag polar may not conform to the quadratic form proposed by Oswald, and there may be additional components of drag, particularly that associated with asymmetric engine-out conditions. However, it seems reasonable to assume that the drag for a typical transport-class airplane is approximately second-order with respect to lift, and thus the gradient increment associated with coordinated, climbing turns should resemble that in equation (29).

We base the entire preceding analysis upon the aircraft having symmetric thrust. For most aircraft configurations, the critical engine-out scenario results in an asymmetric distribution of thrust about the aircraft centerline. In a typical non-turning scenario with an engine inoperative, a combination of rudder deflection and bank angle counteracts this asymmetric thrust. The rudder deflection produces a side-force and a yawing moment. The side-force from the rudder deflection is balanced by a finite sideslip angle. It is worth noting that it is possible to counteract the asymmetric thrust with a combination of rudder deflection and bank such that sideslip is zero, and no side force and associated drag is generated. Conversely, if the configuration possesses sufficient rudder authority, it may be possible to counteract asymmetric thrust with rudder deflection only (i.e., zero bank). Ultimately, there are unlimited combinations of rudder deflection and bank that will produce steady-heading flight, but for any given bank angle, there is only one rudder deflection for steady-heading flight.

The generation of this side-force has implications for turning flight. The formulation for coordinated, turning flight provided above, in which the rotated lift vector is the only force contributing to the turn, is no longer valid if some additional side-force is present. At least one manufacturer asserts that the effect of side-force on OEI turns is significant, and has prescribed adjustments to the turn radius and gradient loss accordingly.[21] This seems like an appropriate conclusion. Consider steady-heading flight: unless the asymmetric thrust is being rejected entirely by rudder deflection, some finite bank angle is required. A similar situation occurs in a turn at a given bank angle. Depending on the rudder deflection, some portion of $L\cdot sin(\Phi)$ may be working in opposition to the force from the rudder. Thus, the radius would not be as expected from a purely coordinated turn.

Curiously, this knowledge has not impacted the regulatory world. In Advisory Circular AC 25.1581-1, which provides guidance for development of the AFM, the FAA states the following:

"Radius of turn, for use in obstacle lateral separation, is not airplane dependent and can easily be calculated from speed and bank angle. Climb gradient decrements, however, are airplane dependent. Climb gradient decrements for bank angles up to at least 15 degrees should be provided in the AFM. Consider providing coverage of higher bank angles as appropriate to the expected operation of the airplane."[18]

An advisory circular "is neither mandatory nor regulatory in nature and does not constitute a requirement."[18] There is no requirement in Title 14 CFR requiring radius or gradient loss in a turn to be demonstrated or provided in the aircraft flight manual.[11]

In Section VI, we will discuss the coupling between stall speed ratio, flight speed, turn radius and turn rate in terms of the operational flight envelope of transport category aircraft.

IV. OBSTACLE CLEARANCE – CLIMB AND TURN PERFORMANCE REQUIREMENTS TO ENSURE FLIGHT SAFETY

Early in the history of aviation, dogfighting and air combat led to a natural interest in the interplay between climbing and turning performance. During the First World War, pilots like Oswald Boelcke and Max Immelmann recognized the necessity of beginning an air sortie from a higher altitude than the enemy.[19] However, it was not until the Second World War that modern Energy Maneuverability Theory came to fruition. Fritz Kaiser, then working in Nazi Germany, published the earliest known derivation in 1944.[20] In the English language, Boyd and Christie published the most important early treatise on this topic.[21] This process required engineers to generate energy maneuverability "sky-maps" (contour plots of performance parameters as functions of speed and altitude) of specific excess power (P_s), climb rate, acceleration, maximum instantaneous turn rate, and maximum sustained turn rate, each plotted as a function of speed and altitude. When comparing two candidate aircraft, Boyd finds superior dog-fighting combat effectiveness where the flight envelope for the aircraft indicates superior usable sustained turn, linear acceleration and/or climb capability as compared against its opponent.[21]

For commercial aircraft, the combined turning-climb problem manifests itself in terms of obstacle clearance.

In 1926, the U.S. congress passed the first laws providing a uniform regulatory framework for aviation.[22] This Act required the Department of Commerce to devise national regulations for aircraft design, construction and operation. Air Commerce Reg. Ch. 7, Sec. 74. "Flying Rules" (from 1929) establishes the rule that outside of initial climb-out or final approach from an airport, aircraft must fly "at a height sufficient to permit of a reasonably safe emergency landing, which in no case shall be less than 1,000 feet" (over congested areas) but no lower "than 500 feet" but does not specify any sort of minimum rate-of-climb capability.[23] Clearly, aircraft are expected to be operated in manner where they clear terrain by at least 500 and preferably 1000 feet.

In 1933, we find the U. S. DEPARTMENT OF COMMERCE, AERONAUTICS BULLETIN No.7-AIRWORTHINESS REQUIREMENTS FOR AIRCRAFT Sec. 76 providing minimum performance guidelines.[24] To be federally certified as airworthy, all passenger carrying airplanes must

(1) Land at a speed not exceeding 65 miles per hour … (where) Landing speed (is) the stalling speed at sea level.

(2) Take-off within 1,000 feet at sea level. [24]

They must also demonstrate minimum climb performance where the climb rate in ft/min in "the first minute after taking off shall [exceed] eight times the theoretical stalling speed in miles per hour, but shall be not less than 300 feet [per minute]."[24]

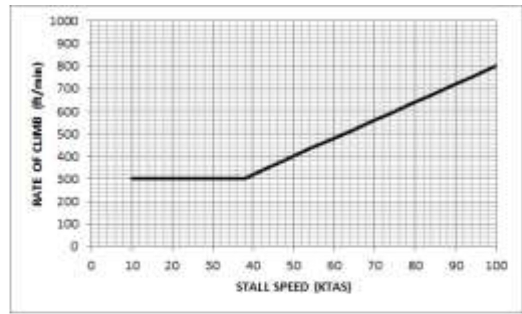

Figure 13 –"Rule of Eight" Minimum Take-Off Climb Performance from early Air Commerce Regulations

The "rule of eight" for climb performance is graphically depicted in Figure 13. As the stall speed increases, so must the climb capability. Because these early regulations do not specify any relationship between the scheduled climb speed and stall it is impossible to define a precise equivalent climb gradient. However, the "rule of eight" effectively specifies a minimum 6.6% climb gradient if the aircraft is scheduled to fly at *1.2 Vs*.

In 1938, the Air Commerce Regulations were more formally codified within Title 14 of the Code of Federal Regulations.[25] Germane regulations for obstacle clearance include the "rule of eight" from 1929:

14 CFR 04.702 Climb (1938) Landplanes shall climb, in feet the first minute after leaving the ground, at least eight times the measured power-on stalling speed (with flaps retracted) in miles per hour, but not less than 300 feet. [26]

Plus the following additional requirements:

14 CFR 04.703 Controllability and maneuverability (1938). All airplanes shall be controllable and maneuverable under all power conditions and at all flying speeds between minimum flying speed and the maximum certified speed. All airplanes shall have control adequate for an average landing at minimum landing speed with power off. [27]

14 CFR 04.723 Emergency ceiling (multi-engine airplanes only) (1938). Multiengine airplanes, … shall be flight tested at the standard weight to determine the usable ceiling which, for this purpose, shall be defined as the highest altitude at which the best rate of climb is 50 feet per minute, with the throttle closed and the ignition switch of one engine on or shut off, whichever results in a lower ceiling. The remaining engine, or engines, shall be operated at not to exceed maximum (except take-off) horsepower. Means shall be provided by which the pilot is suitably informed of such ceiling and the conditions under which it may be realized.[28]

14 CFR 04.731 Climb (1938). The best angle of steady climb and the corresponding speed,

(a) With the throttle of one engine (whichever is critical) closed and the ignition switch on or shut off, whichever results in a lower climb, with the remaining engine(s) operating at not to exceed take-off power, and with the landing gear, if retractable, fully retracted; and

(b) With all engines operating at not to exceed maximum (except take-off) power and with the landing gear, if retractable, fully retracted.[29]

14 CFR 40.232 Aircraft requirements: visual … multi-engine operation over land (1938). Applicant shall show aircraft … to be used on the proposed route or part thereof are capable, with any one engine inoperative, of maintaining level fight with authorized load for the route or part thereof at an altitude of at least 1,000 feet above the highest obstruction to flight on the valley level of such route or part thereof on which the aircraft will be operated. [30]

and

14 CFR 40.250 Aircraft requirements: instrument or over-the-top operation over land (1938) Applicant shall show multi-engine aircraft … to be used on the proposed route, or part thereof, are capable, with any one engine inoperative, of maintaining level flight, with authorized load for the route or part thereof, at an altitude equivalent to 1,000 feet above the highest part of the terrain on the proposed instrument course of the route, or part thereof. [31]

that imply a need to plan all flights (including airport departure and landing) to overfly all possible obstacles with at least 1000-ft of clearance.

By 1952, Title 14 regulations had grown considerably in detail.[32] They specified quantitative minimum flight speeds as well as engine inoperative climb rates:

14 CFR § 4b.114 Take-off speeds. (1952) (b) The minimum take-off safety speed *V2,* in terms of calibrated air speed, shall be selected by the applicant so as to permit the rate of climb required in § 4b.120 (a) and (b), but it shall not be less than:
(1) 1.20 *V8,* for two-engine airplanes,
(2) 1.15 *Vs 1* for airplanes having more than two engines,
(3) 1.10 times the minimum control speed[33]

14 CFR § 4b.117 Temperature accountability (1952). Operating correction factors for take-off weight and take-off distance shall be determined to account for temperatures above and below standard (day). [34]

14 CFR § 4b.120 One-engine–inoperative climb (1952)
(a) *Flaps in take-off position; landing gear extended.* The steady rate of climb without ground effect shall not be less than **50 ft/min**. at any altitude within the range for which take-off weight is to be specified …
(b) *Flaps in take-off position; landing gear retracted.* With the landing gear retracted the steady rate of climb in feet per minute shall not be less than $0.035*VS^2$
(c) *Flaps in en-route position.* The steady rate of climb in feet per minute at any altitude at which the airplane is expected to operate, at any weight within the range of weights to be specified in the airworthiness certificate, shall be determined and shall, at a standard altitude of 5,000 feet and at the maximum take-off weight, be at least $(0.06–0.08/N)*VS^2$ where N is the number of engines installed, with: (1) The landing gear retracted, ... (5) The critical engine inoperative, its propeller stopped, (6) All remaining engines operating at the maximum continuous power available at the altitude. [35]

Figure 14 graphically depicts the implications of the 1952 rules on take-off climb performance. As we saw before with the earlier regulations, as the stall speed increases, so must the climb capability. Because these "mid-century" regulations specified relationships between the scheduled climb speed and the stall speed, we can define a precise equivalent climb gradient.

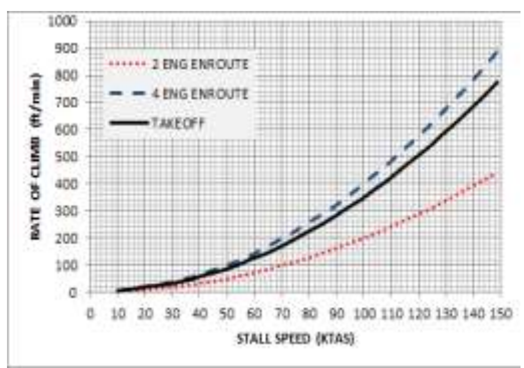

Figure 14 – 1952 era Minimum Take-Off Climb Performance (One-Engine Inoperative)

If we fly both the two-engine second segment take-off climb and the en-route climb at *1.20 * Vs*; and the four-engine second segment take-off climb and en-route climb at *1.15 * Vs*, we can develop the data as found in Figure 15 (overleaf). Here, we see that these regulations require aircraft to have considerable reserve climb capability. A two-engine aircraft with a 100 KTAS stall speed must maintain a 2.9% climb gradient @ *1.20 * Vs* with one engine inoperative during initial second-segment take-off climb and 1.6% climb gradient during en-route climb at 5,000-ft. A larger, four-engine aircraft with a higher 150 KTAS stall speed must maintain a 4.5% climb gradient @ *1.15 * Vs* with one engine inoperative during second segment climb, and a 5.1% climb gradient during en-route climb at 5,000-ft. As we will see, this amount of reserve climb performance is considerably greater than that required today.

In addition, the operations section of the 1952 CFR provides additional dispatch guidance which introduces a 15^o bank angle limitation into departure planning.

14 CFR § 61.215 Take-off limitations to provide for engine failure. (1952) Take-offs shall be made only from such fields in such directions and under such weight limitations that … if the critical engine should fail at any instant after the airplane attains the critical-engine-failure speed, it shall be possible to proceed with the take-off, and attain a height of 50 feet, as indicated by the take-off path data, before passing over the end of the take-off area. Thereafter it shall be possible to clear all 'obstacles either by at least 50 feet vertically, as shown by the take-off path data … In determining the allowable deviation of the flight path in order to avoid obstacles, it shall be assumed that the airplane is not banked before reaching a height of 50 feet, as shown by the take-off path data, and that the maximum bank thereafter does not exceed 15^o.[36]

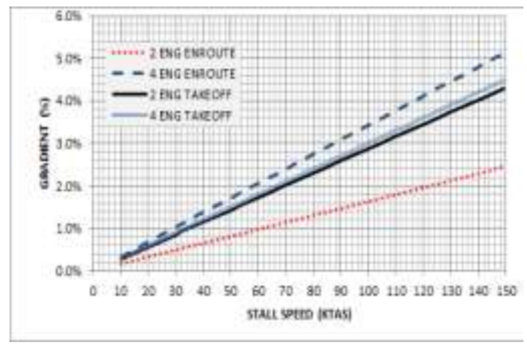

Figure 15– 1952 era Minimum Take-Off Climb Gradient Capability (one-engine-inoperative)

Unfortunately, the agency promulgated this regulation during a time when transparency in decision-making was not particularly valued. Consequently, we have not been able to discover the rationale behind the 15^o bank limitation.

While modern 14 CFR regulations[12] provide guidance to establish minimum flight speeds for take-off and landing (these were summarized above in Table 1), they also provide precise limits to minimum climb performance. The most important regulation for obstacle clearance is:

14 CFR § 25.115 Takeoff flight path. (2015)

(a) The takeoff flight path shall be considered to begin 35 feet above the takeoff surface at the end of the takeoff distance determined in accordance with § 25.113(a) or (b), as appropriate for the runway surface condition.

(b) The net takeoff flight path data must be determined so that they represent the actual takeoff flight paths … reduced at each point by a gradient of climb equal to–

(1) 0.8 percent for two-engine airplanes;
(2) 0.9 percent for three-engine airplanes; and
(3) 1.0 percent for four-engine airplanes.

(c) The prescribed reduction in climb gradient may be applied as an equivalent reduction in acceleration along that part of the takeoff flight path at which the airplane is accelerated in level flight.[37]

This regulation introduces the concept of “net” as opposed to “actual” or “gross” flight path gradients. In a modern 14 CFR 25 certified flight manual, the published climb performance used for obstacle clearance computations has been de-rated in accordance to 14 CFR 25.115.

In addition, 14 CFR specifies the following minimum climb capability with one engine inoperative:

14 CFR 25.121 - Climb: One-engine-inoperative. (2015)

(a) Takeoff; landing gear extended. *(first segment)* In the critical takeoff configuration existing along the flight path (between the points at which the airplane reaches V LOF and at which the landing gear is fully retracted) … the steady gradient of climb must be positive for two-engine airplanes, and not less than 0.3 percent for three-engine airplanes or 0.5 percent for four-engine airplanes, at V LOF and with … the critical engine inoperative

(b) Takeoff; landing gear retracted. *(second segment)* In the takeoff configuration existing at the point of the flight path at which the landing gear is fully retracted, … (1) The steady gradient of climb may not be less than 2.4 percent for two-engine airplanes, 2.7 percent for three-engine airplanes, and 3.0 percent for four-engine airplanes, at V2 with … the critical engine inoperative

(c) Final takeoff. *(fourth segment)* In the en route configuration at the end of the takeoff path *(1500 AGL)* … (1) The steady gradient of climb may not be less than 1.2 percent for two-engine airplanes, 1.5 percent for three-engine airplanes, and 1.7 percent for four-engine airplanes, at VFTO with … the critical engine inoperative

(d) Approach. In a configuration corresponding to the normal all-engines-operating procedure in which VSR for this configuration does not exceed 110 percent of the VSR for the related all-engines-operating landing configuration: … (1) The steady gradient of climb may not be less than 2.1 percent for two-engine airplanes, 2.4 percent for three-engine airplanes, and 2.7 percent for four-engine airplanes, with … the critical engine inoperative[16]

The climb performance required by the later regulations is considerably less restrictive than that found in the 1952 edition. The 1952 regulations would require more than the modern 2.4% second-segment climb gradient on a twin-engine airplane with one-engine-inoperative if the stall speed with take-off flaps deployed exceeds 84 KTAS. Similarly, the 1952 en-route climb performance regulations exceed the modern 1.7% fourth-segment climb gradient minimum for a four-engine aircraft under all circumstances where the flaps-up stall speed exceeds 49 KTAS. Modern jet aircraft typically have stall speeds considerably in excess of 100 KTAS in the cruise as well as take-off configuration.

The modern CFR also includes a "balked landing" all-engines-operating climb limit:

14 CFR § 25.119 Landing climb: All-engines-operating. (2015)
In the landing configuration, the steady gradient of climb may not be less than 3.2 percent… *(all engines operating)* [38]

In addition, 14 CFR 121 provides additional guidance for the aircraft dispatch planner to use when aircraft fly in mountainous terrain.

14 CFR § 121.189 Airplanes: Turbine engine powered: Takeoff limitations. (2015)

(c) No person operating a turbine engine powered airplane certificated after August 29, 1959 (SR422B), may take off that airplane at a weight greater than that listed in the Airplane Flight Manual at which compliance with the following may be shown:

(1) The accelerate-stop distance must not exceed the length of the runway plus the length of any stopway.

(2) The takeoff distance must not exceed the length of the runway plus the length of any clearway except that the length of any clearway included must not be greater than one-half the length of the runway.

(3) The takeoff run must not be greater than the length of the runway.

(d) No person operating a turbine engine powered airplane may take off that airplane at a weight greater than that listed in the Airplane Flight Manual— …

(2) In the case of an airplane certificated after September 30, 1958 … a net takeoff flight path that clears all obstacles either by a height of at least 35 feet vertically, or by at least 200 feet horizontally within the airport boundaries and by at least 300 feet horizontally after passing the boundaries.

…

(f) For the purposes of this section, it is assumed that the airplane is not banked before reaching a height of 50 feet, as shown by the takeoff path or net takeoff flight path data (as appropriate) in the Airplane Flight Manual, and thereafter that the maximum bank is not more than 15 degrees. [39]

and

14 CFR § 121.191 Airplanes: Turbine engine powered: En route limitations: One engine inoperative. (2015)

(a) No person operating a turbine engine powered airplane may take off that airplane at a weight, allowing for normal consumption of fuel and oil, that is greater than that which … based on the ambient temperatures expected en route:

(1) There is a positive slope at an altitude of at least 1,000 feet above all terrain and obstructions within five statute miles on each side of the intended track …

(2) The net flight path allows the airplane to continue flight from the cruising altitude to an airport where a landing can be made … clearing all terrain and obstructions within five statute miles of the intended track by at least 2,000 feet vertically [40]

Thus, the aircraft flight manual expressly disallows flight from airports where either the runway is too short, or the aircraft is too heavy for the weather conditions to meet minimum 14 CFR § 25 engine inoperative climb requirements. In addition, takeoff cannot commence unless the aircraft has enough residual climb performance after engine failure to clear all obstacles in the takeoff flight path. During initial climb out, banks are limited to $\Phi < 15^o$ with a critical engine inoperative.

The FAA has further clarified the meanings of these regulations through Advisory Circular AC-120-91.[41] The FAA holds that:

> "Sections 121.189, 135.379, and 135.398 require that the net takeoff flightpath clears all obstacles by either 35 feet vertically or 200 feet laterally inside the airport boundary, or 300 feet laterally outside the airport boundary. To operate at the required lateral clearance, the operator must account for factors that could cause a difference between the intended and actual flightpaths and between their corresponding ground tracks. For example, it cannot be assumed that the ground track coincides with the extended runway centerline without considering such factors as wind and available course guidance"[41]

Thus, the obstacle clearance window effectively grows in width, as the aircraft heads down range from its lift-off point, especially if the aircraft needs to turn to avoid terrain. Per Figure 16:

> "During departures involving turns of the intended track or when the airplane heading is more than 15 degrees from the extended runway centerline heading, the following criteria apply:
> (1) The initial straight segment, if any, has the same width as a straight-out departure.
> (2) The width of the OAA at the beginning of the turning segment is the greater of:
> (a) 300 feet on each side of the intended track.
> (b) The width of the OAA at the end of the initial straight segment, if there is one.
> (c) The width of the end of the immediately preceding segment, if there is one …
> (3) Thereafter in straight or turning segments, the width of the OAA increases by 0.125D feet on each side of the intended track (where D is the distance along the intended flight path from the beginning of the first turning segment in feet), except when limited by the following maximum width:
> (4) The maximum width of the OAA is 3,000 feet on each side of the intended track."[41]

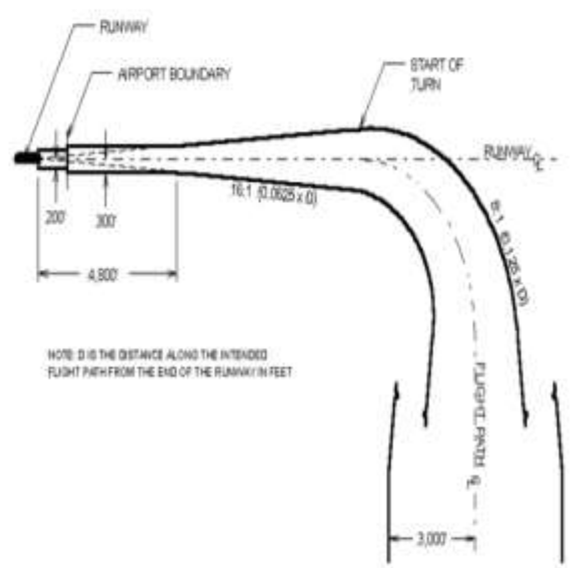

Figure 16 – FAA AC-120-91 Obstacle Clearance Avoidance Area as a function of Downrange Distance

Once the aircraft has attained "the minimum crossing altitude … at a (navigation) fix or the minimum en-route altitude for a route to the intended destination," the en-route rules apply.[41] In an engine-inoperative scenario, the flight path should always clear terrain and obstacles by a minimum of 1,000-ft of vertical height and 5 statute miles (26,400-ft) of horizontal spacing.

In en-route operations, there is no express limitation to bank angle; in practice it must be limited by the stall speed ratio implied by the climb speed schedule. Through AC 120-91, the FAA sets the following policy applicable both the initial departure and en-route operations: 1) no bank is allowed from lift-off until the aircraft attains an altitude of the greater of 50-ft or ½ wingspan above the elevation at the departure end of the runway, 2) $\Phi < 15^o$ from the greater of 50-ft or ½ of the wingspan) to 100-ft altitude; 3) $\Phi < 20^o$ from 100 to 400-ft altitude; 4) $\Phi < 25^o$ for flight above 400-ft.[41] Operators can potentially plan flight at "bank angles greater than the values shown" subject to "additional specific FAA authorization."[41]

Thus, for precise mission planning aircraft designers and operators are motivated to provide precise climb-turn (and possibly descent-turn) performance estimates for all-engines and critical-engine-inoperative cases. In the next section, we will examine how differing commercial aircraft document the performance impact of a climbing-turn.

V. EXAMPLE FLIGHT MANUAL CHARTS

As mentioned in Section III, there is no requirement in the CFR to provide gradient loss for a turn in the AFM. Advisory Circular AC 25.1581-1 provides the guidance that:

"Climb gradient decrements for bank angles up to at least 15 degrees should be provided in the AFM. Consider providing coverage of higher bank angles as appropriate to the expected operation of the airplane."[23]

However, as stated in Section III, advisory circulars provide only guidance, and are neither mandatory nor regulatory. Thus, the gradient loss information found in AFM's is varied in its range and granularity.

Consider the Hawker 800XP AFM,[42] for example, which states: "For a turning climb, up to and including a 15° bank angle, decrease net climb gradient by 1.0 percentage point." This decrement, while simple to apply, seems grossly conservative. Using Equation (29), the worst case gradient occurs at conditions that maximize weight and minimize dynamic pressure. Searching the tabulated V_2 speeds in Ref. 42 showed that the greatest gradient loss occurs at the greatest takeoff flap setting, a sea level pressure altitude, an outside air temperature of 50°C, and a relatively heavy weight of 25,000 lbm. Using the aspect ratio of 7 for the airplane, and assuming (arbitrarily) an Oswald efficiency, *e*, of 0.7, the maximum gradient loss is about -0.55%. This value is much less than the value in the AFM.

In contrast to the simple presentation in the Hawker AFM, consider the gradient loss information found in the Dassault Falcon 2000 AFM.[43] As reproduced in Figure 17, Dassault expresses gradient loss in tabular form as a function of flap and landing gear configuration and bank angle. Given the formulation of Equation (29), one can guess that this table accounts for the worst anticipated weight, outside air temperature, and altitude for each configuration. A quick check of the V_2 speeds in Reference 43 for configuration SF2 shows a worst-case gradient loss at a bank angle of 15° of about -0.54%. This is roughly invariant across the heavier weights and airfield altitudes. Though anecdotal, this single sample is very close to the published value of -0.5%.

GRADIENT LOSS IN TURN (%)

Climb gradients corrections to be applied with bank angle:

BANK ANGLE (°)	CONFIGURATION			
	Landing gear up One engine inoperative			Landing gear down All engines operating
	CLEAN	SF1	SF2	SF3
5	0.05	0.05	0.05	0.05
10	0.1	0.2	0.2	0.2
15	0.2	0.4	0.5	0.4
20	0.4	0.7	0.9	0.6
25	0.6	1.2	1.5	1.0
30	0.9	2.0	2.2	1.6

Figure 17 – A gradient loss table from a Dassault 2000 AFM.[43]

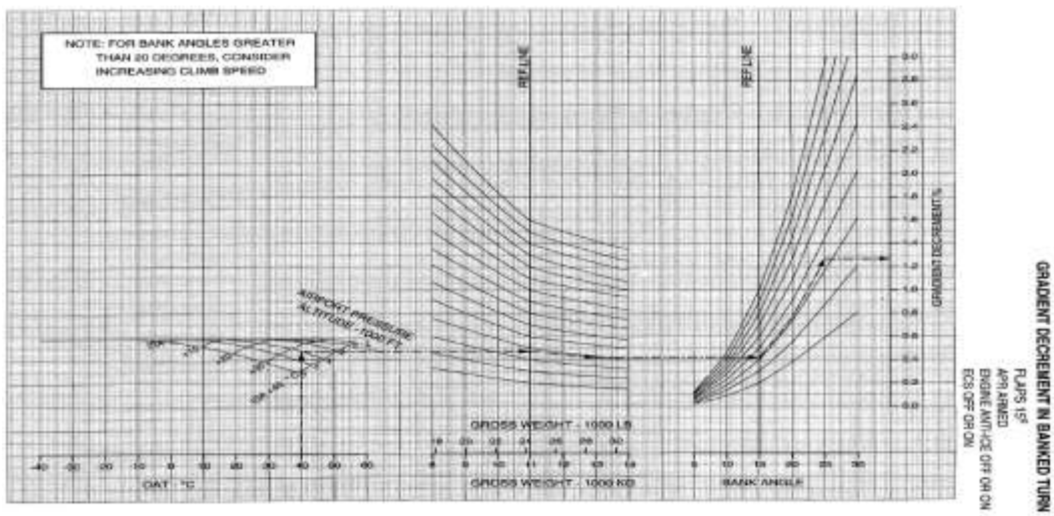

Figure 18 – A complex gradient loss formulation from a Saab 340B flight manual.[44]

For a complex presentation of gradient loss, refer to chart from the Saab 340B AFM[44] in Figure 18. Here the manufacturer has gone so far as to quantify the effects of airspeed, weight on the gradient loss at a given bank angle.

In the case of AFMs that only provide gradient loss for bank angles up to 15°, Advisory Circular AC 120-91[41] prescribes gradient decrements as well as V_2 speed additives for bank angles up to 25°. Figure 19 provides these additional margins, which we believe likely to impose an unnecessarily punitive effect on performance. For instance, for the Hawker 800XP to conduct a 25° bank, the available net gradient would need to be reduced 3%. For reference, recall that the minimum net second segment gradient for a twin-engine airplane is only 2.4%.

BANK ANGLE ADJUSTMENTS

Bank Angle	Speed	'G' Load	Gradient Loss
15°	V_2	1.035	AFM 15° Gradient Loss
20°	V_2+XX/2	1.064	Double 15° Gradient Loss
25°	V_2+XX	1.103	Triple 15° Gradient Loss

Figure 19 – Gradient loss and speed additives for an AFM with bank data limited to 15°. From Advisory Circular AC 120-91.[41]

VI. TRADE STUDIES

In this section, we consider some trade studies to demonstrate the impact of a turning-climb on transport category aircraft. Here, we develop a very simple analytical model to represent an Airbus A320-class twin-engine airliner flown to 14 CFR § 25 standards.

Recall that the CFR no longer calls out climb performance in terms of "rate-of-climb." For obstacle clearance purposes, it describes the required climb performance in terms of the climb gradient. The gradient is the rate-of-climb per distance travelled, expressed in terms of a percentage. When the climb gradient is zero, the airplane neither climbs nor descends. When the climb gradient is 100%, the airplane climbs one foot for every foot it travels down range.

Continuing to follow the small angle approximation, used so-far in this section, the climb gradient may be written as:

$$Gradient \approx \frac{R.O.C.}{V} \approx \frac{T(M,ALT)-D(M,ALT)}{W} = SpExcessThrust \qquad (30)$$

For this simplified model, we define thrust, *T*, in terms of the thrust loading, *T/W*; the wing area, *Sref*, in terms of the wing loading, *W/Sref*; and the drag from a coefficient perspective in terms of the classical parabolic expression, $C_D = C_{D0} + C_L^2 / (\pi AR)$.

The nominal aircraft model has $W/S = 125\ lbf/ft^2$; $T/W_{OEI} = 0.11$; $C_{D0} = 0.0200$; $AR = 8$; $V_{climb} = 150\ KTAS$. We can thus estimate the nominal climb gradient at ~+3.3%. We may also compute the climb gradient for flight at $\Phi = 15^o$; that is at the higher induced drag associated with a load factor, $Nz = 1.035$. We seek to understand the degradation in climb performance associated with banked flight.

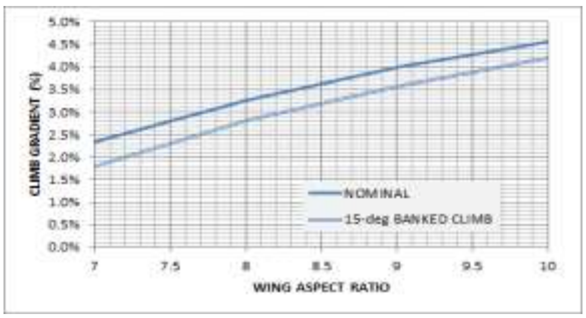

Figure 20– Climb Gradient – Narrow-Body Airliner – Sensitivity to Aspect Ratio

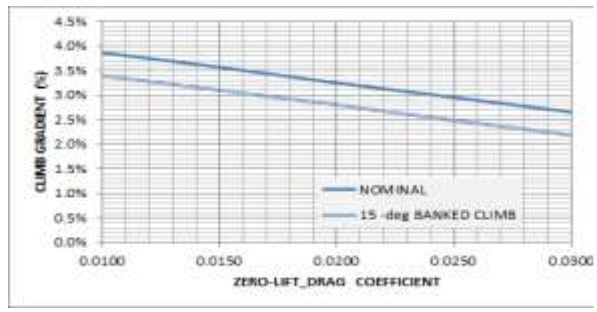

Figure 21– Climb Gradient – Narrow-Body Airliner – Sensitivity to Zero Lift Drag Coefficient

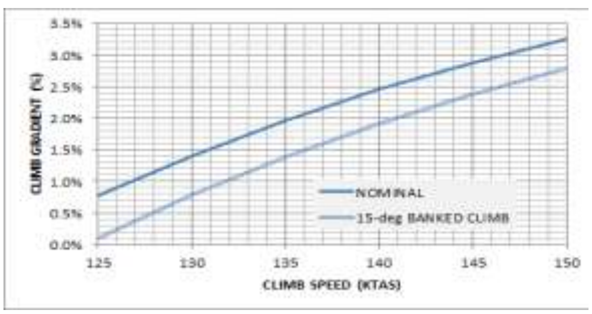

Figure 22– Climb Gradient – Narrow Body Airliner – Sensitivity to Climb Speed

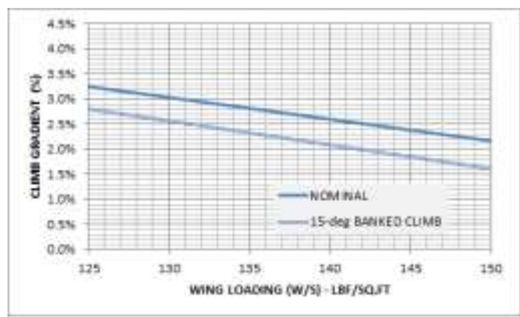

Figure 23– Climb Gradient – Narrow Body Airliner – Sensitivity to Wing Loading

Across Figures 20 through 24 we depict the sensitivity of the climb gradient to various changes in configuration. In each of these studies, only one parameter changes; all others remain constant. In Figure 20, we see how climb gradient improves if the aircraft were to feature a higher aspect ratio (i.e. greater span) wing that develops less induced drag. In Figure 21, we see how the climb gradient decreases as the zero-lift drag coefficient increases. In Figure 22, we see how an increase in climb speed leads to improved climb gradients. In Figure 23, we see how an increase in wing loading *(W/S)* leads to shallower climb gradients. In Figure 24, we see how an increase in thrust loading *(T/W)* promotes steeper climb gradients.

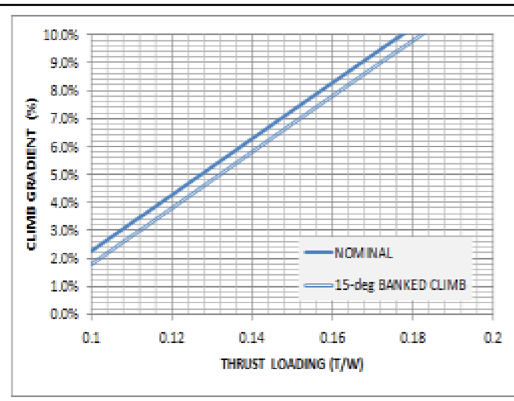

Figure 24– Climb Gradient – Narrow Body Airliner – Sensitivity to Thrust Loading

Taken together, these plots show that a 15° bank angle turn leads to ~0.5% degradation in climb gradient. Because regulation 14 CFR § 25.115[37] requires only a 0.8% decrement in climb performance for take-off obstacle clearance planning, more steeply banked flight without any other form of climb gradient correction may lead to an optimistic prediction of altitude gain. This poses a very real safety-of-flight hazard when scheduling departures where a complex, turning flight path is needed to clear obstacles.

Once en-route climb speeds have been attained, the aircraft is scheduled to fly at a minimum stall speed margin of *1.18 Vs* (corresponding to a potential maximum load factor $Nz=1.39$). During fourth segment and en-route climb (above 400-ft), the Φ=15° bank limitation need no longer apply; the aircraft can bank more steeply. In Figure 25, we examine the change in climb gradient as a function of load factor. The design variations consist of: 1) a smaller wing representing W/S=150 lbf/ft^2 instead of W/S=125 lbf/ft^2, 2) a larger engine where the one-engine-inoperative thrust loading is T/W=0.13 as opposed to T/W=0.11, 3) an increase in zero-lift drag where C_{D0} rises to 0.0250 from 0.0200, 4) lower induced drag associated with AR=9 as opposed to AR=8, 5) an increase in V_2 speed from 150 KTAS to 160 KTAS; and 6) a 10% reduction in weight keeping wing size, engine size and C_{Lmax} constant.

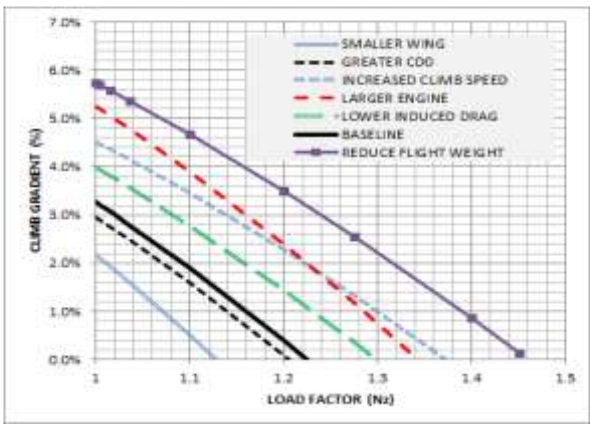

Figure 25– Climb Gradient Sensitivity to Load Factor (various design changes)

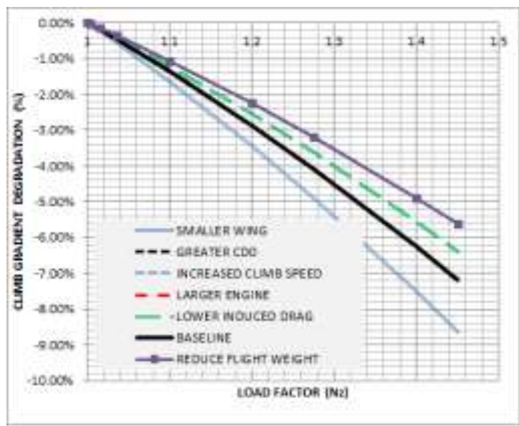

Figure 26– Climb Gradient Degradation - Sensitivity to Load Factor

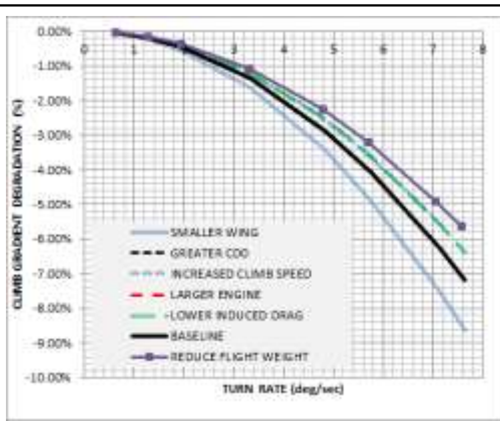

Figure 27– Climb Gradient Degradation - Sensitivity to Load Factor

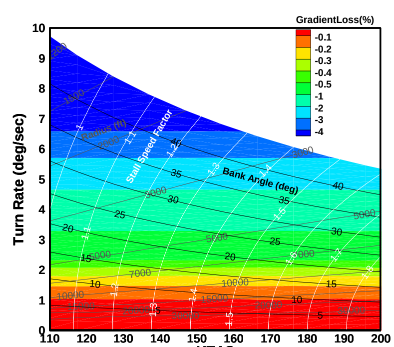

Figure 28– Expression of gradient loss in "doghouse-plot" format

A take-away from Figure 25 is that the slopes of climb gradient as a function of load factor differ for the various perturbed configurations. This is brought into greater relief in Figure 26; the same data is shown but expressed in terms of climb gradient degradation from Nz=1 flight. For some parameters, such as increased climb speed, the response is clearly non-linear. Figure 27 presents the same trade studies expressed in terms of climb gradient degradation from Nz=1 flight but now expressed as a function of turn rate. Thus, the maximum permissible load factor for climb/turn becomes extremely context dependent; the maximum turn rate can neither stall the airplane, nor excessively degrade the climb performance.

Figure 28 takes turn radius, turn rate, and bank angle and superimposes them with the gradient loss in the classic "doghouse" format.

VII. CONCLUSIONS

In this work, we developed an analytical formulation for gradient loss in turning and climbing flight that is based on the small-angle approximation of excess thrust. We show that this formulation gives gradient loss which is purely a function of the increase in induced drag associated with the increase in lift required for a coordinated turn. This formulation is rooted in classic energy-maneuverability theory. Its output can be expressed graphically in the form of a traditional doghouse plot, in which the relationship between turn radius, turn rate, and bank angle can be superimposed with the available stall margin and climb capability of the airplane. The formulation does not account for any side-force incurred in balancing the asymmetric thrust in the engine-out configuration.

A review of the regulations relating to climb gradient reveals that the rules are not immutable as engineers often think, and in fact, the regulatory environment has changed considerably over the years. The current requirements in the CFR's and the guidance in the advisory circulars are not consistent as they pertain to gradient loss.

References

[1] http://www.boeing.com/commercial/safety/manufacturers_role.html#controlledFlight (accessed June 29, 2011)
[2] Shaw, R.L. *Fighter Combat: Tactics and Maneuvering*, Naval Institute Press, Annapolis, MD, 1985.
[3] Anon., *Legislative History of the Air Commerce Act of 1926*, United States Government Printing Office, Washington, DC, 1928.
[4] De Bothezat, G., "Theory of the Steady Motion of an Airplane," NACA TR-97, 1920.
[5] Oswald, W.B., "General Formulas and Charts for the Calculation of Airplane Performance," NACA TR-408, 1936.
[6] Millikan, C. B., *Aerodynamics of the Airplane*, Wiley, 1941.
[7] Jones, B.M., *Elements of Practical Aeronautics*, 2nd Edition, Wiley, New York, 1939.
[8] Dwinnell, J.H., *Principles of Aerodynamics*, McGraw-Hill, 1949.
[9] Anon., *Jet Transport Performance Methods*, Boeing Flight Operations Engineering Training Document D6-1420, 7th Edition, Boeing, Seattle, WA, May 1989.
[10] *See* 14 C.F.R. § 25.333, "Flight Maneuvering Envelope" (2015).
[11] *See generally* 14 C.F.R. "Aeronautics and Space" (2015).
[12] *See* 14 C.F.R. § 25.149 "Minimum Control Speed" (2015).
[13] *See* 14 C.F.R. § 91.117 "Aircraft Speed" (2015).
[14] *See* 14 C.F.R. § 25.107 "Takeoff Speeds" (2015).
[15] *See* 14 C.F.R. § 25.125 "Landing" (2015).
[16] *See* 14 C.F.R. § 25.121 "Climb: One Engine Inoperative" (2015)
[17] *See* 14 C.F.R. § 25.123 "En Route Flight Paths" (2015)
[18] Advisory Circular (AC) 25.1581-1, *Airplane Flight Manual,* United States Department of Transportation, Federal Aviation Administration (FAA), Change 1, 16 October 2012
[19] Bertin, J.J., and Cummings, R.M., *Aerodynamics for Engineers*, Pearson Prentice-Hall, 2009.
[20] Merritt, S.R., Cliff, E.M. and Kelley, H.J., "Energy-modelled Climb and Climb-Dash – The Kaiser Technique," Automatica, Vol 21, No. 3, pp. 319-321, 1985.

[21] Boyd, J. R., Christie, T. P., and Gibson, J. E., "Energy Maneuverability," APGD-TDR-64-28, Vol. 1, March, 1966.
[22] Air Commerce Act of 1926, Pub. L. No. 69-254, ch. 344, 43 Stat. 568, 568, (1926).
[23] Anon., "Air Commerce Regulations, Aeronautics Bulletin No.7," Dept. of Commerce – Aeronautics Branch, U.S. GPO Washington , effective Dec. 31, 1926.
[24] Anon., "AERONAUTICS BULLETIN No.7-AIRWORTHINESS REQUIREMENTS FOR AIRCRAFT." Dept. of Commerce – Aeronautics Branch, U.S. GPO Washington, 1933.
[25] *See generally* 14 C.F.R. "Civil Aviation" (1938)
[26] *See* 14 C.F.R. § 04.702 "Climb" (1938).
[27] *See* 14 C.F.R. § 04.703 "Controllability and Maneuverability" (1938).
[28] *See* 14 C.F.R. § 04.723 "Emergency Ceiling" (1938).
[29] *See* 14 C.F.R. § 04.731 "Climb" (1938).
[30] *See* 14 C.F.R. § 40.232 "Aircraft Requirements: Visual … Multi-Engine Operation over Land" (1938).
[31] *See* 14 C.F.R. § 40.250 "Aircraft Requirements: Instrument or Over-the-Top Operation over Land" (1938).
[32] *See generally* 14 C.F.R. "Civil Aviation" (1952)
[33] *See* 14 C.F.R. § 4b.114 "Take-Off Speeds" (1952)
[34] *See* 14 C.F.R. § 4b.117 "Temperature Accountability" (1952)
[35] *See* 14 C.F.R. § 4b.114 "One-Engine-Inoperative Climb" (1952)
[36] *See* 14 C.F.R. § 61.215 "Take-Off Limitations to provide for Engine Failure" (1952)
[37] *See* 14 C.F.R. § 25.115 "Take-Off Flight Path" (2015)
[38] *See* 14 C.F.R. § 25.119 "Landing Climb: All Engines Operating" (2015)
[39] *See* 14 C.F.R. § 121.189 "Airplanes: Turbine engine powered: Takeoff Limitations" (2015)
[40] *See* 14 C.F.R. § 121.191 "Airplanes: Turbine engine powered: En-Route Limitations" (2015)
[41] Advisory Circular (AC) 120-91, *Airport Obstacle Analysis*, United States Department of Transportation, Federal Aviation Administration (FAA), May 5, 2006
[42] Anon., Hawker 800XP FAA Approved Airplane Flight Manual, P/N 140-590032-005A4, Rev. A4, Hawker Beechcraft Corporation, Wichita, KS, Mar. 2007.
[43] Anon., Dassault Falcon 2000EX EASy, 2000DX, 2000LX, 2000S, 2000LXS Airplane Flight Manual (AFM), DGT88898, Rev. 17, Dassault Aviation, Saint Cloud, France, Mar. 2013.
[44] Anon., Saab 340B LFV Approved Airplane Flight Manual, Doc. No: 72LKS6174, Rev. 18, Saab Aircraft AB, Linkoping, Sweden, Apr. 2015.