A History and Commentary on Thrust/Drag Bookkeeping

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This paper reviews the history of thrust / drag bookkeeping. It clarifies the mathematical basis behind common definitions of *uninstalled thrust* in light of air vehicle thrust-drag accounting methodology. This physics-based control volume approach to propulsive force accounting discovers differences in the application of control volume methods from different classic sources. We find that certain combinations of thrust and installation corrections, that are seemingly tempting to use, lead to force accounting mistakes that can propagate when engineers use legacy aerodynamics and propulsion codes.

Nomenclature

Variables:

A	=	Area
$\dot{m}$	=	Mass flow rate
q	=	Arbitrary quantity per unit mass
Q	=	Arbitrary quantity
ρ	=	Fluid Density
u, V	=	Fluid Velocity
F	=	Force
T	=	Thrust
τ	=	Viscous Stress
P	=	Pressure Stress
S	=	Control Surface
$\tilde{V}$	=	Control Volume
f	=	Fuel Mass Flow Ratio
D	=	Drag
C	=	Correction Coefficient

Subscripts:

c	=	Free Stream Capture
e	=	Exhaust Plane
∞	=	Free Stream
1	=	Inlet Plane
s	=	Stream tube
f	=	fuel
x	=	x-direction

I. Introduction

THE net force delivered to a flight vehicle is not difficult to conceptualize at first glance. Tricky nuances appear only after developing analytic models of systems. Fundamentally, air vehicle performance should be agnostic to force-accounting standards so long as it captures every force only once; we realize that the semantics of a computational model cannot affect the real world. Since inaccurate computational models can lead to faulty design, it is vital to capture consistent physics when developing predictive models. If there is a substantial error of omission between aerodynamics and propulsion, the vehicle will be underpowered. Those who do not know history, become doomed to repeat it.

We abundantly recognize that the idea that two control volumes with identical freestream conditions and identical flow exit conditions produce substantially different thrust estimates is controversial. AIAA Fellow

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and Propulsion expert Paul Bevilaqua exclaimed "F = M A is a law of Physics. If F ≠ M A in a Thrust Equation there has to be an error somewhere!" [1] Yet we realize that the propulsion system is but one of several systems in a flight vehicle. There is usually a unique (often independent) model created or adapted to model propulsion. Engineers use the output from such models to predict air vehicle performance. While there are infinite ways to analyze a propulsion system and predict possible performance, many are ultimately impractical.

In this paper we detail a few of the most common definitions and derivations of thrust. We provide a summary of aero-propulsion force accounting and corrections associated with engine installations. Last, we discuss key differences in the methods for calculating *uninstalled thrust*, detail a potential missing correction, and propose a couple general solutions to aero-propulsive force accounting.

There is no one "correct" way to establish thrust/drag force accounting. All approaches can be technically sound so long as they correctly capture the physics at work. At the same time, the engineering community is well known to define and use "legacy methods" for decades or even centuries. When convention loses its association with proper rigor, its meaning fades. Thus, it becomes easy to misapply methods and compromise inherent physical accuracy. Even when history doesn't repeat, the present often rhymes with the past.

II. Common Definitions and Derivations of *Uninstalled Thrust*

In this paper, we derive thrust in several ways which are all perfectly acceptable for the control volume they apply to. In this section we will first concern ourselves with the *uninstalled thrust* produced by a propulsion system. In latter sections we will examine common terminology and corrections used to estimate the *installed thrust* produced by the propulsion system.

A. A 1-D Control Volume Analysis is the Basis of the "Classical" Free-Stream-to-Tail Thrust Equation

Many sources, including popular textbooks and manuals, have the same equation and the same inferred definition of thrust:

$$T = \dot{m}_i[(1+f)V_e - V_\infty] + (P_e - P_\infty)A_e \tag{1}$$

That is that thrust is the sum of the change in axial momentum flux of the flow going through the engine and the net pressure difference between freestream static pressure, P_∞, and the static pressure at the engine exhaust, P_e.

In our experience, not all sources rigorously derive the equation for thrust. If the control volume analysis isn't carefully described, it is easy for propulsion to omit significant forces which other disciplines also ignore.

In this work, we base our discussion on Arizona State University's propulsion faculty member Professor Werner Dahm's Air Breathing Propulsion class notes [2]

To begin, consider the "Classic" propulsion system control volume; see FIGURE 1 as well as Refs [3-9]. We call this control volume the "Swallowed Flow Control Volume" or *CV1*.

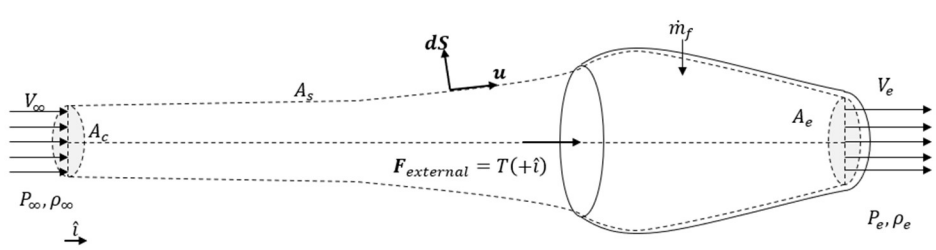


FIGURE 1. The "Swallowed Flow Control Volume", *CV1*

CV1 concerns itself with the flow that passes through the propulsion system, the "propulsion stream tube." It is a "free-stream-to-tail" control volume that begins with free-stream conditions passing through a capture area A_c and ending at the exit plane of the propulsion system A_e. The surface A_s is defined by the flow streamlines that pass through the propulsion system separating the internal and external flow. This resembles and works well for a podded engine which will remain the generality.

The body composed of solid lines in *CV1* does not necessarily represent a nacelle structure but rather represents the casing that encloses the propulsion system including the swallowed flow. This body's internal surface is wetted by the internal flow but is not included within the control volume definition. This applies to surfaces like the casing inner wall on a turbojet which houses the propulsion system hardware and swallowed flow. In contrast, the control volume considers "internal" propulsion system components (rotor blades, stator blades, fuel injectors, etc.) internal to the defined control volume. However, the control volume diagram does not depict such complicated hardware.

To estimate uninstalled thrust, we begin with the generic "Reynolds Transport Theorem" in Eq. (2) which we use to apply to mass and momentum conservation. First, we discard the unsteady term for this analysis as the focus is on steady state operation. This equation is generic concerning some arbitrary quantity Q within a specified volume of fluid. The rate of change of that quantity $\dot{Q}_{net}$ relates to the time rate of change of Q per unit volume, q, within the control volume V and the flux of Q across the control surface S using the fluid velocity vector component normal to the control surface $\boldsymbol{u} \cdot \boldsymbol{dS}$. This is the foundation of control volume analysis which itself is an accounting problem rooted in conserving a given quantity. Note that $\boldsymbol{dS}$ is the differential control surface normal vector and $\boldsymbol{u}$ is the fluid velocity vector.

$$\frac{d}{dt}\int_{\tilde{V}} (q)dV + \int_{S} (q)\boldsymbol{u} \cdot \boldsymbol{dS} = \dot{Q}_{net} \tag{2}$$

Apply this to mass conservation in (3) where the quantity q is the mass per unit volume, or density ρ. Recall that we neglected the unsteady term in this analysis.

$$\frac{d}{dt}\int_{\tilde{V}} (\rho)dV + \int_{S} (\rho)\boldsymbol{u} \cdot \boldsymbol{dS} = \dot{m}_{net} \tag{3}$$

This equation expands to (4) when applied to *CV1* and the three primary surfaces depicted above.

$$\int_{A_c} (\rho)\boldsymbol{u} \cdot \boldsymbol{dS} + \int_{A_e} (\rho)\boldsymbol{u} \cdot \boldsymbol{dS} + \cancel{\int_{A_s} (\rho)\boldsymbol{u} \cdot \boldsymbol{dS}} = \dot{m}_f \tag{4a}$$

$$-\rho_\infty V_\infty A_c + \rho_e V_e A_e = \dot{m}_f \tag{4b}$$

Note the definitions of mass flow rate into and out of the stream tube are $\dot{m}_i \equiv \rho_\infty V_\infty A_c$ and $\dot{m}_e \equiv \rho_e V_e A_e$, respectively. Also realize that the mass flow through the surface A_s is zero by the definition of a streamline (the constructive element of the stream tube boundary). This simplifies to (5).

$$-\dot{m}_i + \dot{m}_e = \dot{m}_f \tag{5}$$

Next, we define the fuel mass flow ratio as $f \equiv \frac{\dot{m}_f}{\dot{m}_i}$ to transform equation (5) to (6):

$$\dot{m}_e = \dot{m}_i(1 + f) \tag{6}$$

Now, we apply the transport theorem to conserve momentum in the x-direction ("x-momentum"). An axisymmetric control volume will not have any net force in the other directions, especially if the mass flow into and out of the system is along the same longitudinal axis. If present, these forces are comparatively small and not very important for this analysis; they become important when considering thrust vectoring

nozzles which is not the goal here. Now the quantity Q is momentum which means that $\dot{Q}$ is a force and q is volume specific momentum. Again, neglect the unsteady term to reach (7) for the full vector equation. The rest of this analysis will focus on the x-direction only.

$$\int_S (\rho \boldsymbol{u})\boldsymbol{u} \cdot d\boldsymbol{S} = \boldsymbol{F}_{net} \tag{7}$$

This general equation states that the net force on the fluid control volume $\boldsymbol{F}_{net}$ is equal to the momentum flux through the boundaries of the control volume. This equation alone cannot determine any propulsive force as this requires more definition of terminology. The force term $\boldsymbol{F}_{net}$ includes a few constituent parts: 1) the "surface" force and 2) the "internal" force. The "surface" force is the result of integrating the full stress tensor in (8) on every applicable surface of the control surface $\boldsymbol{S}$. Recall that τ_{ij} is the viscous stress tensor, P is the local pressure, and δ_{ij} is the Kronecker delta matrix allowing pressure to act normal to the surface.

$$\sigma_{ij} = \tau_{ij} - P\delta_{ij} \tag{8}$$

Thus, the "surface" force (in the x-direction) is:

$$(F_x)_{surface} = \int_S \tau_{xj} dS_j - \int_S P\delta_{xj} dS_j \tag{9}$$

The second force, the "internal" force, is a replacement for the integration of the full stress tensor σ_{ij} on any surface that is internal to the control volume. This is a subtle part of control volume analysis where the body of interest (sometimes an airfoil, sometimes a propulsion system component) is *included* inside the control volume boundary and the force on the solid body is interpreted as the "internal" force in place of carrying out a tedious integration along every rotor blade, stator blade, fuel injector, etc. inside the volume.

For a body within this defined fluid volume, referred to as a "control volume", there are two commonly used options to account for forces on that body:

1) complete a direct stress integration on every surface exposed to flow or
2) include the entire body within the control volume and account for those surface stresses in an internal force term.

The force on the body does not change, this is just a different way to account for it.

A great representation of this is found in Lozano's Fluid Mechanics public lecture notes [10] where the first image in FIGURE 2 demonstrates a complete accounting of the present body in surface stress integration, and the second image demonstrates the accounting for the surface integration as a reaction force which eventually leads to the computation of drag from a momentum decrement in the wake. **We note that this common derivation contains an inherent error, general flows do not represent a flow with constant static pressure conditions on all surfaces of the control volume. Inviscid subsonic flows maintain constant total pressure outside of the viscous wake, but not within the viscous wake.**

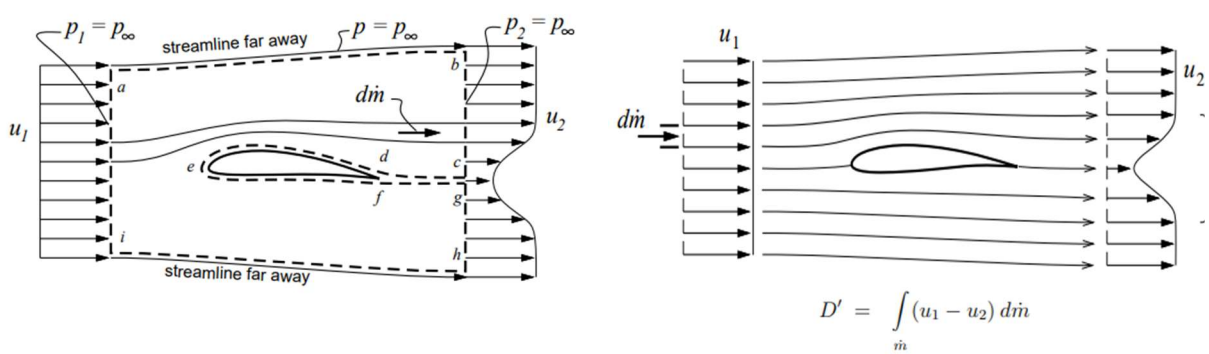


FIGURE 2. Two Classic Approaches to Surface Influences in Control Volumes [10]

The same approach applies to the conventional propulsion control volume where the integration of the stress tensor is completed on all exterior surfaces (A_c, A_s, A_e) and all surface stress integration internal to

the volume (including all internal propulsion system components) is represented as the "internal force". For this specific analysis, that internal force is *defined* as the net propulsive "thrust" from the propulsion system. This is an important point concerning this analysis; thrust is *defined* as the net force due to integration of the stress tensor on all surfaces *internal* to the propulsion system.

$$T \equiv (F_x)_{internal} \tag{10}$$

This thrust force is not the actual force transmitted to the flight vehicle from the propulsion system because this is the analysis to determine *uninstalled thrust*. This same *uninstalled thrust* is the force holding this specific control volume stationary relative to the moving propulsion system (and flight vehicle) which is an application of D'Alembert's Principle which converts a dynamic problem to a static one [11] (this is also the $\boldsymbol{F_{external}}$ in the *CV1* diagram). Installation effects will change the thrust delivered to the flight vehicle and the selection of thrust definition is a critical part in determining what *installed thrust* corrections the analysis should include.

Simplify Equation (11) to get (12).

$$\int_{A_c} (\rho u)\boldsymbol{u}\cdot\boldsymbol{dS} + \cancel{\int_{A_s} (\rho u)\boldsymbol{u}\cdot\boldsymbol{dS}} + \int_{A_e} (\rho u)\boldsymbol{u}\cdot\boldsymbol{dS} = (F_x)_{surface} + T \tag{11}$$

$$\int_{A_c} (\rho u)\boldsymbol{u}\cdot\boldsymbol{dS} + \int_{A_e} (\rho u)\boldsymbol{u}\cdot\boldsymbol{dS} = \int_S \tau_{ij} dS_j - \int_S P dS_x + T \tag{12}$$

The left side of (12) resolves to (13) just like the mass conservation analysis.

$$\int_{A_c} (\rho u)\boldsymbol{u}\cdot\boldsymbol{dS} + \int_{A_e} (\rho u)\boldsymbol{u}\cdot\boldsymbol{dS} = -\rho_\infty V_\infty A_c V_\infty + \rho_e V_e A_e V_e = -\dot{m}_i V_\infty + \dot{m}_e V_e \tag{13}$$

The right side of equation (12) requires further definition of the "surface" force that we initially defined in (9). This has two parts, but first consider the viscous stresses and note that this integration on the forward and aft external surfaces is zero due to the uniform velocity profiles and no velocity gradient in the x-direction. This is a common simplification of uniform velocity profiles that is not expected to induce significant error, but it is still a simplification.

$$\int_S \tau_{xj} dS_j = \cancel{\int_{A_c} \tau_{xj} dS_j} + \int_{A_s} \tau_{xj} dS_j + \cancel{\int_{A_e} \tau_{xj} dS_j} \tag{14}$$

The result from (14) is then (15).

$$\int_S \tau_{xj} dS_j = \int_{A_s} \tau_{xj} dS_j \tag{15}$$

Now consider the pressure stresses of the "surface" force in (16).

$$\int_S P dS_x = \int_{A_c} P dS_x + \int_{A_s} P dS_x + \int_{A_e} P dS_x \tag{16}$$

This reduces to (17) for the inlet and exit plane noting that the normal vector on A_c is in the $-\hat{\imath}$ and the normal vector on the A_e surface is in the $+\hat{\imath}$ direction. This assumes constant pressure profiles along those surfaces which is an approximation but not necessarily a poor one. This is most often true for the capture area and a reasonable approximation for the exit area.

$$\int_S P dS_x = -P_\infty A_c + \int_{A_s} P dS_x + P_e A_e \tag{17}$$

Now, substitute the results of (13), (15), and (17), into (12) which gives (18).

$$T + \int_{A_s} \tau_{xj} dS_j - \int_{A_s} P dS_x = \dot{m}_i[(1+f)V_e - V_\infty] + (P_e A_e - P_\infty A_c) \tag{18}$$

This is as far as the analysis can go using *CV1* and this definition of thrust, all of which is perfectly acceptable. To complete the analysis and isolate the thrust term in a simple algebraic equation, as opposed to the integral one, we must then consider a second control volume

This "Spilled Flow Control Volume," or *CV2*, only includes flow that does *not* enter the propulsion system. It has a different inlet and outlet surface (A_1 and A_2), a cylindrical surface A_3, and the same stream tube surface A_s as *CV1* but with the opposite orientation (pointing "inward"); see FIGURE 3 (overleaf). Note that the conditions at A_1 and A_2 are identical to *CV1*. The goal of this control volume is to eliminate the integral terms in (18) which we cannot evaluate directly.

The choice to make the conditions at A_1 and A_2 the same is a conscious one that effectively neglects any installation effects and isolates the theoretical uninstalled engine performance. This may appear odd; a real engine will not see this exact flow field (specifically the velocity gradients in the A_2 plane due to the external surface of whatever body houses the propulsion system). Neglecting them here makes for a simple analysis of *uninstalled thrust*; the installation effects that are affected by and produce those changes in the flow fields can be accounted for elsewhere.

We note important features of this control volume. The "internal" force term discussed earlier is not present even though there is clearly some flight vehicle hardware within the control volume diagram which may represent parts of a nacelle, fuselage, or other external body. This means that any solid body not expressly within *CV1* is **not** directly accounted for in the analysis of *CV2*. Tradition holds this as acceptable in the search for *uninstalled thrust*.

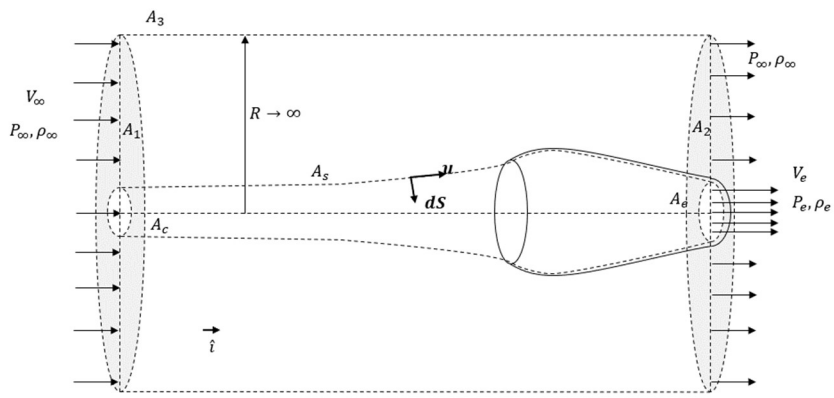


FIGURE 3. The "Spilled Flow Control Volume", *CV2*

In the "Spilled Flow Control Volume," the mass flux and associated force due to flow across A_3 is nonzero and accounts for the possible difference between the capture area (A_c) and exit flow area (A_e) considering both mass and momentum conservation. Analytically, this means the cylindrical surface radius must extend to infinity and use mass conservation to complete the *x*-momentum conservation. Lastly, the surface A_s in *CV2* is *opposite* to that in *CV1* such that the following in (19) is true.

$$(\boldsymbol{dS})_{A_s,alternate} = -(\boldsymbol{dS})_{A_s,original} \tag{19}$$

The result of this analysis in (20) is the "puzzle piece" needed to complete the analysis from *CV1*.

$$\int_{A_s} \tau_{xj} dS_j - \int_{A_s} P dS_x = P_\infty (A_e - A_c) \tag{20}$$

Thus, when we substitute (20) into (18) the result is (21) which we refer to as the "Classical Thrust Equation" as it is the equation that most textbooks present:

$$T = \dot{m}_i[(1+f)V_e - V_\infty] + (P_e - P_\infty)A_e \tag{21}$$

We note that many textbooks and references establish Equation (21) as the thrust equation even if the derivation is different or not as detailed. Conversely, if an analysis of a propulsion control volume reaches a different conclusion with an analogous CV and variable set, then it means something different with different implications.

B. A Second 1-D Control Volume Analysis Forms the Basis of the Inlet-to-Tail Thrust Equation

There is a second interpretation of thrust worthy of discussion. It hinges on a difference in the *definition* of thrust. Since there is nothing in physics that decides what we *must* call "thrust" from a control volume perspective, we can equally define a control volume that physically matches the hardware of a jet engine. The reader should always remember that we would like to define thrust in such a way that it represents the force transferred to the flight vehicle.

Consider the control volume in FIGURE 4, which we call the "Truncated Control Volume" or *CV3*. This is identical to *CV1* except the front portion is neglected (or truncated). The inlet plane of the control volume is now the entry plane of the propulsion system that we have arbitrarily defined. This surface A_1 is best to fix by geometry rather than using the division between flow that passes through and around the propulsion system which can vary drastically through the flight envelope. The exit plane is identical to that of *CV1* and the stream-tube surface is the same except for the flow ahead of the engine entry plane. The surfaces internal to the control volume are identical to *CV1* as well. A variation of this control volume shows up in [5].

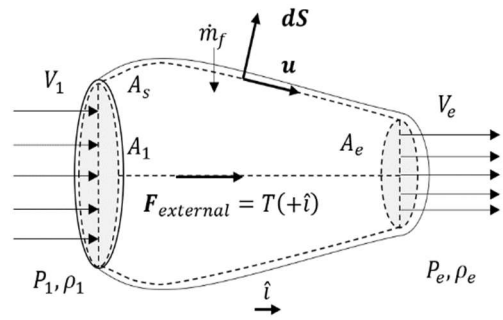


FIGURE 4. The "Truncated Control Volume", *CV3*

The same analysis used in *CV1* is perfectly valid for this control volume using analogous variables for the inlet plane of the control volume and will have an analogous result. The main issue from this analysis is that the surface integral terms ($\int_{A_s} \tau_{xj} dS_j - \int_{A_s} P dS_x$) are not easy to solve for using a Spilled Flow Control Volume like *CV2*. However, that analysis uses one definition of thrust that does not have to be the same definition for this analysis.

Consider the *x*-momentum conservation for *CV3* noting that the mass conservation is **exactly** the same as *CV1*.

$$\int_S (\rho u)\, \boldsymbol{u} \cdot \boldsymbol{dS} = (F_x)_{surface} + (F_x)_{internal} \tag{22}$$

In (22), the term $(F_x)_{internal}$ is identical to the thrust from the classical thrust equation which is the net force due to the integration of σ_{ij} on all surfaces *internal* to the CV (same "internal" surfaces here). In *CV3*, the surface A_s *only* touches parts of the flight vehicle wetted by swallowed flow that contain the "internal" propulsion system components. These surfaces (like the internal inlet wetted area, internal nozzle surface, etc.) are not "internal" surfaces in a control volume context but are surfaces that contact internal propulsion system *flow* (relating to $(F_x)_{surface}$ instead of $(F_x)_{internal}$). This is a subtle distinction that it is easy to lose in semantics, but it is very important to include. The stresses (σ_{ij}) on A_s are not only acting on the fluid passing through the CV (which is why we include it in the CV analysis) but also acting on the flight vehicle surfaces outside of the defined control volume. So, when applying Newton's 3rd law, we choose to define the net propulsive force (thrust, *T*) to include the stresses on A_s in addition to the thrust from the classic thrust equation.

The definition of thrust for *CV3* is in (23) below; note the difference from (10) for *CV1*.

$$T \equiv (F_x)_{internal} + \int_{A_s} \tau_{xj} dS_j - \int_{A_s} P dS_x \tag{23}$$

With that definition, consider how it works in the *x*-momentum equation focusing on the force terms in (24).

$$(F_x)_{surface} + (F_x)_{internal} = \int_S \tau_{xj} dS_j - \int_S P dS_x + (F_x)_{internal} \tag{24}$$

Use the results of (24) and analogous results from (25) and (26) from *CV1* to simplify the integrals in (27).

$$\int_S \tau_{xj} dS_j = \int_{A_s} \tau_{xj} dS_j \tag{25}$$

$$\int_S P dS_x = \int_{A_s} P dS_x - P_1 A_1 + P_e A_e \tag{26}$$

$$(F_x)_{surface} + (F_x)_{internal} = \int_{A_s} \tau_{xj} dS_j - \int_{A_s} P dS_x + (F_x)_{internal} + (P_1 A_1 - P_e A_e) \tag{27}$$

Notice that thrust is in the right-hand side of (27).

$$(F_x)_{surface} + (F_x)_{internal} = T + (P_1 A_1 - P_e A_e) \tag{28}$$

Like the *CV1* analysis, simplify the momentum flux terms noting that $\dot{m}_i \equiv \rho_1 V_1 A_1 = \rho_\infty V_\infty A_c$.

$$\int_{A_c} (\rho u)\boldsymbol{u} \cdot \boldsymbol{dS} + \cancel{\int_{A_s} (\rho u)\boldsymbol{u} \cdot \boldsymbol{dS}} + \int_{A_e} (\rho u)\boldsymbol{u} \cdot \boldsymbol{dS} = -\rho_1 V_1 A_1 V_1 + \rho_e V_e A_e V_e = -\dot{m}_1 V_1 + \dot{m}_e V_e \tag{29}$$

Now substitute (28) and (29) back into (22) and rearrange to get what we call the "Alternate Thrust Equation" in (30).

$$T = \dot{m}_i[(1+f)V_e - V_1] + (P_e A_e - P_1 A_1) \tag{30}$$

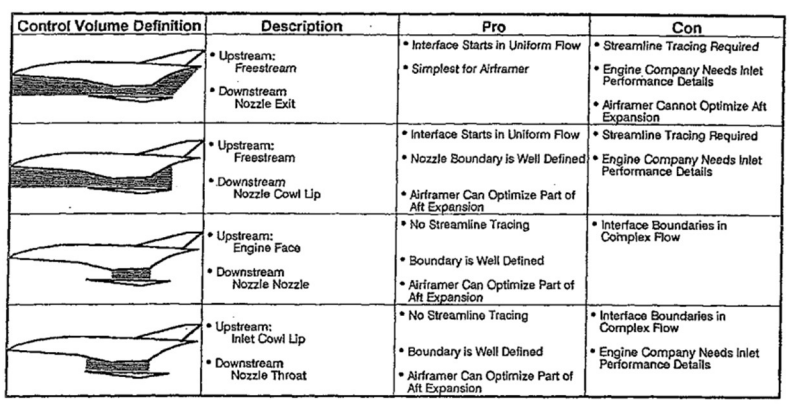
Note that this equation is not as simple to implement as the "Classical" thrust equation because it depends on the flow conditions at the inlet. This makes things more difficult as the inlet conditions are dependent on the flight conditions, the mass flow requirements of the propulsion system, and the propulsion system geometry. It is possible (for some applications) to simplify the equation further and express either P_1 or V_1 in terms of the other variable and free stream conditions, but I will not include that here. This is still an *uninstalled thrust* equation as it does not account for any typical "installation effects" but just uses a different definition of thrust. Like the previous approach, any result that is identical to this with perfectly analogous variables has the same meaning with the same implications. This approach may also leave out the presence of any forebody from a shock cone or similar whereas the classical thrust equation may not leave that out.

III. Thrust/Drag Bookkeeping Develops Installed Thrust from Uninstalled Thrust

To compute aircraft performance, engineers must develop consistent models of thrust and drag. In steady level flight, thrust must oppose drag while lift must oppose weight. If engineers define thrust and drag to differing or competing standards, they might balance in a numerical simulation while they might significantly disconnect during flight test. Since a propulsion house and airframe prime can define their control volumes in arbitrary ways, it is easy to see how ad-hoc and inaccurate air vehicle force accounting systems can be an unfortunate "industry standard." Practicing engineers must give the force accounting scheme great respect.

A. Keith Numbers Documented Many Thrust/Drag Bookkeeping Conventions Used by Industry

In the early 1990s, Keith Numbers at AFRL [12] surveyed American aerospace companies concerning air vehicle force accounting. He learned of a large variety of usable accounting systems as well as evidence of a lack of standardization. FIGURE 5 [12] illustrates the diversity in thrust/drag accounting schemes.



Control Volume Definition	Description	Pro	Con
	• Upstream: Freestream • Downstream Nozzle Exit	• Interface Starts in Uniform Flow • Simplest for Airframer	• Streamline Tracing Required • Engine Company Needs Inlet Performance Details • Airframer Cannot Optimize Aft Expansion
	• Upstream: Freestream • Downstream Nozzle Cowl Lip	• Interface Starts in Uniform Flow • Nozzle Boundary is Well Defined • Airframer Can Optimize Part of Aft Expansion	• Streamline Tracing Required • Engine Company Needs Inlet Performance Details
	• Upstream: Engine Face • Downstream Nozzle Nozzle	• No Streamline Tracing • Boundary is Well Defined • Airframer Can Optimize Part of Aft Expansion	• Interface Boundaries in Complex Flow
	• Upstream: Inlet Cowl Lip • Downstream Nozzle Throat	• No Streamline Tracing • Boundary is Well Defined • Airframer Can Optimize Part of Aft Expansion	• Interface Boundaries in Complex Flow • Engine Company Needs Inlet Performance Details

FIGURE 5. A Selection of Possible Control Volume Definitions for Propulsion System Analysis from [12]

Since there is no single way to approach force accounting and the practice is highly non-standard throughout the US aerospace industry, it is essential for engineers on a specific project select one scheme early and do not "flip-flop" between two or more to reduce confusion.

Numbers defined the division of the vehicle surfaces between the aerodynamics team and propulsion system team as the "Aero-Propulsion Interface" (API) [12] which is the primary division between the systems. There is also discussion of a so called "Engine-Airframe Interface" (EAI) that represents the control volume used for engine analysis, or the portion of the propulsion system the engine company is responsible for. This is useful in practice but introduces another division that can make thrust/drag bookkeeping even more confusing. For simplicity in this paper, we consider the API and EAI coincident.

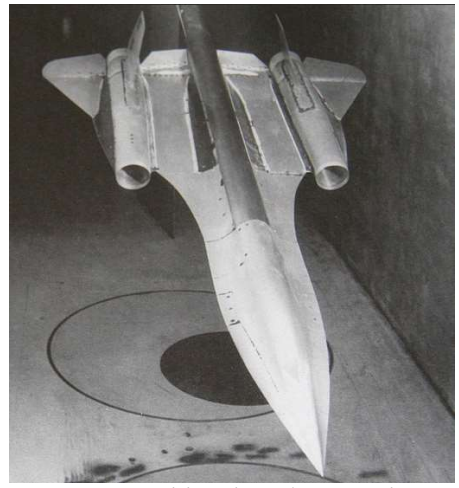


FIGURE 6 – Lockheed SR-71 Aerodynamic Performance Wind Tunnel Model

Numbers [12] highlighted the concept of "Aero-Reference Conditions" as another key factor in thrust/drag bookkeeping. These are the "baseline" aerodynamic conditions around the flight vehicle where aerodynamic and propulsion system data is prepared. Since a performance wind-tunnel model often has simplified, fixed geometry (see FIGURE 6) the aerodynamics team often defines this basis to be the reference. For the example shown in FIGURE 6, a Lockheed SR-71, the aerodynamics reference omits the large spike inlets completely and has the engine represented as a sharp leading-edge constant area duct. Thus, the propulsion system (where the formation of shockwaves, the presence or absence of spilled flow and the existence of variable inlet geometry) will change the pressures and tractions around the reference flight vehicle throughout the flight profile. Therefore, aerodynamics wants to characterize these effects either as buried within an *installed thrust* model and/or as secondary correction terms relative to the reference aerodynamics. Under this common scenario, axial terms are buried in *installed thrust* (which differs significantly from *uninstalled thrust*) while the aerodynamics model will account for other throttle dependent pitching, rolling and yawing effects.

B. *Installed Thrust* Corrections Increment or Decrement the *Uninstalled Thrust* Defined by Simple Control Volume Analysis to Render Propulsion Performance Consistent with Aerodynamics Modelling Conventions

Despite this history, thrust/drag bookkeeping remains a messy affair that is highly non-standardized. This environment serves as breeding ground for potential mistakes, misunderstanding and malfunctions.

Our literature review found consensus between textbook authors on what standard is acceptable to use. Covert [8] describes this well and depicts it in FIGURE 7. Lehrach [13] and Ball [14] examine multiple propulsion control volume options echoing this as well. Some *installed thrust* corrections [8,14,15] use reference propulsion conditions to measure the propulsion system performance against. Rather than explicitly deriving a meaningful estimate of propulsion system performance at every condition, it is common for these authors to measure the net thrust (*installed thrust*) at a specific reference condition and then alter individual parameters to examine how the propulsion system performs in off-design conditions.

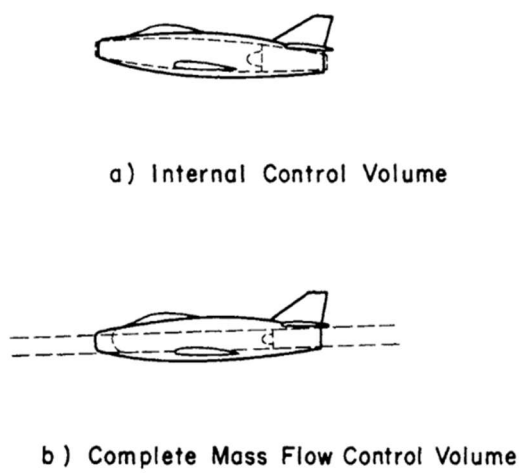


FIGURE 7. Two Common Control Volume Definitions [8]

Installed thrust corrections allow engineers to "account for everything" in force accounting which includes nuances that may be absent from a simplified control volume analysis. As the name implies these "tools", or a combination of them, mature an *uninstalled thrust* estimate to an *installed thrust* estimate to predict

vehicle performance more accurately. Ideally, engineers apply these to a specific propulsion system on a specific flight vehicle as not all the corrections apply to every situation. The engineer must select them piecemeal to describe the situation.

Ball [14] and Rooney [8] approached *installed thrust* prediction in different ways; compare Ball (FIGURE 8) to Rooney (FIGURE 9). In Ball's breakdown, the "net thrust" does not include the "separate" effects of the inlet or nozzle so we can infer that this is what we would call *uninstalled thrust*. Meanwhile, Rooney describes this breakdown in much more detail factoring in the inlet/nozzle effects, reference, operating, and actual conditions. Rooney is particularly sensitive to the different sources of wind tunnel data: 1) overall force & moment data from a complete configuration tested with a simplified (usually flow through) inlet over a wide range of speeds and attitudes, 2) flow distortion, pressure recovery and stability data from a detailed inlet model tested over a range of speeds, ramp geometry, bleeds and bypass door settings, 3) afterbody plume effects documented in terms of changes to lift, drag and moments based on a "jet effects" wind tunnel model tested over a range of speeds, attitudes and cold-flow conditions.

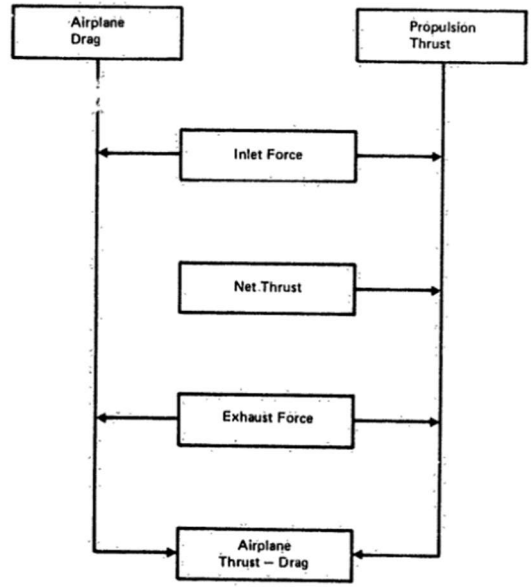


FIGURE 8. Thrust/Drag Accounting Methods from Ball [14]

While these two authors approach the problem from different viewpoints, their end goal remains the same: that is to adjust both the aerodynamic model and the propulsion model to be consistent with one another. Both authors recognize that direct as well as indirect forces arising from propulsion system operation, the inlet and engine exhaust plumes need to be apportioned between **both** aerodynamics and propulsion.

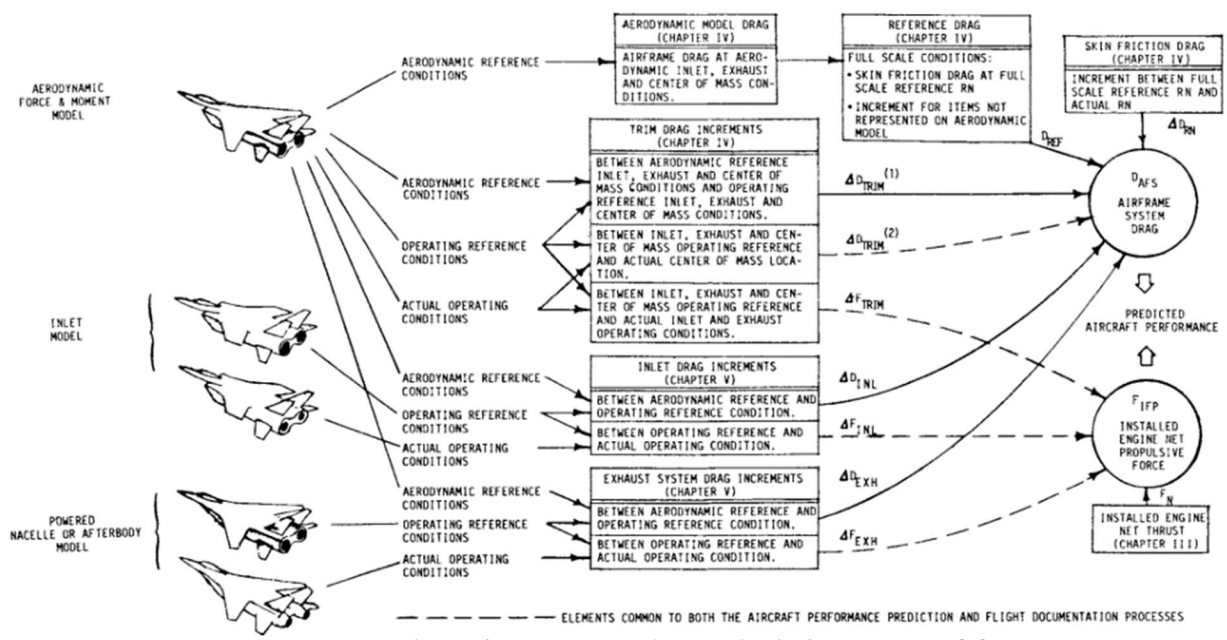


FIGURE 9. Thrust/Drag Accounting Methods from Rooney [8]

IV. Additional Thrust/Drag Bookkeeping Concepts

A. Since the Choice of API Directly Impacts Installed Thrust, Inlets Need to Be Characterized by More than Their Impact to Total Pressure Recovery at the Fan Face

The major issue the propulsion performance engineer must face is how to compute an *installed thrust* that is compatible with the aerodynamic drag reference. The paradigm shown above in FIGURE 6 (previous) is the most common aerodynamic reference; that requires that *installed thrust* must consider all forces acting upon the internal control volume shown in FIGURE 7a (previous).

We believe that Sibulkin [16] correctly notes that the inlet-to-tail paradigm and "alternate thrust equation" best represents the actual forces imparted by the propulsion system on the airframe. However, to implement this control volume to produce a numerical estimate of thrust we must further subdivide it into two regions: a region comprising the interior of the inlet (the region between Station 1, the inlet cowl, and Station 2, the fan face) as well as the traditional engine (Keith Numbers' EAI called out in **RED**); see FIGURE 10.

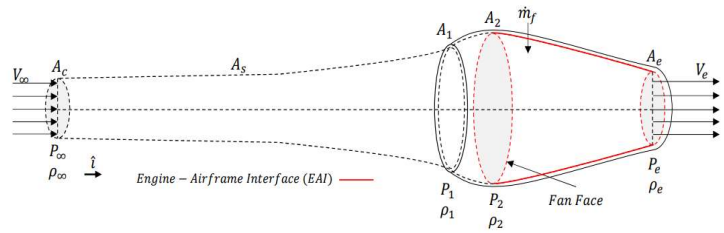


FIGURE 10. The "Inlet-to-Tail Control Volume", *CV3 comprising the entire internal flowpath. We show the location of the engine fan face and the boundaries of the EAI (in red)*

Thus, the effective thrust of the system includes the traditional engine thrust (a cycle analysis appropriate for the specific EAI) based upon pressures, densities and temperatures found at the fan face (Station 2) **as well as** integrated axial projections of static pressures and surface tractions found in the physical inlet.

Real world inlets need not swallow shock waves to produce internal compression. Internal compression is associated with any form of flow diffusion that occurs between the inlet entry point and the engine fan face. Any increase in cross-sectional area moving from inlet cowl to fan face will produce an axial pressure gradient which acts upon forward projected areas. Takahashi & Cleary [17] and Chaudhari & Takahashi [18] estimated the magnitude of these axial pressure gradients for subsonic, simple supersonic and supersonic mixed compression (i.e. swallowed shock) inlets and found that these terms can vary from almost negligible (for a simple subsonic inlet at low speeds) to substantial (for the supersonic mixed compression inlets). As speeds increase for inlets with significant internal diffusion, this "inlet buoyancy" internal force can easily equal or exceed the force predicted by the classic control volume analysis.

Returning to Paul Bevilaqua's exclamation that if "F ≠ m a, something must be wrong!" We note that the existence of this propulsion system induced force is not otherwise obvious in the derivation of the classic thrust equation using *CV1* or even the alternative control volume in *CV3*. Yet, this is the force that gives rise to the statements that at the supersonic cruise point half or more of the cruise thrust of the GE4 on the B2707 [18] or the Olympus on Concorde [19] arises from the inlets. If any forces due to internal compression (what occurs between Stations 1 and 2 in FIGURE 10) are omitted from *installed thrust*, the thrust lapse and TSFC of supersonic propulsion systems will appear to be abnormally high compared to quoted cruise values from these legacy programs. Such installed thrust data will also be incompatible with aerodynamic data derived using a passive flow through model as the aerodynamic reference.

B. Boundary Layer Bleed Drag Represents the Momentum Flux of Air Which Enters the Inlet But Is Diverted Into a Boundary Layer Control Plenum

To improve system performance, many propulsion systems "bleed" boundary layer (BL) air from key inlet surfaces. This occurs typically before the fan face often for the purpose of supporting a desired shock structure or minimizing "messy" (i.e. distorted) flow at the fan. Engineers typically design inlet ducts with perforated (or slotted) plates that "vacuum" off the boundary layer; refer to FIGURE 11. This air can sit in a bleed "plenum" which is connected to a low static pressure region elsewhere on the airframe.

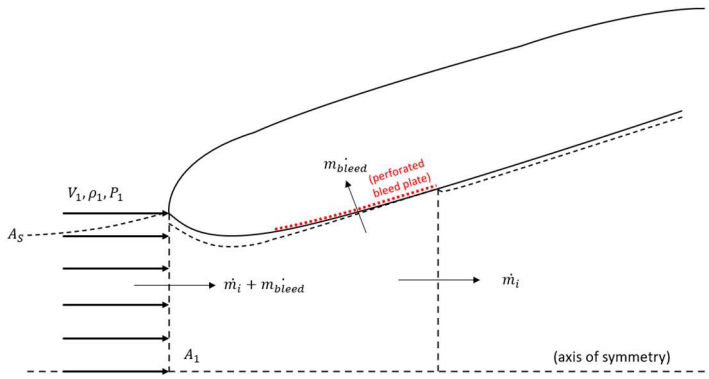


FIGURE 11. Example Diagram for BL Bleed

In Ref. [15], Bowers describes the resulting "bleed drag" as the "wind direction component of total momentum loss from freestream to [the] exit station of the bleed air, plus the incremental change in external drag, at constant inlet airflow, from no bleed to the operating bleed." Many who use BL bleed forego a complex analytic solution for a simple correction based on the mass flow through the bleed system to augment system drag (or drag coefficient depending on the accounting scheme).

This bleed air has a force consequence (either adding to aerodynamic drag or reducing installed thrust) as the mass flow "dumped" overboard has gone through irreversible processes (i.e. passed through a loss-producing slot or porous plate). It is common to assume a fixed percentage loss (i.e. 90% total pressure loss) of freestream momentum loss and apply it to the bypass mass flow to determine the bypass drag force.

C. Bypass Drag Represents the Momentum Flux of Air Which Enters the Inlet but Is Diverted "Overboard" to Compensate for Excess Mass Flow Supplied by the Inlet Which Cannot Be Ingested By the Engine

An engine / inlet system must always conserve mass. Engine mass flow demand is a function of power-lever-angle and flight conditions. Inlet mass flow supply is a function of inlet geometry and flight conditions. Engineers size inlets to prevent "choking" the inlet (where the engine mass flow demand exceeds the inlet mass flow supply). This typically occurs at a single operational point in the flight envelope; thus, inlets are inherently "over-sized" and over supplies mass flow to the engine at most "off design" flight conditions.

Engineers design movable bypass doors to "dump" excess inlet mass flow overboard to match the mass flow requirements between the engine and the inlet at specific flow conditions. Such mechanisms are needed for most supersonic external compression or internal compression inlets because the desired shock pattern can only form over a narrow range of inlet mass flow conditions. If the inlet mass flow is forced to identically match engine demand, the shock structure will misbehave – leading to inlet "buzz" or "unstarts."

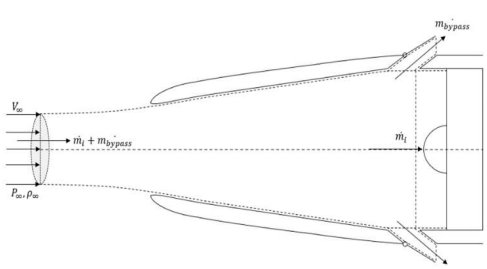


FIGURE 12. Sample Diagram for Bypass Drag

Bowers [15] defines bypass drag as the "wind direction component of total momentum loss from freestream to exit station of the bypass air **plus** the incremental change in external drag, at constant inlet airflow, from no bypass to the operating bypass." This bypass air has a force consequence (either adding to aerodynamic drag or reducing *installed thrust*) as the mass flow "dumped" overboard has gone through irreversible processes (i.e. passed through normal and/or oblique shock waves) and may well unfavorably alter the external flow field of the aircraft as an indirect consequence of it being vented into an otherwise low static pressure zone.

FIGURE 12 has a diagram of a potential control volume. It is common to assume a fixed percentage loss of freestream

momentum loss (i.e. 50% total pressure loss) and apply it to the bypass mass flow to determine the bypass drag force.

D. Integrated Cowl Pressure Forces Can Increase (Rather Than Reduce) Installed Thrust

A discussion of the need to consider the axial integrated components of the Cowl Lip pressures, also known as "Leading Edge Suction", appears in works by Pearcey [19], Seddon [6], Farokhi [5], Bowers & Tamplin, [8] Küchemann [21] and others; see FIGURE 13. These forces are not directly addressed by the "classic" control volume, *CV1*, discussed above.

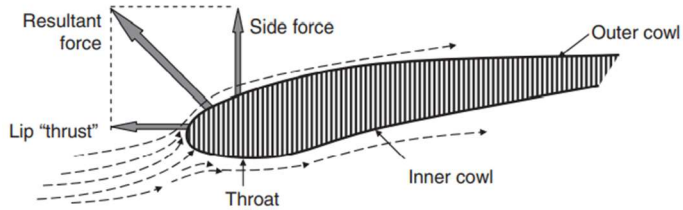


FIGURE 13. Cowl Lip Suction from Farokhi [5]

Wings may experience this same force. The force is real, and it can be estimated if one integrates the full stress tensor in the axial direction (body axis). Pressure differences between forward and aft facing surfaces can lead to a net axial force (which reduces the overall drag of the wing) which points forwards and opposes (and even overwhelms viscous skin friction). In classic wing theory, these forces are often idealized (i.e. Joukowsky theory) or even neglected (i.e. "induced drag is the vector projection of the normal force developed by the wing parallel to the incoming airflow"). Takahashi & Ou [22] discuss how these forces are or are not accounted for in classic airfoil theory. They also demonstrate how sensitive leading-edge suction forces are to nuances of leading edge geometry.

Cowl lip forces may be lost in standard force accounting for *installed thrust* corrections, but cannot be forgotten. If the cowl is shaped with specific features and the operating conditions are right, cowl lip forces can increase *installed thrust*, or reduce airframe drag. Consider FIGURE 14 from Küchemann [21] and notice how the pressure distribution arrows point outward and often align with the axial direction. When isolating the inlet portion of the propulsion system and integrating the forces due to pressure alone, this would create a net forward force contributing an increase in *installed thrust*.

Küchemann's drawing also shows how nuance of the cowl highlight (most forward geometry) to throat geometry impacts the control volume analysis. The top sketch in FIGURE 14 has the cowl highlight area equal to the inlet throat area; there all forward projected area in the cowl region is external to the highlight. The lower sketch in FIGURE 14 as the cowl highlight area being greater than the inlet throat area; forward projected area exists both external and internal to the highlight. Consequently, details of the cowl geometry impact both *installed thrust* and drag.

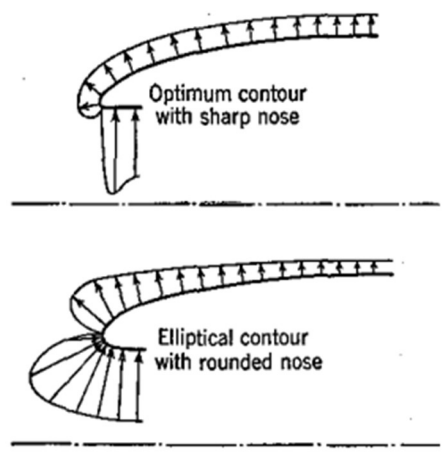


FIGURE 14. Diagram Demonstrating Pressures for Cowl Lip Suction. [21]

Details of the thrust / drag bookkeeping convention further complicate matters. All propulsion systems divide oncoming flow into internal and external flow components. Since the location of the stagnation point changes as a function of flight conditions (speed, altitude and attitude) and power lever angle setting, the geometry of the inlet "wetting" the swallowed flow does not remain constant. Moreover, while there is no theoretical issue dividing aerodynamics (drag) and propulsion (thrust) along the stagnation point in the flow, a practical API and EAI is best kept constant.

On a real airplane program, it is far more practical to divide the two accounting areas using points fixed by geometry (a specific point on the flight vehicle that is known). Consequently, some of the flow that passes through the propulsion system may touch the control volume defined by the aerodynamics team as drag. Numbers [12] notes this as a trade-off when selecting the vehicle API and EAI.

Turn next to FIGURE 15 to visualize how the flow-defined API might move based on flight conditions and a geometry-defined API might cause some cross-accounting issues. Numbers [12] suggests that the propulsion company needs to be responsible for what happens within the EAI so any surfaces not included by the airframe company's external aerodynamics team or the propulsion company's EAI may be accounted for by a propulsion aerodynamicist at the airframe company.

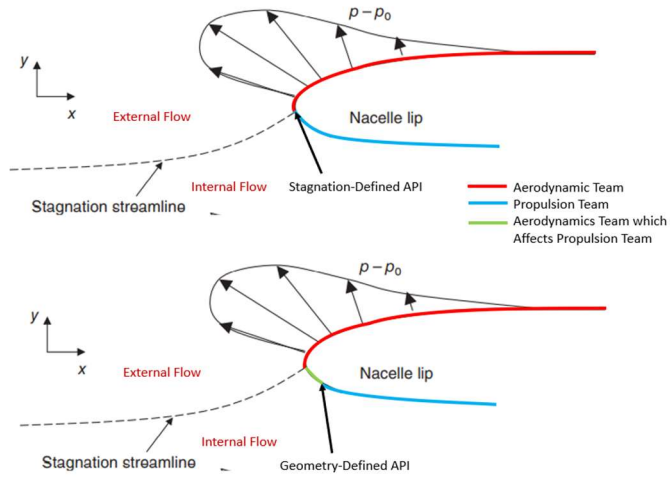


FIGURE 15. API Nuance Associated with Cowl Lip Suction (Adapted from [5])

This recommendation is still fraught with challenges; examine FIGURE 15 more closely. In the upper sketch, the API boundary is given at the inlet highlight area. Thus, both aerodynamics and propulsion models include significant forward facing areas. Leading edge suction forces may then reduce airframe drag **and** increase i*nstalled thrust*. In the lower sketch, the stagnation point moves outside the API boundary. Here, leading edge suction forces increasingly will manifest themselves in terms of airframe drag BUT the propulsion system will see an increase in effective contraction ratio between highlight and fan face due to the region colored green.

Conventional wisdom solely characterizes steady state inlet performance by its impact on total pressure recovery at the fan face. Because unsteady buzz and flow distortion limits influence operational stability and service life of the turbomachinery, these metrics are used more to define "no fly" zones in the operational envelope rather than *uninstalled* or *installed thrust*.

Note that the classical thrust equation technically includes these effects on the stream tube flow (like inlet pressure recovery) but does not include the force due to surface stress and pressure integration on the inlet geometry.

Consider next whether we can every directly measure uninstalled thrust in a test cell. Refer to FIGURE 16 [5] to see the schematic installation of an engine in a typical test cell. The load cell measures the thrust produced by this system which includes the cowl pressure forces associated with the bellmouth inlet and afterbody forces due to entrained flow around the nozzle. Farokhi [5] calls out that the integrated pressure and surface tractions on the bellmouth increases measured thrust during a static engine run. Thus, static test cell data does not actually measure *uninstalled thrust*. Indeed, it measures *installed thrust* for a non-flightworthy geometry. This may be why authors like Jakobsson [23] believe that thrust measured in this manner "is of little value unless it is obtained from full-scale tests of aircraft with completely representative engine nacelles.

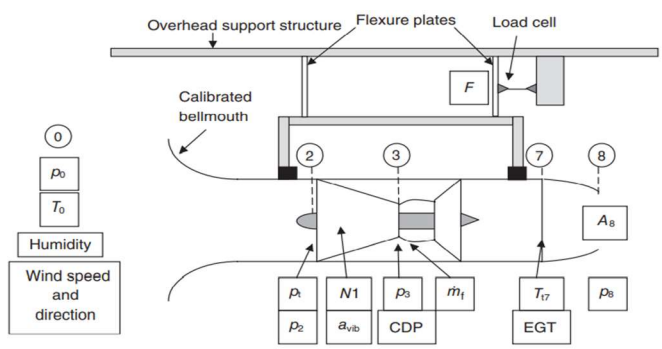


FIGURE 16. Air breathing Engine Test Stand Diagram from [5]

Hence, we highlight a problem inherent in the system. To be useful, installed thrust must represent the propulsion system performance of the actual flight geometry. Yet, tradition has engineers lump correction after correction upon reference thrust values not representative of flight inlet geometry. For example, Ball [24] seems to leave the inlet (and nozzle) effects out of the thrust definition so he can add inlet performance effects (pressure recovery, fan face distortion, etc.) to the *uninstalled thrust* afterward. To properly define

installed thrust we must include forces due to axial surface pressure and surface traction integration to a precise inference of *uninstalled thrust*.

E. Angle of Attack Effects Need to Be Considered in Terms of Impact on Pressure Recovery, Flow Distortion, Buzz Limits AND other API Related Forces

As the angle of attack changes, the free stream conditions the engine experiences which changes somewhat. At subsonic speeds, the cowl suction forces will depend on the inbound angle of attack. If the angle of attack grows high enough, cowl lip flows may separate. Refer to FIGURE 17a to examine the cowl geometry of the Eurofighter Typhoon; we see the movable cowl lip whose geometry is scheduled as a function of angle-of-attack to improve inlet flow during high-α maneuvering. Alternatively, with an external compression inlet (see FIGURE 17b, for a F-15) the shock formation will be a function of angle of attack, ramp angle and Mach number.

FIGURE 17. Production Aircraft Inlets. a) Eurofighter Typhoon Variable Cowl Lip, b) F-15 2D external compression inlet.

Ball notes that it is difficult to generalize these corrections. He offers that many aircraft are rather insensitive to small angle changes. [24] For those aircraft, we can omit these corrections without significantly degrading our understanding of installed thrust.

For other aircraft, these corrections are essential both in terms of impact to airframe drag, engine operational envelopes and installed thrust. Return to FIGURE 17b and consider the inlet of the F-15; we note that the moveable ramp hinge is located aft of the leading edge of the inlet. Thus, the first shockwave formed in its external compression inlet is keyed directly to a ramp whose incidence relative to the oncoming supersonic flow is a direct function of angle of attack. Thus, inlet flow mass flow as well as total pressure recovery depends upon the angle of attack independent of any sort of adjustment of the moveable ramp.

F. "Additive Drag" or "Pre-Entry Drag" Is a Thrust Correction Often Conflated with Spill Drag, but Is Formally Distinct and Easy to Misapply

Additive drag is a confusingly named *installed thrust* correction term. Understanding its formalism is key to understanding how the different definitions of uninstalled thrust shown in Section II represent the same propulsive flow.

We realize that there are many interpretations of additive drag. That is understandable because the name is not a helpful descriptor.

- Flack [7] describes this as the drag due to flow spilling around the inlet cowl from a mismatch between the required mass flow of the engine (fixed by internal engine attributes like nozzle or thermal choking) and the flow provided by free stream flight conditions. This description is much like "Spill Drag" which we discuss later in this paper.
- Ball, in the Boeing Manual for the Propulsion Installation and Table Assembly Program (PITAP) [24], describes it as the drag due to poor mass flow rate matching specifically compared to a reference condition.
- Bowers & Tamplin [8] describe this as a "static pressure force exerted, in the wind direction, on the inlet stream tube, between freestream conditions and the inlet stagnation point, with the inlet operating at zero external bleed flow."
- Sibulkin [16] describes this drag as a force already "credited" to thrust at a low MFR flight condition that must be subtracted from the thrust prediction.

That makes for several somewhat related but noticeably different descriptions of a correction using the same name. That has and will greatly add to confusion.

We understand that "additive drag" is a force that accounts for the differences between 1) the force delivered to the flight vehicle and 2) a specific thrust force for *CV1*. Our definition aligns with Sibulkin [16] which appears to be the most rigorous. Wyatt [25] seems to agree with this noting that "conventional definitions of thrusts and drag do not predict the propulsive forces on an engine installation when the inlet mass-flow is not unity."

The other definitions, in our opinion, relate more to spill drag so we should consider them under that category. Note that Ball discusses additive drag in the manner I've described but uses the same analytical expression as Sibulkin which increases the possibility for confusion. Recall *CV1* the "Swallowed Flow Control Volume" refer back to FIGURE 1.

To establish Additive Drag we carry out a similar control volume analysis as before. Remember that the surface stresses along A_s are included in the definition of thrust resulting in the following Eq. (31) which we call the "Full Thrust Equation" as it includes the most forces with all else the same.

$$T = \dot{m}_\iota[(1+f)V_e - V_\infty] + (P_e A_e - P_\infty A_c) \tag{31}$$

If the engineer hopes to calculate the *uninstalled thrust* force delivered to the flight vehicle, this equation is not sufficient because it includes forces that are not acting on the flight vehicle (mainly the forces on the "pre-entry" flow or the surface stresses acting on the surface A_s before the propulsion system inlet plane). Thus, we require a correction to subtract off forces the Full Thrust equation "credits" as thrust but to not act on the flight vehicle.

Consider the "pre-entry" control volume in FIGURE 18 that we call "Pre-Entry Control Volume" or *CV4*. Note here that *CV3* and *CV4* combine to form *CV1* but remember that these control volumes can have different definitions of "thrust" or "net propulsive force" applied to them. The inlet plane station (subscript 1) represents the aft fact of the control volume.

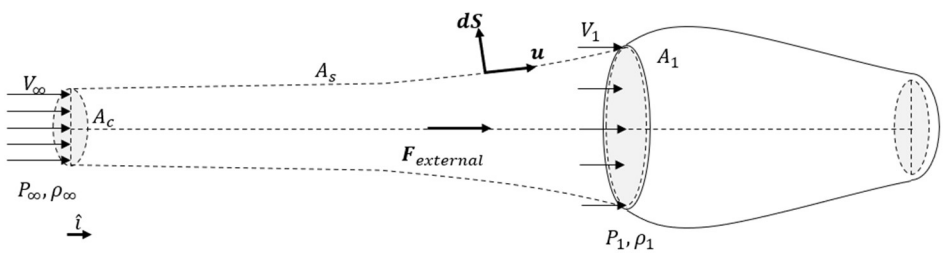


FIGURE 18. "Pre-Entry Control Volume", *CV4*

Let us now work through the control volume to derive "additive drag" in a similar was as Sibulkin in Reference [16]. Just as before, the unsteady terms are discarded when considering mass conservation in (32).

$$\cancel{\frac{d}{dt}\int_{\tilde{V}} (\rho) dV} + \int_S (\rho)\boldsymbol{u} \cdot \boldsymbol{dS} = \dot{m}_{net} \tag{32}$$

The result in (33) comes from the flow rate into the control volume $\dot{m}_\iota$ matching the flow rate out of the control volume $\dot{m}_1$ thus $\dot{m}_{net} = 0$. There is only mass flow across the capture area and inlet plane with no flow across streamlines and no fuel addition.

$$\dot{m}_1 = \rho_1 V_1 A_1 = \dot{m}_\iota = \rho_\infty V_\infty A_c \tag{33}$$

Now consider conservation of x-momentum in (34).

$$\cancel{\frac{d}{dt}\int_V (\rho u) dV} + \int_S (\rho u)\boldsymbol{u} \cdot \boldsymbol{dS} = \cancel{(F_x)_{internal}} + (F_x)_{external} \tag{34}$$

In this case, there are no internal control volume surfaces so there is no force due to internal surfaces. Expand the left side of (34) to produce (35) resulting in (36) just like the analysis of *CV1* and *CV3*.

$$\int_S (\rho u)\boldsymbol{u} \cdot \boldsymbol{dS} = \int_{A_c} (\rho u)\boldsymbol{u} \cdot \boldsymbol{dS} + \int_{A_1} (\rho u)\boldsymbol{u} \cdot \boldsymbol{dS} + \cancel{\int_{A_s} (\rho u)\boldsymbol{u} \cdot \boldsymbol{dS}} \tag{35}$$

$$\int_S (\rho u)\boldsymbol{u} \cdot \boldsymbol{dS} = -\rho_\infty V_\infty^2 A_c + \rho_1 V_1^2 A_1 \tag{36}$$

Next, consider the expansion of the right side of (34) in (37). Here, we infer the presence of the force F_{ext} which represents the external force imparted on this control volume to keep it stationary relative to the (potentially) moving flight vehicle; another application of D'Alembert's Principle. Unlike thrust, this force does not actually act on the flight vehicle which is precisely why we are interested in it. We hope to find an analytical solution to this force and subtract it from the thrust computed from the Full Thrust Equation.

$$(F_x)_{external} = \int_S \sigma_{ij}\, dS_j + F_{ext} \tag{37}$$

The surface stress tensor integration expands to (38) resulting in (39).

$$\int_S \sigma_{ij}\, dS_j = \cancel{\int_{A_c} \tau_{xj}\, dS_j} + \cancel{\int_{A_1} \tau_{xj}\, dS_j} + \int_{A_s} \tau_{xj}\, dS_j - \left(\int_{A_c} P\, dS_x + \int_{A_1} P\, dS_x + \int_{A_s} P\, dS_x\right) \tag{38}$$

$$\int_S \sigma_{ij}\, dS_j = \int_{A_s} \tau_{xj}\, dS_j - \int_{A_s} p\, dS_x + P_\infty A_c - P_1 A_1 \tag{39}$$

Now, combine the results of (36) and (39) into (34) to make (40).

$$-\rho_\infty V_\infty^2 A_c + \rho_1 V_1^2 A_1 = \int_{A_s} \tau_{xj}\, dS_j - \int_{A_s} p\, dS_x + P_\infty A_c - P_1 A_1 + F_{ext} \tag{40}$$

Like the derivation of the "Full Thrust" equation, we define the force of interest to include both the force holding the control volume still (F_{ext}) and the integration of surfaces stresses along the stream tube surface. We call this force the "Pre-Entry" force which is often called "Additive Drag".

$$F_{pre-entry} \equiv \int_{A_s} \tau_{xj}\, dS_j - \int_{A_s} p\, dS_x + F_{ext} \tag{41}$$

Rearrange (40) using (41) to create (42) the analytic expression for "Additive Drag". This result is identical to that from Sibulkin [16], Ball [22], and others that use the same convention. This expression is meant to be subtracted from the "Full" thrust equation.

$$F_{pre-entry} = D_{add} = \dot{m}_1 V_1 + P_1 A_1 - \dot{m}_\iota V_\infty - P_\infty A_c \tag{42}$$

We believe this definition and convention to be best for "Additive Drag". Yet it is distinctly different from the forces that relate to flow spillage around the inlet as proposed by Flack. [6]

The key thing is that additive drag is not an actual force experienced by the flight vehicle nor is it a "drag force" suitable for inclusion with airframe aerodynamics under conventional understanding.

We believe that additive drag is a correction term used on a very specific *uninstalled thrust* definition to represent forces on the flight vehicle more accurately. As such, it is **not valid** to apply this correction for every application. Sibulkin [16] and Wyatt [25] seem to agree with this opinion as well.

Wyatt also notes that the force implication from momentum change at the inlet can arbitrarily be applied to **either** thrust or drag. [25] We agree with this sentiment even though forces like "Additive Drag" are considered with thrust in this analysis. Wyatt states that if "the conventional thrust definition. . . is considered to be an absolute definition of thrust, then the force must be considered a drag. If, on the other hand, it is considered that drag must wholly be represented by pressures on the external surface, then the force must obviously be a thrust." [25] There is no "correct" way to allocate this force, but depending on the thrust definition of a given application it cannot be wholly ignored.

G. Spill Drag Is Distinctly Different from Additive Drag; It May Be Accounted for Either as an Airframe Drag Correction or As a Correction to Installed Thrust.

Although spill drag is often conflated with additive drag; it is a separate but related force.

Spill drag has to do with a mismatch of the flow required by the engine and provided by free stream flow at the current flight conditions causing flow to "spill" around the propulsion system inlet and increase the overall system drag. This correction often uses propulsion reference conditions to compute where spill drag is defined to be zero at a reference mass flow ratio. Thus, spill drag changes based on the mass flow ratio between what the engine requires and what the flight conditions provide. Compared to the reference conditions, flow will likely be "spilling" over the external installation of the propulsion system to meet mass flow requirements. Flow distortion related to the spilled from Its byproducts may have an associated drag penalty (especially if it triggers shock waves or distorts the flow over the wing) but might also provide a performance improvement if it increases cowl lip suction forces above and beyond those found at the reference condition.

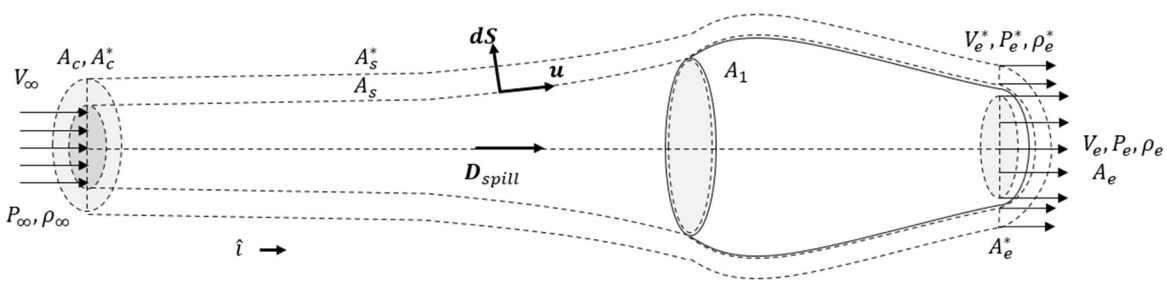


FIGURE 19. Potential Control Volume for a Spill Drag Derivation

FIGURE 19 displays a possible control volume to analyze spill drag which is defined as the flow just outside of the propulsion system affected by spilled flow extending radially out to the point where external flow is unaffected by the spilled flow. Defining the volume there is arbitrary but seems logical. This diagram shows a capture area relating to the area for the reference condition A_c^* and the area that is captured by the propulsion system A_c. A_c need not be smaller than A_c^*. Similarly, the aft end accounts for the spilled flow at the end of the propulsion system. Spill drag would result from integration of the stress tensor along the surface exposed to the spilled flow stream tube (incremented beyond the reference condition). Obviously, this control volume is only for nacelle configurations so an integrated propulsion system would need a different control volume. For the scope of this writing, we will not complete this analysis, but we believe it is required to define this with more rigor. Just like previously mentioned corrections, this one is typically handled by using an empirical drag increment based on the mass flow ratio of the engine and the free stream flow.

Authors commonly use a correction term K_{add} to describe spill drag, which scales the theoretical additive drag coefficient. Turn to FIGURE 20 to see equations from Ball [24] and Bowers & Tamplin [8] for spill drag and additive drag. Bowers & Tamplin seem to define the spill drag as the actual additive drag (whatever that means to them) subtracting off the cowl lip suction compared to the theoretical additive drag. Ball [24] considers this a correction "based on experimental data, to account for the

$$K_{\mathrm{ADD}} = \left(D_{\mathrm{ADD}} - \Delta D_{\mathrm{Lip}} \right) / D_{\mathrm{ADD}} \text{ (theoretical)}$$

$$C_{D_{SPILL}} = K_{ADD} \; C_{D_{ADD}}$$

FIGURE 20. Equations from Bowers and Tamplin [8] (top) and Ball [24] (bottom) Concerning Spill Drag Corrections

configuration effects" which include "cowl lip shape, bluntness, and sideplate cutback." Both authors assume that 1) K_{add} is proportional to mass flow and 2) additive drag and spill drag are directly related.

We agree with the Ball [24] and Bowers & Tamplin [8] that additive drag (as discussed) is related to spill drag because their respective control volumes share a surface. We also agree that the accounting scheme must factor in Cowl Lip Suction, and it appears convenient to do this with spill drag.

We do not think it is best to relate spill drag and additive drag so strongly; they are separate phenomenon.

We believe it is better to combine Spill Drag and Cowl Lip Suction into a single mass-flow dependent correction as there will be opposing effects from both due to increased spilled flow around a given nacelle. Generally, this correction should be applied to installed thrust rather than to airframe drag.

H. Integrated Nozzle Pressure Forces May Further Improve Installed Thrust

Nozzle performance corrections to *installed thrust* tell a similar story to the inlet duct. At one level, the nozzle geometry can impact the total pressure recovery between the turbine exit stages and the atmosphere. At a second level, integrated axial projections of surface pressures and tractions over internal nozzle surface impact thrust. The same considerations apply here: 1) total pressure losses are often included in the thrust definition and considered in the thrust equation (often resulting from a Brayton cycle analysis) and 2) the force from surface stress integration is omitted as with the classical thrust equation.

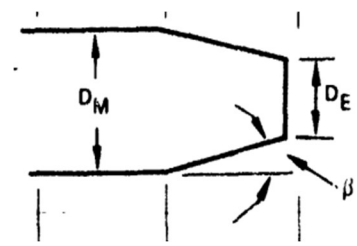


FIGURE 21. Boat Tail Diagram from Ball [24]

As with inlets, it depends on what the baseline the engineer uses. If the baseline does not include the production nozzle, the reference thrust value will not represent the actual *installed thrust* and may not even represent a coherent *uninstalled thrust*. In general, the flow within the nozzle will exist at a higher static pressure than the surroundings, thus integration of pressures across any diverging nozzle will result in a forward propulsive force which increases *installed thrust*.

I. Boat Tail Drag Forces are Throttle Dependent Forces that May Be Bookkept With Either Airframe Aerodynamics or With Installed Thrust

This force concerns the propulsion system effects on aft vehicle pressures. The "Boat Tail" region is the aft facing external surfaces of the airframe that lie in proximity to the nozzle. In ordinary circumstances, they are exposed to free stream flow – entrainment of external flow by the engine exhaust may express itself as throttle dependent forces. Boat Tail drag can arise from these integrated pressures acting over the exposed parts of the nozzle as well as other parts of the airframe. Because entrained flow tends to have lower static pressure than would naturally develop and since the projected area is aft facing, throttle dependent effects express themselves as a drag force which impacts *installed thrust;* see FIGURE 21.

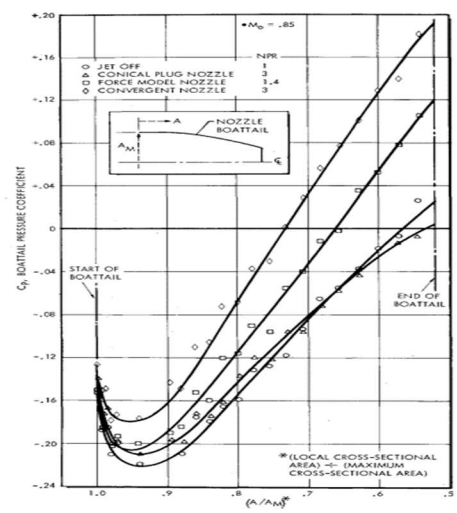


FIGURE 22. Pressure Distribution on Boat Tail from Bergman [26].

Boat Tail drag depends on flight conditions, throttle position, and geometry. In Reference [24] Ball notes that an increase in the "Boat Tail" angle relates to higher drag which is further increased by an increase in Mach number. According to Ball [24], Bergman [26], and others the boat tail drag is also dependent on the shaping of the post-exit exhaust plume from the propulsion system; they further affect the pressure of aft vehicle surfaces. Bergman also notes that flow entrainment from the propulsion system exhaust is a significant contributor to boat tail drag [26]. Examine FIGURE 22 for the drag penalty from Bergman concerning different geometry boat

tails. This force is highly throttle dependent and a consequence on the airframe due to the propulsion system operation making it a very coupled *installed thrust* correction.

J. Interference Drag Terms Bookkeep Miscellaneous Installation Effects Arising From Engines Operating In Close Proximity to One Another into Installed Thrust

Interference drag is a multi-engine drag consideration as the interaction of multiple propulsion systems in proximity can augment predicted aircraft net thrust. This is primarily considered by Ball in the PITAP manual [24][14] who notes that the Mach number affects the engine spacing ratio (spacing to diameter ratio) that relates to the largest interference drag coefficient; see FIGURE 23. Note that the coefficient is normalized by the ideal gross thrust. This correction is an overall vehicle configuration concern and should be considered an *installed thrust* correction only for multi-engine aircraft.

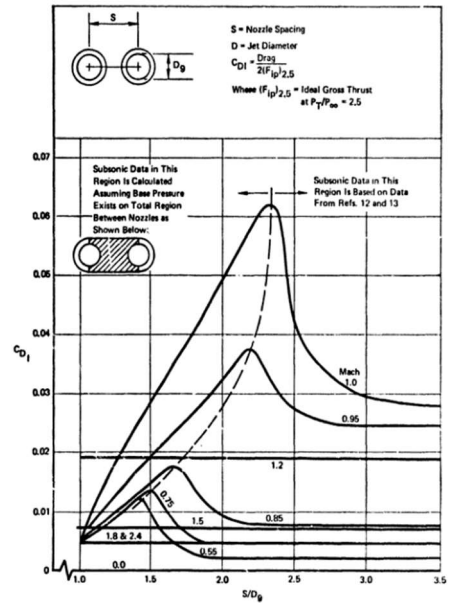


FIGURE 23 Interference Drag Coefficient for Multi-engine Configurations [14][24]

K. Base Drag Forces Are Other Axial Forces Arising from Pressures Acting On Aft Facing Steps that may be Bookkept with Either Airframe Aerodynamics or with Installed Thrust

This drag is commonly used in the aerodynamic analysis concerning aft-facing areas and the drag associated with separation on these surfaces. Takahashi describes this well [27] as a zero-lift drag relating to aft facing areas and the pressure on those surfaces imposed by flight conditions; see FIGURE 24. Usually, all the base area on a vehicle is considered that is not negligibly small. Ball [24] notes that to get bookkept with propulsion "the base area must be located where it is also affected by the propulsive jet effects which vey with nozzle pressure ratio." This creates an interaction between the installed propulsion system and other parts of the flight vehicle.

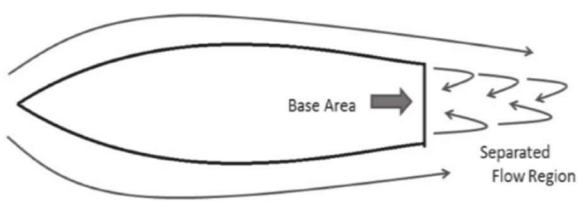


FIGURE 24. Basic Base Drag Definition

Base drag pressures are well documented as a function of Mach number from the X-15 and Space Shuttle Program so the drag predictions are reliable given the Mach number. This drag would change for aft facing areas affected by the propulsive jet effects meaning it may be throttle dependent. This is another aerodynamic consideration that may affect the flight vehicle that is due to the pressure field created by the propulsion system and the exhaust jet.

L. Bypass Airflow from Some Turbofan Engines May Flow Over Other External Features of the Airframe Resulting in A Scrubbing Drag Which May Be Bookkept with Either Airframe Aerodynamics or with Installed Thrust

Scrubbing drag, much like others in this section, is not a good descriptor of the intended force and may have multiple definitions person to person. In this paper, this terminology relates to turbofan propulsion systems and the drag associated with the bypass flow that "scrubs" on the walls of the bypass flow duct and core casing before reaching the exit plane [24]. This force may or may not be included in the definition of thrust but if it is not, a correction of this type is required.

This only seems to matter for turbofan propulsion systems with little discussion of how the bypass flow thrust is included in thrust definitions. This drag is composed of the stress tensor integration on these surfaces specifically. Ball notes that this drag depends on the bypass nozzle pressure ratio. The nomenclature here is confusing considering the name implies drag primarily due to viscous stresses but it depends on the bypass nozzle pressure ratio. [24]

V. Thrust/Drag Bookkeeping as Implied By Legacy Engineering Methods

In this section, we will discuss the thrust/drag bookkeeping conventions implied by several common engineering tools used by both the aerodynamics and propulsion communities.

A. NASA's NPSS Propulsion Modelling Software Computes Uninstalled Thrust Using the Classical Thrust Equation and Skims Over Installation Effects

Numerical Propulsion Simulation System (NPSS) is a 1-D aero/thermodynamic modelling environment that can track propulsion and thermal power system components to predict the performance of an uninstalled propulsion system. [28] NPSS computes state parameters within the propulsion control volume (EAI) using gas tables and matching power requirements to produce a comprehensive 1-D model of the propulsion system.

The basic NPSS scripts lack a lot of nuances regarding *installed thrust* prediction.[28] This program focuses on *uninstalled thrust* using the classical thrust equation and does not focus on installed propulsion systems. Like any *uninstalled thrust* model, it is not optimal to use it alone but to use in combination with other methods to correct for *installed thrust*. Given the coupled nature of propulsion systems and airframes, this is still limiting.

According to the NPSS script files from Reference [28], it appears to compute net thrust (thrust from the classical thrust equation) by computing the nozzle gross thrust and subtracting the "ram drag" which accounts for the "jet thrust" and "pressure thrust" that compose the classical thrust equation. The equations below translate the code from NPSS and the result is the classical thrust equation in (43).

$$\begin{gathered} Nozzle \text{ Gross Thrust: } F_g = \dot{m}_e V_e * C_{mixCorrect} C_v C_{angle} + (P_e - P_\infty) A_e \\ Inlet \text{ Ram Drag: } F_{ram} = \dot{m}_\iota V_\infty \\ Net\ Thrust\ (Uninstalled\ Thrust)\ F_n = F_g - F_{ram} \\ T = \dot{m}_e V_e - \dot{m}_\iota V_i + (P_e - P_\infty) A_e \end{gathered} \tag{43}$$

The code applies corrections to the exit velocity relating to internal propulsion system losses ($C_{mixCorrect}, C_v, C_{angle}$) to compute the "actual" exit velocity and considers a separate "ram drag" force on the inlet. It also includes some small corrections for pressure losses from the freestream to fan face engine stations. The common output of NPSS is a tabular data set detailing the thrust and fuel consumption of a propulsion system based on the Mach number, altitude, and throttle setting. It is best practice to further correct this data for installation effects.

B. The NASA/ Lockheed EDET Drag Prediction Code Ignores All Forces Associated with Flow that Passes into the EAI

EDET [29] was written by Feagan, et.al in 1978 at Lockheed California under contract with NASA. It predicts total aircraft configuration drag using aircraft geometry, theoretical solutions, and empirical relations. When combining this with propulsion data, the engineer can accurately estimate aircraft performance. This program is used in the famous FLOPS program by NASA and the method just described is used in the classes taught by Takahashi [27] to estimate aircraft performance often coming within a few percent of published values.

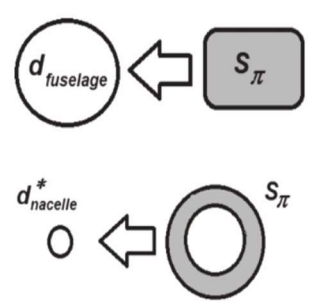


FIGURE 25. EDET Procedure for

Despite the impressive accuracy, EDET has issues when considering how it accounts for the engine-airframe interface. EDET computes drag of fusiform bodies using the wetted area, base area, length, and the form factor which depends on the body's length to equivalent diameter ratio. The equivalent diameter is the diameter of an equivalent circular cross section area at the maximum area cross section. See FIGURE 25 from Takahashi [27] for how this applies to a nacelle body where the inlet capture area is subtracted from the cross section. There is little guidance in the documentation for dealing with propulsion systems other than to entirely subtract out the inlet capture area to compute the drag. This is the most common approach used by a common aerodynamic drag tool to divide areas of concern for aerodynamics and propulsion.

This practice of removing the inlet capture area implies a direction for the accounting scheme that the system will use. Reducing a nacelle (for podded engines, not always the case) to an equivalent diameter effectively ignores the internal surface that will interact with the stream tube flow. For example, a nacelle is reduced to an equivalent body of revolution rather than a flow through nacelle with the proper blockages and flow conditions imposed by a propulsion system. Thus, the responsibility of accounting for the forces on the internal nacelle surfaces (those exposed from subtracting out the capture area) lands on the propulsion team.

More issues arise if the propulsion team uses a thrust definition that does not include the stress integration on that exposed surface or does not include that in an *installed thrust* correction. EDET is clearly an aerodynamics program and demonstrates this by ignoring nuance in the airframe-propulsion interaction.

C. The Harris Wave Drag Program Also Ignores All Forces Associated with Flow Which Passes into the EAI

This famous 1960's era program was written by Harris [30] for NASA to predict the "wave drag" of an aircraft flying at supersonic speeds. This methodology can estimate the difference in vehicle drag coefficient between the subsonic, incompressible estimate and at supersonic speeds.

The code relies on Slender Body Theory [31] which reduces a 3D body to an axisymmetric body with the equivalent longitudinal cross section area distribution. It then uses the axisymmetric body to predict supersonic drag performance using simplified potential flow / inviscid theory. For the airframe models, this takes out a lot of nuances in exchange for results that "qualitatively match" reality but not exactly just like results from Reference [32].

Similar nuance is lost for the propulsion accounting with the same pitfalls as EDET. View FIGURE 26 for a diagram from Harris [30] differentiating the actual aircraft and the mathematical model. Notice how the propulsion stream tube (or a constant-area version of it) is not included in the drag model. Then, the body is reduced to an equivalent body of revolution that changes with Mach number to compute drag [30]. This, again, neglects all nuance in the surface stresses on the internal and external surfaces of the nacelle due to the propulsion system. By itself this implies that much of the nuance is left to the propulsion group.

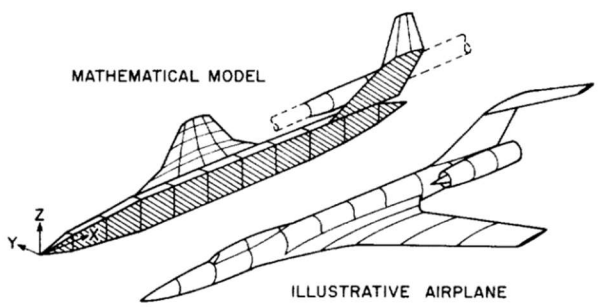


FIGURE 26. Harris Wave Drag Procedure for Computing Configuration Supersonic Drag

D. The PITAP Method Estimates Many but Not All Uninstalled To Installed Thrust Corrections for Air Breathing Propulsion Systems

The Propulsion Installation and Table Assembly Program (PITAP) was written by Boeing for the US Air Force to estimate the *installed thrust* of air breathing propulsion systems. It includes many *installed thrust* corrections mentioned in the previous section (BL bleed drag, bypass drag, additive drag, spill drag, angle of attack effects, inlet/nozzle internal performance, Boat Tail drag, interference drag, base drag, and scrubbing drag) and they are included in my explanations. This program appears to be a precursor to the following program PIPSI focusing primarily on describing and modelling common installation corrections rather than documenting a procedure to correctly predict *installed thrust*.

Ball, in the PITAP manuals – References [14] and [24], directs the user in thrust/drag bookkeeping by including all baseline effects of the engine, inlet, and exhaust system in "net thrust" and including all incremental throttle-dependent forces on the inlet and exhaust system in separate terms. Examine FIGURE 27 for a breakdown if all included effects in PITAP and FIGURE 28 for a summary of aft vehicle geometry correction "drag" terms.

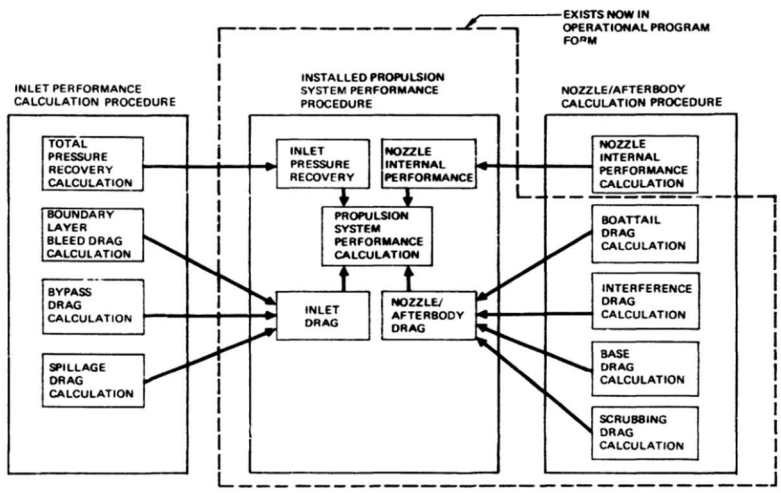


FIGURE 27. PITAP Procedure Map for Computing Installed Propulsion System Performance

We agree with Ball's stance on thrust-drag accounting that "from a performance calculation standpoint it is immaterial how the spit is made between thrust and drag provided that all forces exerted on the airplane system are accounted for once and only once as either a drag force or a thrust force." This inspires confidence but we have doubts on the interaction of Ball's thrust definition and associated installation corrections detailed in the PITAP manual.

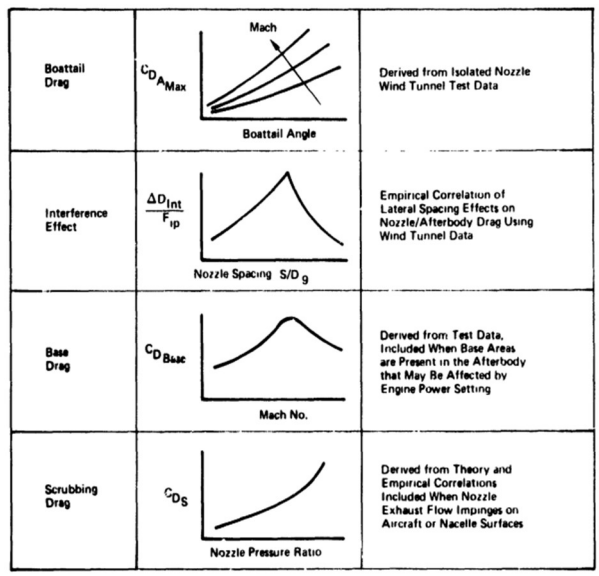


FIGURE 28. Summary of Aft Vehicle Throttle Dependent "Drag" Corrections

Ball in Reference [14] defines the reference thrust as "established from a static full scale thrust measurement with inlet internal pressure and exhaust system altitude condition reproduced in the test cell." There is little explanation of what the thrust measurement is (load cell, strain gauge, or computation using methods from sources like Reference [31]) or what is included in the propulsion system. Lack of clear definition to the level of rigor in the first section of this paper leaves questions as to exactly what force they are discussing. The same manual states that the engine net thrust "is defined to be the difference between the gross thrust of the exhaust system in quiescent air, at a specified pressure ratio, and the ram drag on the engine stream tube at the specified flight condition." This is very similar to the NPSS thrust definition but lacks any equation from which we could extract control volume implications. It additionally notes that "the effects of inlet internal performance, i.e., inlet total pressure recovery and steady-state and dynamic distortion, are accounted for in the engine net thrust." As written, these definitions are indistinguishable between the classical thrust definition and the alternate thrust definition.

E. The PIPSI method estimates many but not all inlet and nozzle effects needed to model air breathing propulsion systems

The Performance of Installed Propulsion Systems Interactive (PIPSI) computer program [34] was developed by Boeing for the US Air Force to predict inlet and nozzle/afterbody performance using compiled performance libraries. According to Takahashi & Cleary [17] "it contains data representative of a wide range of generic high-speed inlets included fixed geometry pitot, fixed geometry external compression, variable geometry external compression and variable geometry mixed compression inlets" which are derived from "wind tunnel experiments of real-world supersonic inlets."

PIPSI includes many of the installations effects we discussed earlier like inlet pressure losses, BL bleed drag, bypass drag, spill drag, and mass flow limits imposed by the colloquial "buzz" and "distortion" limits. It can then output the total pressure recovery and total inlet drag to an accompanying engine program. It also attempts to predict nozzle/afterbody performance from engine outputs that, presumably, includes some of the corrections we have mentioned above. FIGURE 29 depicts the independent subprograms for each subsystem.

The included installation corrections and associate performance maps imply that the inlet and nozzle are separate from the rest of the propulsion system. Like other programs, this limits available accounting schemes for the airframe and propulsion systems. PIPSI documentation seems to imply that the user should couple it with an engine performance program that is reasonable given the coupled nature of a propulsion system. However, if the program is not connected to an engine model with an appropriate accounting scheme or thrust definition, then it will not be "all-inclusive", and the engineers are left to add on more corrections themselves.

Takahashi & Cleary [17] note that pairing NPSS with PIPSI produces uninspiring results for installed system performance compared to what is observed in the real world. This indicates that NPSS is not an ideal companion to PIPSI. Additional corrections will be needed to develop a more representative value for installed thrust.

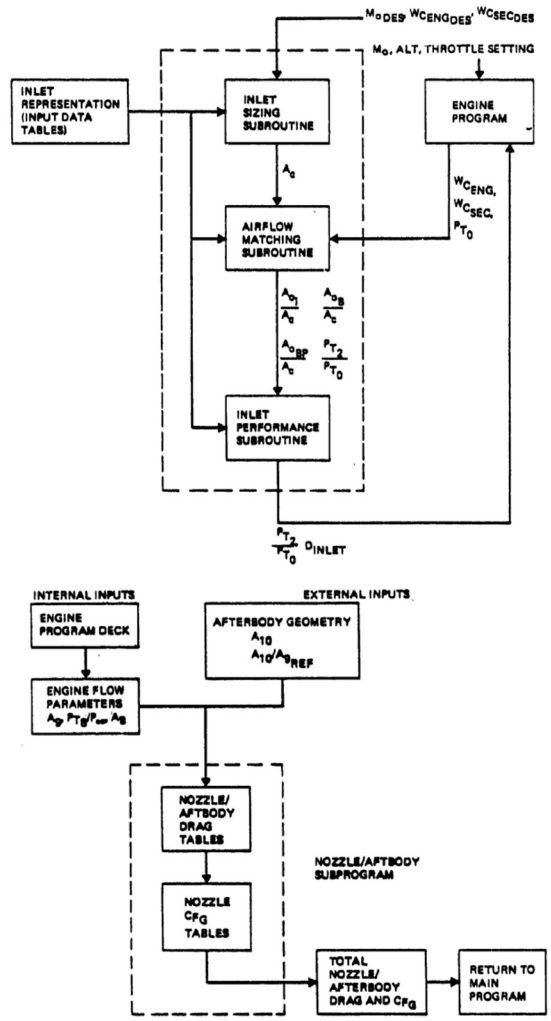


FIGURE 29. PIPSI Inlet (top) and Nozzle/Afterbody (bottom) Subprograms

VI. Sibulkin's Additive Drag Breaches the Gap Between Freestream-to-Tail and Inlet-to-Tail Uninstalled Thrust

Recall the two common methods to define and describe thrust (in equation form) coming from a practically infinite pool of potential options. We refer to these as the "Classical" and "Alternative" thrust equations, (21) and (30), respectively.

However, the two definitions of thrust and equations mean different things using identical control volumes. As such, using the disparate results interchangeably, either intentionally or not, is inappropriate.

Sibulkin [16] understood the difference between defined thrust and the force on the flight vehicle. Sibulkin combines additive drag on what we call the "Full" or "Freestream-to-Tail" thrust equation to better represent the force delivered to the flight vehicle. This definition of additive drag when applied to this specific uninstalled thrust converts the "Freestream-to-Tail" thrust into "Inlet-to-Tail" thrust. Sibulkin's significance is the observation that the "Freestream-to-Tail" thrust doesn't properly represent the force acting on the flight vehicle; the "Inlet-to-Tail" thrust better represents it.

Recall that the "Inlet-to-Tail" equation includes the internal force and stream tube stress tensor force from the inlet plane to the exhaust. The additive drag term converts between the two by subtracting the force on the pre-entry flow from the "Full" equation to result in the "Alternate" equation. Note how (44), the "Full" equation, results in (46), the "Alternate" equation when subtracting off additive drag in (45).

$$T_{full} = \dot{m}_\iota[(1+f)V_e - V_\infty] + (P_e A_e - P_\infty A_c) \tag{44}$$

$$D_{add} = \dot{m}_1 V_1 + P_1 A_1 - \dot{m}_i V_\infty - P_\infty A_c \tag{45}$$

$$T_{alternate} = T_{full} - D_{add} = \dot{m}_i[(1+f)V_e - V_1] + (P_e A_e - P_1 A_1) \tag{46}$$

We can similarly note that the "Full" thrust equation does not inherently represent the force on the flight vehicle less so than the "Alternate" thrust equation. The "Classical" equation uses a thrust definition that includes the internal force term from freestream to exhaust but does not include the stream tube stress tensor force. In fact, it expressly subtracts it from the corresponding definition of thrust.

To support this, consider completing the derivation of thrust using the "Classical" and "Alternate" definitions on a purely theoretical propulsion system that has identical flow to *CV1* and *CV3*, respectively, but does not have any solid body encapsulating the propulsion system; see FIGURE 30. The entry plane (subscript 1) is inferred to be identical to *CV3* in this diagram.

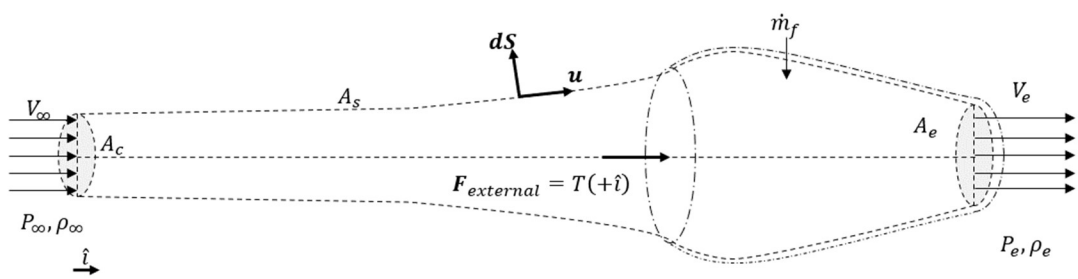


FIGURE 30. The "Generalized Swallowed Flow Control Volume", *CV1* without Solid Boundaries

This is not a real scenario, but if rigorously completed, the *CV1* result would be identical for the "Classical" equation but the *CV3* result would be different compared to the "Alternate" equation. In the "Alternate" equation, "thrust" makes a broader attempt to capture the force delivered to the flight vehicle compared to the "Classical" equation. Using the same definition of thrust as before, there would not be a body that the stream tube stresses act on as part of the thrust definition. Thus, the result would be different from a logical application of the same principles. This only demonstrates that the defined thrust in the "Classical" thrust equation explicitly does not include the stream tube surface stresses.

An important issue is that there is not a commonly used method to convert between the "Classical" and "Alternate" thrust equations like additive drag for the "Full" and "Alternate" equations.

A final observation is that there is no implied preferred sign to the external force, $F_{external}$. It may either enhance or detract from the basic momentum thrust.

VII. The Integration of Surface Pressures and Tractions Where the Propulsion Stream Tube "Wets" the Airframe or Inlet Needs to be Accounted

In section VI, we covered the differences in the common "thrust" definitions for what they do and do not include. In Section IV we noted that none of the common *installed thrust* corrections specifically account for the surface stress integration along the generalized internal stream tube (beyond the cowl lip). Thus, we realized that there is a "missing" *installed thrust* correction that applies to thrust definitions neglecting this force. This correction is not really "missing" as it is included in other thrust definitions and equations, but if using the "Classical" equation is used, the engineer must further account for the stream tube surface pressures and tractions.

Much like additive drag is considered the conversion between the "Full" and "Alternate" thrust equations, we need to consider a similar conversion between the "Classical" and "Alternate" thrust equations as an *installed thrust* correction. Jakobsson, in 1951, called the forces imparted to the airframe due to diffusion upstream of the fan face as "Front External Thrust."[23] This "produces an aerodynamic loading on the outside of the nacelle, most of which is probably on the lips of the intake."[23] Takahashi & Cleary [17] refer to a similar force as "Inlet Buoyancy" representing the force due to pressure stress integration along the diffusor surface that can positively augment *uninstalled thrust*. This assumes that the thrust force it augments is defined without those stresses included. Chaudhari & Takahashi [18,35] further note the

possible exploitation of this force on historical supersonic aircraft. A description of the propulsion system on Concorde holds that "at takeoff and during subsonic flight, 82% of the thrust is developed by the engine alone with 6% from the nozzles and 21% from the intakes." [19] It also claims that during supersonic cruise "8% of the power is derived by the engine with the other 29% being from Nozzles and an impressive 63% from the intakes."[19] Results from this show a convincing correlation augmenting the *uninstalled thrust* prediction that excludes stream tube stress forces. Jakobsson does not agree fully with Takahashi & Cleary in theory. However, in a numerical example Jakobsson predicted "Front External Thrust" to make up 13% of the flight thrust at Mach 1.4. [23] Contrast that to Chaudhari & Takahashi [18] noted that "Inlet Buoyancy" should comprise about 20% of the flight thrust at the same speed on the GE-4 engines proposed for the Boeing 2707 at 55,000-ft; Takahashi & Cleary [17] postulated "Inlet Buoyancy" could comprise up to 55% thrust on a purely hypothetical engine with an aggressive 2:1 subsonic diffusor at 40,000-ft. These percentages, without considering fine details, support the assertion that forces like "Inlet Buoyancy" and "Front External Thrust" are significant enough not to be ignored.

This "Inlet Buoyancy" force is related to but not the same as the one we consider here in that it ignores viscous losses and other irreversible losses in the internal flow between the inlet plane and the engine fan face. "Front External Thrust" is also not identical to the proposed force correction in this section, so a new name is required. We will refer to the full force as the "Stream tube Stress Force" $F_{streamtube}$ for the lack of a better term.

The equations for "Classical" and "Alternate" thrust are *uninstalled thrust* equations relating to effectively the same control volume. Therefore, $F_{streamtube}$ is the difference between the "Alternate" and "Classical" equation so that one may add $F_{streamtube}$ to the later to get the previous. The result is in (49) below for this conversion.

$$F_{streamtube} = T_{alternate} - T_{classical} \tag{47}$$

$$F_{streamtube} = \dot{m}_\iota[(1+f)V_e - V_1] + P_e A_e - P_1 A_1 - \{\dot{m}_\iota[(1+f)V_e - V_\infty] + A_e(P_e - P_\infty)\} \tag{48}$$

$$F_{streamtube} = \dot{m}_\iota(V_\infty - V_1) + A_e P_\infty - P_1 A_1 \tag{49}$$

VIII. Airframe and Propulsion Engineers need to Rigorously Define the Bookkeeping Convention to Prevent Mismatch

Over the course of this paper, we highlight that the ambiguous nature of *installed thrust* corrections can easily make the resulting engine model inaccurate. Using common methods like establishing drag from a flow through duct wind tunnel model and propulsion using PIPSI and NPSS without further correction will produce discordant data; as shown by Takahashi & Cleary [17] as well as Chaudhari & Takahashi [18,35]. Wyatt puts it best that the "fundamental aspects of the force analysis problem have become obscured by the use of conventional definitions. When conventional definitions are applied to regions outside their original scope, confusion often arises. . . with the confusion in some cases leading to total neglect of those forces." [25]

In an ideal world, we would prefer to begin with the "Alternate" ("inlet-to-tail") thrust definition and equation. Yet, we realize that most uninstalled engine data suppliers will predict thrust using the "Classical" thrust equation; so this data requires application of both Sibulkin's additive drag correction and the stream tube "buoyancy" as well as stream tube friction drag correction before further processing.

We would then select a vehicle API and EAI that would be coincident. The division between the responsibility of the aerodynamics and propulsion teams would define the propulsion system control volume. This should reduce complexity imposed by allowing some surfaces bookkept with propulsion to be excluded from the engine control volume. Then, the engineers would use the discussed conversions and installation corrections to translate the *uninstalled thrust* model (from the inlet-to-tail) to an *installed thrust* model appropriate to interact with aerodynamic data to accurately estimate aircraft performance.

This paper does not formulate a sweeping generalization to propulsive force accounting, but we hope that we have exposed how the process to estimate installed thrust can descend into a "witches brew" of confusing, erroneous and ad-hoc corrections.

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