

We Wonder Why We Wallow: Impacts of Trim Error on Speed and Flight Path Stability

Mathew Delisle¹ and Timothy T. Takahashi²
Arizona State University, Tempe, Arizona, 85287

This paper investigates the flight dynamics associated with out-of-trim (destabilized) flight. Pilots may excite oscillatory motions by trying too hard to maintain a constant, but inappropriate, angle of attack; this excites motions similar to the classical Phugoid mode. When pilots fly aircraft trying to maintain a constant, but inappropriate, horizon attitude, entirely different oscillatory modes develop that in no way resemble Phugoid-like modes. To improve aviation safety, pilots need to be trained to identify the source of wallowing flight and select an appropriate recovery technique.

Nomenclature

α	=	angle of attack (deg)
γ	=	flight path angle (deg)
ω	=	frequency
ζ	=	damping ratio
L	=	Lift, lbf
D	=	Drag, lbf
L/D	=	Aerodynamic Efficiency
VKTAS	=	Velocity in nm/hr true airspeed
ROC	=	Rate of Climb, ft/min
T	=	Thrust, lbf
W	=	Weight, lbf

I. Introduction

GROUND SCHOOL teaches pilots that aircraft are affected by both pilot imposed control force inputs and outside external aerodynamic and weather forces that may disturb it from its natural flight attitude. They also teach that the tendencies of an airplane, once disturbed from an attitude, depend upon its aerodynamic design. [1]

In this paper, we as engineers seek to move beyond a classical quasi-steady model of aircraft performance, to learn more about the dynamic motion of an aircraft that is flown in imperfect trim. Favorable aerodynamic design leads to aircraft with positive longitudinal static stability and positive longitudinal dynamic stability. There is much more to this than meets the eye of the typical aeronautical engineer.

A student pilot quickly learns that it is easy to “wallow;” to fly in oscillatory rather than steady manner, when the aircraft is imperfectly trimmed. If the aircraft is not perfectly balanced in pitch, and the pilot lacks the steadiest of hands, it quickly develops a dynamic response in pitch, speed and trajectory.

Classic texts, like Roskam define stick-fixed longitudinal stability as the “tendency of the airplane to develop forces or moments which directly oppose an instantaneous perturbation of a motion variable from a steady-state flight condition.” [2] Thus, when the nose of an aircraft is disturbed above its natural equilibrium point, the aircraft develops a nose-down moment that returns the nose to its original attitude.

Similarly, Roskam [2] introduces the classic concepts of inherent speed stability. Any increase in forward airspeed should be met by an opposing force that resists the increase in airspeed. Since drag typically

¹ M.S. Candidate, Aerospace Engineering, Arizona State University, P.O. Box 876106, Tempe, AZ.

² Professor of Practice, Aerospace Engineering, Arizona State University, P.O. Box 876106, Tempe, AZ.
Associate Fellow AIAA

increases in proportion to the dynamic pressure that, in-turn, is proportional to the square of the indicated airspeed; this criteria is fairly easy to meet unless the aircraft has unusually high induced drag and/or a propulsion system whose thrust increases (rather than lapses) with increasing airspeed.

Roskam [2] also introduces the concept of the longitudinal Short-Period Mode. This is a naturally developing damped oscillatory mode where the aircraft angle-of-attack varies in time with no change in airspeed. The aircraft typically “wobbles” within 10-ft of its nominal altitude at a frequency that is typically far less than 1-hz. Analysts consider the short-period mode as a “nuisance mode,” but pilots find an excessively slow short-period frequency coupled with an airframe that is unusually responsive in pitch difficult to hand-fly.



Figure 1 – Short Period Mode Schematic [3]

Roskam [2] finally introduces the concept of the longitudinal Phugoid Mode. This is another naturally developing oscillatory mode. In the Phugoid, the aircraft “wobbles” about a nominal straight-and-level trajectory with larger changes in speed, altitude and angle-of-attack. The period of the Phugoid Mode is measured in 10’s of seconds, with altitude variations on the order of 100’s of feet.

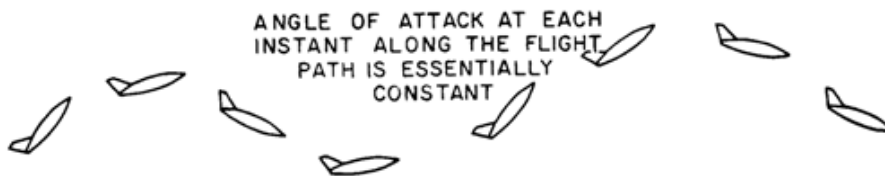


Figure 2 – Phugoid Mode Schematic [3]

Roskam [4] also covers the concept of the human-pilot-plus-airframe acting as a closed loop system. If the pilot can provide control inputs with zero “transportation delay,” he “would have no difficulty controlling pitch.” [4] Any amount of “lead” in inputs to counteract the inherent aircraft oscillatory behavior is beneficial. But Roskam states that increasing transportation delay, a reactive – the proverbial “drunk” pilot - as opposed to a proactive pilot is likely to be unable to maintain control. As the pilot reactive gain or transportation delay to disturbances increases, the system stability decreases.

So what flight dynamics do we experience when a pilot is hand flying an airplane that is imperfectly trimmed in pitch? We can conceive of situations that pilots may experience during initial take-off climb, after a major change in power setting, after a major change in aircraft drag (flap retraction), and/or after a major change in flap setting (pitching moment). We can conceive of even more destabilizing events occurring during a bailed landing “go-around.” During a go around, the pilot must quickly transition from descending flight at a constant airspeed, part-power, and flaps and gear deployed to a climbing flight at a different airspeed at full power, with flaps and gear retracted. If an engine fails during a go-around, it may suddenly introduce lateral-directional as well as additional longitudinal trim challenges to the pilot.

Thus, an out-of-trim aircraft with a mechanical failure introduces cockpit confusion and disarray that can lead to an increase in pilot reaction time and gain. Both of these factors can further destabilize otherwise stable, but dynamic flight modes, introduced by the trim-error.

In our research group, performing observations of pilots flying in a CRJ-200 simulator, we noted significant speed and flight path instability during many approaches. [5] During a bailed landing, with an engine failure during the “go-around,” we noted several occasions where pilots completely destabilized the aircraft – situations that triggered the stick-pusher and ultimately led to a crash.

Delisle & Takahashi came to realize that such events are triggered when pilots provoke and then attempt to recover from a destabilizing flight event. [6][7][8] In the case of a balked landing followed by an engine failure, the aircraft is nominally trimmed for descending flight at the final approach speed V_{ref} with landing flaps and gear deployed. Stick pressure will be needed to maintain a stabilized climb with the engines advanced to full power. Thus, any inappropriate stick force will lead to out-of-trim unstable flight. Unstabilized flight may arise when pilots fly the aircraft to visual cues (from instruments such as the flight-director or artificial-horizon) but lose perception of trim. Unstabilized flight also arises when pilots fly aircraft to specific angle-of-attack targets, but again lose perception of trim.

In this paper we seek to find insight in the dynamic behavior arising from: 1) classic mis-trim (constant-angle-of-attack trim error) as well as 2) flight where pilots slavishly follow the flight director on the artificial horizon (constant horizon attitude trim error). We will see that some piloting strategies provoke “Phugoidal” modes while others induced entirely different dynamic behavior. Piloting techniques needed to suppress or recover from these modes differ widely.

II. Classic Flight Dynamics - Phugoid Approximation

We turn to the Boeing Stability & Control short course notes [9] to best explain the industrial viewpoint of the aircraft Phugoid mode.

This short course correctly states that the fundamental equations of motion may be solved, in general, for the transfer function of response to a forcing function. They explain that the response can be divided two ways: 1) into the steady-state response due to the forcing function and 2) into the initial transient response due to the inherent stability characteristics of the airplane.

The transient mode typically expresses itself as a damped oscillation. For most aircraft, the oscillatory modes manifest themselves as one of long period (the Phugoid mode) with low damping and one of short period (the Short Period Mode) with heavy damping.

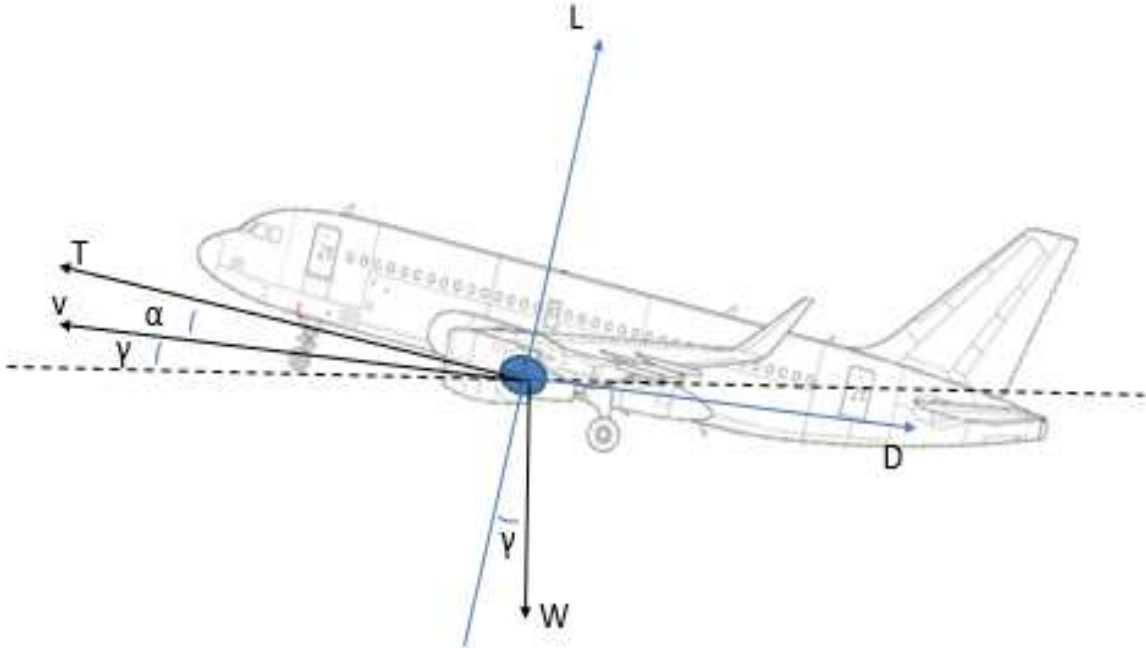


FIGURE 3 – FREE BODY DIAGRAM FOR AIRCRAFT FLIGHT FORCES

The short period mode causes transient motion in angle-of-attack (α) and load factor (N_z) but occurs at a high enough frequency where the airspeed does not change. Holding the aerodynamic design fixed, the frequency of the short period mode increases as the mass properties move the CG position forwards. The short period frequency grows higher as the indicated airspeed increases. The damping tends to decrease as

the CG position moves forwards. Interestingly, an aircraft with an aft mounted horizontal tail inherently has positive damping of the short period mode.

The long period mode causes transient motion in airspeed (VKTAS, KCAS and KEAS) and pitch attitude (γ) as the aircraft exchanges altitude for speed in a trade between potential and kinetic energy. If only long-term low frequencies are considered, the fundamental equations of motion reduce to describe an oscillatory mode with frequency:

$$\omega_{Phugoid} = \frac{g}{VKTAS} \sqrt{2} \quad (1)$$

and damping ratio:

$$\zeta_{Phugoid} = \frac{1}{\sqrt{2} \left(\frac{L}{D} \right)} \quad (2)$$

Thus, the frequency is inversely proportional to the flight speed, VKTAS, and the damping ratio is inversely proportional to the aerodynamic efficiency (L/D). In other words, the faster the aircraft travels the slower the Phugoid frequency and the greater the aerodynamic efficiency, the weaker the damping.

The Boeing handbook notes that the actual aircraft response to a step change in elevator deflection, a change in trimmed angle-of-attack, should result in damped oscillatory motion that should stabilize about a new airspeed.

III. Numerical Simulation Basics

We compute climb performance for this study using a time-step integrating point-mass simulation [10]. The code uses a tabulated aerodynamic performance output file, a 5-column propulsion data file, and a mission file indicating the flight path and procedure. This study used the portion of the code that simulates flight under nominal trimmed conditions as well as the extended code that permits flight under out-of-trim conditions [6][7][8].

The simulation's basis is from balancing all aerodynamic forces on the aircraft; see Figure 3 (previous page). The main modes used from the simulation code are climbing at constant indicated airspeed which balances the forces and keeps the aircraft trimmed, a level acceleration which keeps the aircraft at a constant altitude and allows the aircraft to accelerate, and a climb at a specified angle of attack or pitch angle which allows that aircraft to be flown in destabilized out-of-trim flight.

Given accurate aerodynamic performance data and five-column propulsion data, this numerical simulation will accurately represent the flight specified in the input mission file. The results produced by the point-mass-simulation accurately obtains results from conservation of mass, momentum, and energy equations.

A320 aerodynamic model

This paper will consider the effects of various aerodynamic, propulsive, mass-properties and human factors uncertainties upon the landing distances of a notional narrow-body twin-engine commercial airliner, reminiscent of an Airbus A320.

Transport category aircraft may have many different flap settings. At minimum, aircraft have three flap settings representing cruise, takeoff and landing. The Airbus A320, for example, has five settings: **UP**, **CONF 1+F**, **CONF 2**, **CONF 3** and **CONF FULL**. Engineers design the aircraft so that the FLAPS UP setting has ideal lift and drag divergence characteristics for en-route as well as high speed flight. CONF 1+F deploys the takeoff leading edge slats and minimally deploy the trailing edge flaps; this is one possible setting for takeoff. CONF 2 further deploys the trailing flaps, increases the maximum lift coefficient (and reduces the stall speed); it is a typical takeoff setting for this aircraft. **CONF 3** extends the leading edge slats to a landing position and fully extends the trailing edge flaps; this provides a further increase in maximum lift coefficient but with some drag penalty. Finally, the **CONF FULL** flap setting offers maximally deflected the trailing edge flaps to provide the slowest stall speed possible, but with a further increase in drag.

The aerodynamic model used in this simulation was re-created from reverse-engineered A320 flight performance model developed by Beard [12] and utilized in previous works [6][7][8]. While attempting to match Airbus published climb performance, Beard derived zero-lift-drag increments appropriate for a variety of flap settings (**CONF 1+F**, **CONF 2**, **CONF 3** and **CONF FULL**), all-engines-operating and one-engine-inoperative flight with a windmilling engine, and flight with landing gear extended.

We reverse-engineered an aerodynamic model for multiple flap configurations; see Figure 4. The aerodynamic model accurately represents the increases in lift and drag for increasing flap extension. The maximum coefficient of lift for the clean configuration is 1.4 while deploying the flaps for takeoff at CONF 2 will allow for a maximum lift coefficient of 2.67.

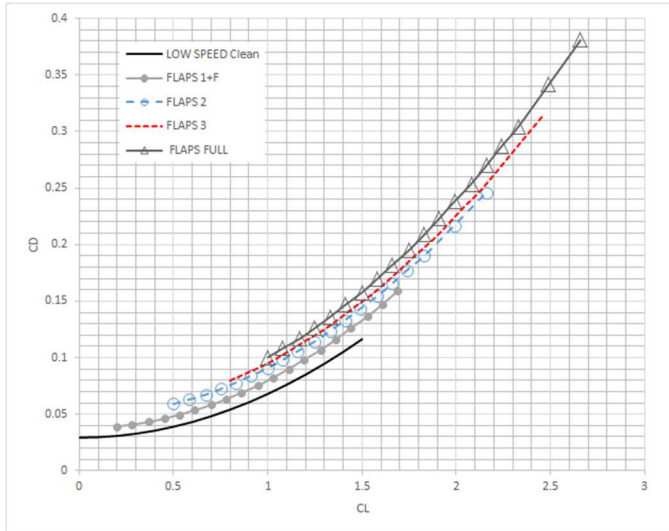


Figure 4 – Drag polars inferred from A320 published climb performance (gear up, AEO)

Beard [12] estimated that the drag increases due to an engine failing results in an increase in coefficient of drag of 0.0134. The model also takes in to account the drag increases for having the gear extended. Having gear extended will increase drag similarly to an engine inoperative. Beard [12] estimated that the drag increases due to landing gear extension causing an increase in coefficient of drag of 0.100.

We developed the propulsion data used in this simulation using NPSS [13]. Our model simulated the V2527 engine using the default two-shaft turbofan compressor-fan maps supplied by the software vendor, along with published values for reference bypass-ratio, fan-pressure-ratio and maximum turbine inlet temperature [13]. This model develops a realistic thrust-velocity and thrust-altitude lapse profile.

When we combine an accurate dynamic simulation basis with well-calibrated aerodynamic and propulsive performance data, we can perform comprehensive trade studies. We can study how the aircraft preforms under different flight conditions.

IV. Trade Studies

This paper will consider the effects of various aerodynamic, propulsive, mass-properties and human factors uncertainties upon the flight characteristics of a “hand flown” notional narrow-body twin-engine commercial airliner, reminiscent of an Airbus A320. We will pose and answer the following questions

- What is the response of the aircraft when flown out-of-trim in angle-of-attack by 0.5, 1.0, and 1.5 degrees with all engines operating?
- What is the response of the aircraft when flown out-of-trim in horizon-pitch-attitude by 0.5, 1.0, and 1.5 degrees with all engines operating?

- What is the response of the aircraft when flown out-of-trim in angle-of-attack by 0.5, 1.0, and 1.5 degrees with one engine inoperative?
- -What is the response of the aircraft when flown out-of-trim in horizon-pitch-attitude by 0.5, 1.0, and 1.5 degrees with one engine inoperative?

For this trade study, we begin the simulation with the aircraft climbing at V2+15 knots (151-KIAS) and a flight weight of 160,000-lbm. This represents flight at a lift coefficient, $CL \sim 1.57$. The one-gee stall speed under these conditions is 120 KIAS.

Turning to Figure 4, we estimate the drag coefficient as $CD \sim 0.16$ for CONF 2 flaps gear-up and all-engines-operating. Thus, the aerodynamic efficiency is $L/D \sim 9.8$.

Following the Boeing equations [Eqn 1 and 2], we can determine that the approximate Phugoid period and damping ratio for this aircraft would then be $\omega_{Phugoid} \sim 35.16$ seconds and $\zeta_{Phugoid} \sim 0.07$. We can thus expect the aircraft to demonstrate a long period and very lightly damped Phugoidal motion.

Figures 5 through 8 below shows the response of the aircraft that would be climbing at constant indicated airspeed (trimmed) as well as the response if the aircraft were to climb +0.5, +1.0, or +1.5 degrees out-of-trim in angle of attack. Figure 5 is a time history plot of aircraft altitude; Figure 6 is a time history plot of airspeed, Figure 7 is a time history plot of horizon-pitch-attitude and Figure 8 is a time history plot of load factor. In all plots, we set the major gridline spacing to 35.16-sec to represent the estimated Phugoid period. In each simulation, the nominal angle-of-attack for AEO trimmed climb is $\alpha=11.7^\circ$. Thus a +0.5-degree error is a climb at $\alpha=12.2^\circ$; a +1.0-degree error is a climb at $\alpha=12.7^\circ$ and a +1.5-degree error is a climb at $\alpha=13.2^\circ$.

In Figure 5, we see that that destabilization triggered from being out-of-trim due to an error in elevator setting (out-of-trim by various degrees in angle-of-attack) excites a clear Phugoidal mode (in time constant) for the first couple of oscillations. The relative motions exhibit a phase shift as time goes on. Interestingly all three responses for out-of-trim flight have the same period for the first couple of oscillations but diverge as time progresses.

When looking at Figures 5 through 8 together, we can see how the aircraft initially begins to ascend with an increased pitch angle and begins to lose indicated airspeed as soon as the aircraft is destabilized in angle of attack. As the aircraft decreases speed, it loses lift and eventually pitches down to descend and gain airspeed. As the aircraft gains speed, the lift goes up, pitching the aircraft up and returning it to the previous state. This process then repeats, just as Roskam describes the classical Phugoid response.

Of course, this behavior is incredibly unsettling to the pilot. The horizon attitude and airspeed swings wildly back and forth. At some points in this wallowing climb trajectory, the aircraft has a negative horizon attitude (pilot looking at the ground) and at other points the airspeed gauge will drop below the 1-g posted stall speed. If the stall warning horn is triggered by airspeed alone, it would sound. The reader should note that the aircraft is in no danger of actually stalling; as the airspeed dips below the 1-gee stall speed (120-KIAS), the load factor reaches its minimum value of well under 1.0. Of course, these wild oscillations are likely to provide substantial pilot inputs. A pilot attempting a pull-up at low airspeed will actually induce a stall, this is far worse than going along for the ride. The risk of further pilot induced oscillations is high.

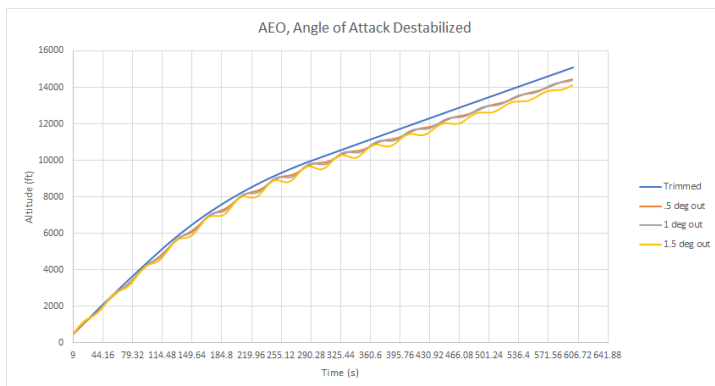


Figure 5 – Takeoff Flight Path (AEO) with mis-trim in angle-of-attack (stick-fixed)

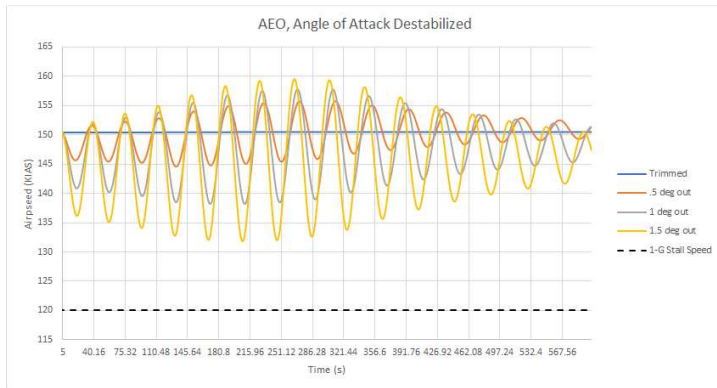


Figure 6 – Takeoff Flight history (AEO) with mis-trim in angle-of-attack (airspeed variations)

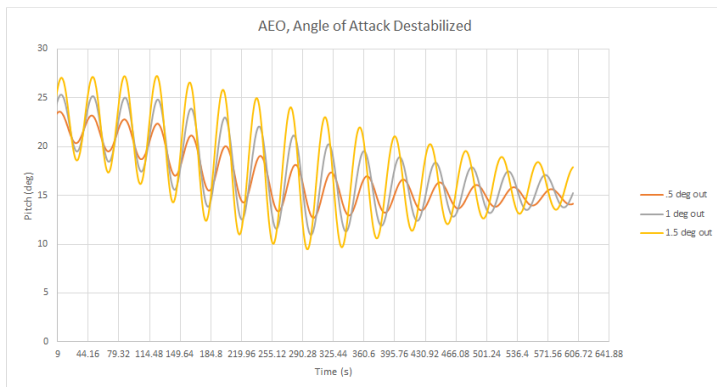


Figure 7 – Takeoff Flight history (AEO) with mis-trim in angle-of-attack (horizon attitude variations)

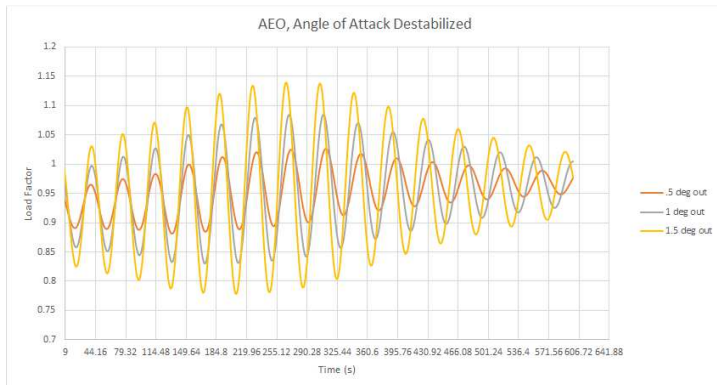


Figure 8 –Takeoff Flight history (AEO) with mis-trim in angle-of-attack (Nz variations)

In Figures 9 through 11, we simulate the alternative source for mis-trimmed flight; flight at a constant but inappropriate horizon attitude. If the horizon attitude is too high, we see a different means to a crash. Nominal flight is at an angle of attack $\alpha=11.7^\circ$ and a horizon angle of $(\alpha+\gamma)= 22.6^\circ$. $+0.5^\circ$ mis-trim would result in flight at a horizon angle at $(\alpha+\gamma)=23.1^\circ$. Mis-trim by $+0.5^\circ$ would result in flight at a horizon angle at $(\alpha+\gamma)= 23.6^\circ$

The reader is invited to compare Figure 5 to Figure 9. Mis-trim from an inappropriate horizon-attitude includes an inherent mechanism for pilot feedback (the stick is continuously adjusted to maintain horizon-attitude). As such, the time history response is seemingly smooth and in no way resembles the Phugoid mode.

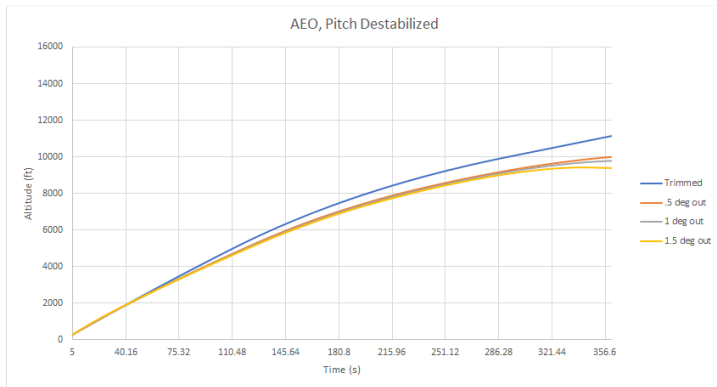


Figure 9 – Takeoff Flight Path (AEO) with mis-trim due to flying constant horizon attitude

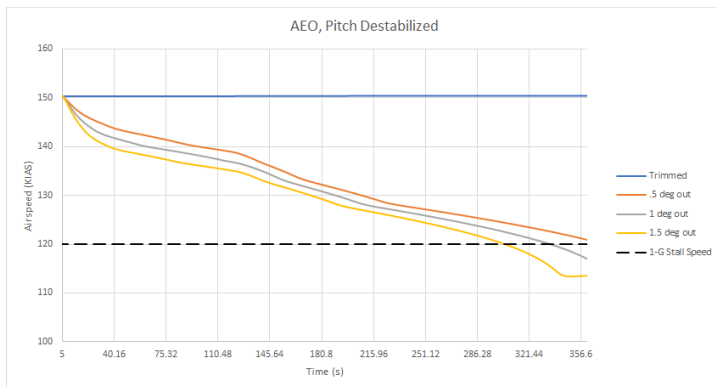


Figure 10 – Takeoff Flight history (AEO) with mis-trim due to flying constant horizon attitude (airspeed variations)

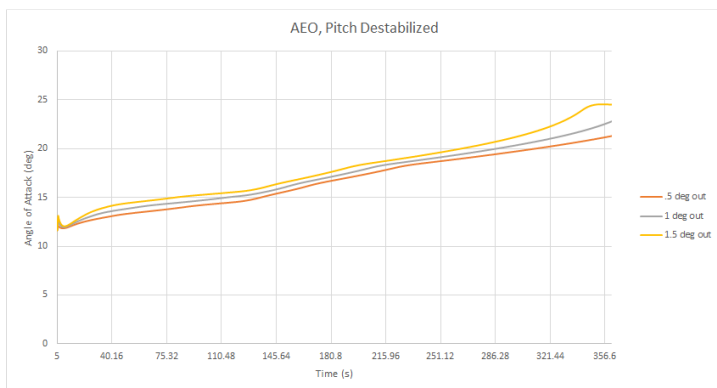


Figure 11 – Takeoff Flight history (AEO) with mis-trim due to flying constant horizon attitude (angle-of-attack variations)

Figures 10 and 11 show an overall decline in airspeed with some slight “wobble” and a corresponding increase in angle-of-attack. About three minutes into the simulation, the pilot reaches incipient stall; the angle-of-attack cannot be increased further. Any further attempts to maintain horizon attitude would be met with a stick-shaker, stall warning horn. The good news here involves the time scale of the impending stall; three minutes at a typical climb rate of ~3000/ft to bleed off speed to reach stall. Because of the long period predicted here, it is unlikely that the dynamic problems from a foreseeable, but minor mis-trim in horizon attitude will become a safety hazard with all engines operating.

Now let us repeat these simulations, but with flight with a critical engine inoperative.

Figures 12 through 14 show the response of an aircraft with one engine inoperative that is destabilized in angle of attack. The nominal angle-of-attack here is $\alpha=12.4^\circ$. With the reduced thrust and added drag of the windmilling engine, the overall climb performance is much weaker than before as is the energetics of the oscillatory flight. The period of the first couple of oscillations is closely predicted by the Phugoid approximation. The damping (probably due to the lower aerodynamic L/D from the windmilling engine) is much greater. As we increase the nose-high trim attitude, the overall climb performance declines somewhat. The airspeed wobbles are much smaller than before, and in none of our simulations ever approached the 120-KIAS 1-gee stall speed of the aircraft.

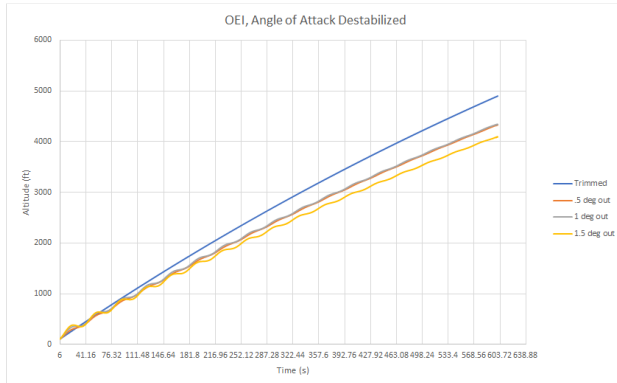


Figure 12 – Takeoff Flight Path (OEI) with mis-trim in angle-of-attack (stick-fixed)

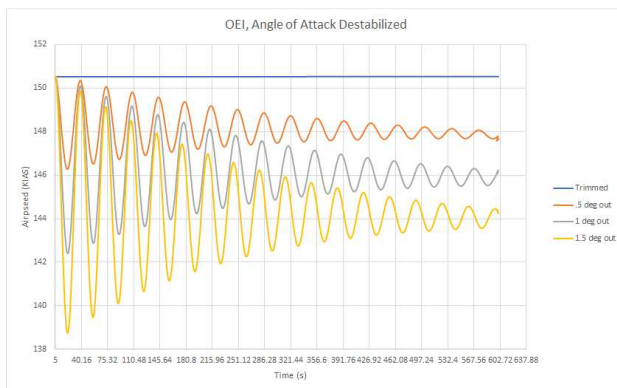


Figure 13 – Takeoff Flight History (OEI) with mis-trim in angle-of-attack (stick-fixed) (airspeed variations)

In Figures 14 and 15, we simulate engine-inoperative flight at horizon attitude. It is immediately clear that these trajectories do not exhibit any sort of Phugoidal action.

If the pilot flies the aircraft to the AEO trimmed horizon attitude ($\alpha+\gamma$) = 22.6° , the aircraft will rapidly decelerate beneath its 1-gee stall speed within ~25-seconds. As the aircraft decelerates, it will climb. Upon stall, if the pilot continues to attempt to maintain the nose-up attitude, the aircraft will eventually impact the ground. Thus, to recover from such an excursion, the pilot needs to reduce horizon attitude to regain airspeed.

Conversely, if the pilot flies the aircraft to the OEI trimmed horizon attitude ($\alpha+\gamma$) = 14.7° , the aircraft will climb at constant airspeed. If the pilot files the aircraft to a slight mis-trim, for example a $+1.0^\circ$ mis-trim, will have the pilot attempt to maintain a constant horizon attitude ($\alpha+\gamma$) = 15.7° . Under such circumstances, our simulation shows a moderate airspeed loss of ~7 KIAS and an overall slight reduction in climb performance. Such minor mis-trim is easy to correct by depressing the nose to a lower horizon attitude, whereby the aircraft would gain some speed.

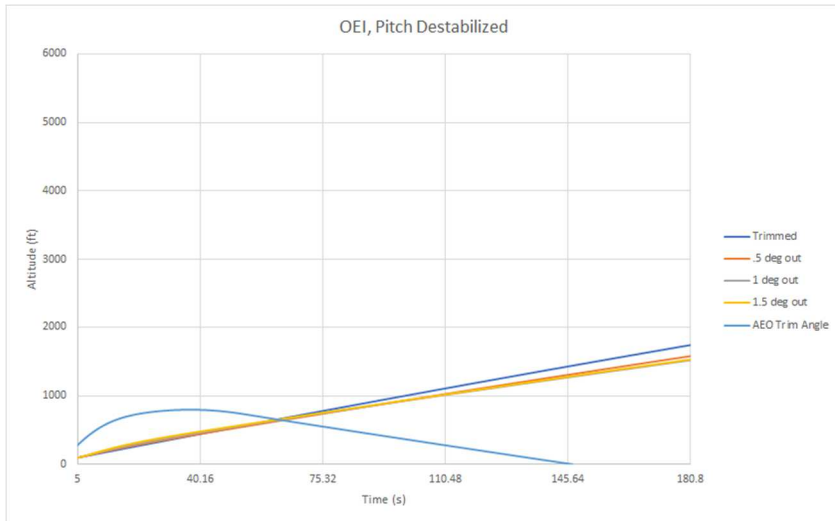


Figure 14 –Takeoff Flight Path (OEI) with mis-trim due to flying constant horizon attitude

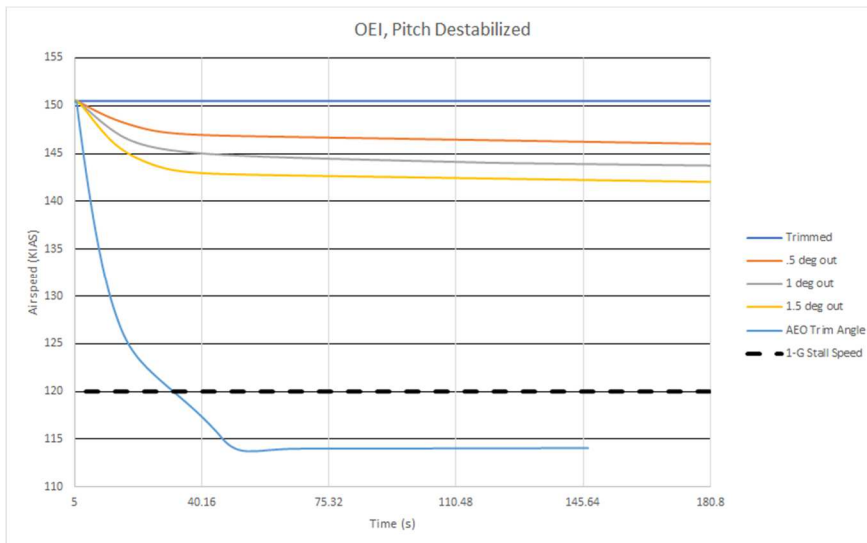


Figure 15 –Takeoff Flight History (OEI) with mis-trim due to flying constant horizon attitude (airspeed variations)

V. Summary and Conclusion

The initial reaction of the airframe to a trim-error is to change airspeed, attitude and altitude. If a pilot is unable to distinguish the source of the out-of-trim flight, his reactions to attempt to damp the motion may prove to exacerbate the situation.

We see, from the simulations presented, that aircraft destabilized by seemingly minor constant-angle-of-attack trim errors will develop Phugoid-like oscillatory response in climb. This can be a wild ride; although the aircraft does not stall, the magnitude of Phugoidal oscillations increases and the relative damping decreases as thrust levels increase. A pilot trying to hold controls fixed will not actively suppress this mode; but they will eventually damp out.

At the same time, constant-horizon-attitude flight will small trim errors produce slow variations in speed and climb performance that are easy to mitigate. Large errors in horizon-attitude, for instance pitching to a familiar attitude for AEO climb but flying with a failed engine, can result in a swift drop in airspeed leading to stall.

In either case, the initial reaction of the airframe is to change airspeed while continuing to climb. If a pilot is unable to distinguish the source of the out-of-trim flight, his reactions to attempt to damp the motion may prove to exacerbate the situation. Our earlier research [5] leads us to believe that flight dynamics tending towards an accidental stall due to major horizon-attitude error is a byproduct of pilot training and flight manuals that call out horizon attitude cues on the artificial horizon. If a pilot attempts to “ride out” a horizon-attitude error, we have seen several simulation runs with both AEO and OEI that result in airspeed loss that ends in stall.

Acknowledgements

This paper derives from unsponsored work performed in conjunction with Mr. Delisle’s thesis research for a M.S. degree in Aerospace Engineering at Arizona State University.

References

- [1] Sanderson, Private Pilot Manual, Jeppesen & Co, Denver, Co, 1972.
- [2] Roskam, J. Airplane Flight Dynamics, Part I, DAR Corp, 1982.
- [3] Hurt, H.H. Jr, (1960), *Aerodynamics for Naval Aviators* Chapter 4, A National Flightshop Reprint, Florida.
- [4] Roskam, J. Airplane Flight Dynamics, Part II, DAR Corp, 1982.
- [5] Wood, D.L. and Takahashi, T.T., "The Effect of Piloting Practices Upon Actual as Opposed to Scheduled Landing Field Performance," AIAA 2018-1756, 2018.
- [6] Delisle, M. and Takahashi, T.T., "(Un)stabilized Approach - An Introduction to Dynamic Flight Conditions during Takeoff and Landing Climb," Accepted for publication at AIAA AVIATION 2018.
- [7] Delisle, M. and Takahashi, T.T., "Speed Stability and Obstacle Clearance During Engine Inoperative Takeoff," Accepted for publication at AIAA AVIATION 2018.
- [8] Delisle, M. and Takahashi, T.T., "Floating Home: Speed Stability and Inadvertent Stalls During a Balked Landing," Accepted for publication at AIAA AVIATION 2018.
- [9] Anon. "Stability and Control in Transport Design" (Boeing S&C short course notes), Boeing D6-42433T, Boeing, Seattle, WA , 1975.
- [10] Takahashi, T.T., Aircraft Performance and Sizing Vol. I, Momentum Press, 2016.
- [11] Takahashi, T.T., Aircraft Performance and Sizing Vol. II, Momentum Press, 2016.
- [12] Beard, J.E. "Takeoff Obstacle Clearance Procedures: The Feasibility of Extended Second Segment Climb " M.S. Thesis, Arizona State University, April 2017.
- [13] NPSS, Numerical Propulsion System Simulation, Software Package, Ver. 2.3.0.1, Ohio Aerospace Institute, Cleveland, OH, 2010.