Stability and Convergence Analysis of a Decentralized Proportional-Integral Control Strategy for Collective Transport

Hamed Farivarnejad and Spring Berman

Abstract-In this paper, we study the stability and convergence properties of a decentralized proportional-integral velocity controller for collective transport by a team of pointmass robots that are rigidly attached to a payload. The controller only requires robots' velocity measurements, and the only information provided to the robots is the target speed and direction of transport. We prove that the closedloop system with proportional control alone is exponentially stable, and we derive the system's rate of convergence to the desired transport velocity. We analyze the parameters that affect this convergence rate and characterize its dependence on the robots' distribution around the payload. We add an integral controller to the proportional controller to compensate for any drift from the desired transport path and prove asymptotic stability in this case. We validate our analytical results for the proportional controller through simulations with three different robot distributions around the payload. These simulations demonstrate that the robots' distribution has the predicted effect on the convergence rate, which influences the load's rotation and drift from the desired path during the transient phase of transport. We also confirm through simulation that the proportional-integral controller drives this drift to zero while achieving the desired transport velocity.

I. INTRODUCTION

Cooperative payload manipulation by multi-robot systems has a variety of potential applications, including construction and manufacturing, assembly in space and underwater, search-and-rescue operations, and disaster response. We aim to design robot controllers that can achieve cooperative manipulation with a quantifiable degree of predictability in unknown, remote, and hazardous environments with limited data and communication. Our approach is inspired by group food retrieval in ants [1], [2], [3], a striking example of decentralized collective transport in which the transport team members do not follow predefined paths, use explicit communication, or have prior knowledge about the payload, the distribution of teammates around it, and the location of obstacles in the environment. While the ants know the direction to their nest, it is likely that their activities during transport are influenced only by their local information.

Our work in this paper constitutes an effort toward multirobot implementation of ant-like collective transport with all of these properties. We consider a team of identical pointmass robots that move on a planar surface and are rigidly attached to a payload in an arbitrary configuration, as shown in Figure 1. We assume that each robot can measure its own velocity. The robots do not have global localization or communication capabilities, and they lack information about the payload dynamics, the number of robots in the transport team, and the robots' distribution around the payload. Subject to these constraints, the robots must transport the payload to a goal location along a straight path at a regulated velocity. We assume that each robot knows the target direction to the goal and the desired transport speed.

Various control strategies have been previously proposed for cooperative manipulation in other scenarios that are not subject to all of these constraints. In some recent control approaches, such as [4], a supervisor (a human or a central computer) observes the motion of the system and communicates appropriate control commands to the robots in order to guide the payload toward the goal. Decentralized control strategies have also been proposed to improve the system's robustness to errors, failures, and disturbances. Some of these methods are leader-follower algorithms in which the leader takes the main role in planning the payload trajectory and controlling its motion to the goal, while the other robots contribute to the transport motion in a coordinated manner using consensus [5] or an estimation method based on force sensing [6]. In other works, all robots in the transport team are assumed to be identical. In [7], a decentralized approach is proposed in which robots push a large load to a goal when their line of sight to the goal is occluded by the load. In other methods, robots communicate their measurements to each other in order to estimate parameters of an unknown payload [8], [9]. Other approaches do not rely on communication or prior information about the payload's dynamics, but they require a supervisor to define trajectories beforehand for the robots and the payload [10], [11], [12]. A strategy inspired by formation control is proposed in [13] for the case of a flexible payload that requires regulation of contact forces. In [14], [15], [16], [17], [18], adaptive robust control approaches are proposed that combine a stabilizing robust term with a regression term in the controller. These approaches require prior information about the robots' distribution around the payload.

The work in [19] proposes a decentralized approach to the problem that we address, in which each robot applies a force to the payload that is defined as a proportional velocity controller. It is demonstrated that the payload moves in a straight line toward the goal with no more than 180° of rotation. However, there is no stability analysis that guarantees the convergence of the system's dynamics to the desired motion. In this paper, we investigate the stability properties

This research was supported by ONR Young Investigator Award N00014-16-1-2605.

Hamed Farivarnejad and Spring Berman are with the School for Engineering of Matter, Transport and Energy, Arizona State University (ASU), Tempe, AZ 85281, USA {Hamed.Farivarnejad, Spring.Berman}@asu.edu



Fig. 1. Illustration of a collective transport team with four point-mass robots and the associated coordinate systems.

of a proportional control scheme that is a modification of the one presented in [19], in which the robots apply a force only toward the goal. In our control scheme, the robots also exert a force component perpendicular to the target direction, in order to maintain a straight transport trajectory. We prove that the closed-loop system is exponentially stable with this controller, and we characterize the rate of the system's convergence to the target transport velocity in terms of the robots' distribution around the payload. In addition, we introduce an integral controller to drive any drift of the payload from the desired path to zero. We note that our analysis on pure proportional control also applies to our previous decentralized strategy for collective transport [20], during the phase when the system trajectories enter the boundary layer defined in the sliding mode controllers.

II. DYNAMICAL MODEL

We model the dynamics of the system in Figure 1, a load that is transported by N point-mass robots, which we studied in our previous work [20]. Here, we derive the equations of motion for the entire system, comprised of both the load and the robots, whereas in [20] we derived the dynamics of each robot. We define m_r as the mass of each robot, m_o as the mass of the payload, and I_o as the load's moment of inertia about the axis normal to the plane of the motion and passing through its center of gravity. We also define \mathbf{r}_c as the vector from the center of mass of the entire system (CM_o) to the load's center of mass, and \mathbf{r}_i as the vector from CM_o to the attachment point of robot *i*. Both \mathbf{r}_c and \mathbf{r}_i are expressed in the inertial reference frame shown in Figure 1, defined such that the x-axis points in the target direction of transport. Then, the mass m and moment of inertia I of the entire system are given by:

$$m = m_o + Nm_r,$$

$$I = I_o + m_o \|\mathbf{r}_c\|^2 + m_r \sum_{i=1}^N \|\mathbf{r}_i\|^2.$$
 (1)

Each robot *i* applies an actuating force $\mathbf{u}_i = [u_{i,x} \ u_{i,y}]^T$ to the payload. We denote the vector of all applied forces by

 $\mathbf{u} = \left[(\mathbf{u}_1)^T \cdots (\mathbf{u}_N)^T \right]^T.$

We define the position of CM_o in the inertial reference frame as $[x_o y_o]^T$ and the load's orientation in this frame as θ_o . We will use $\mathbf{q}_o = [x_o y_o \theta_o]^T$ as generalized coordinates that describe the motion of the entire system. Then we can write the equation of motion of the system as:

$$\begin{bmatrix} m\mathbf{I} & 0\\ 0 & I \end{bmatrix} \ddot{\mathbf{q}}_o = \begin{bmatrix} \mathbf{I} & \cdots & \mathbf{I}\\ \hat{\mathbf{r}}_1 & \cdots & \hat{\mathbf{r}}_N \end{bmatrix} \mathbf{u},$$
 (2)

where \mathbf{I} is the identity matrix and $\hat{\mathbf{r}}$ is a skew-symmetric matrix defined by $\mathbf{r}_i \times \mathbf{u} = \hat{\mathbf{r}}_i \mathbf{u}$.

Let \dot{x}_i and \dot{y}_i be the speed of robot *i* along the *x* and *y* axes of the inertial frame. We define the components of \mathbf{u}_i for each robot *i* as proportional velocity controllers:

$$u_{i,x} = k(v_{des} - \dot{x}_i), \quad u_{i,y} = k(-\dot{y}_i),$$
 (3)

where k is the controller gain and v_{des} is the desired transport speed. This controller drives each robot's velocity to v_{des} along the desired direction of transport, with no velocity component perpendicular to this direction. When all robots attain this velocity, the load moves in the target direction at speed v_{des} with zero angular velocity.

Using the kinematic equations of the payload, we can obtain expressions for \dot{x}_i and \dot{y}_i in terms of $\dot{\mathbf{q}}_o$ and then rewrite Equation (3) as:

$$u_{i,x} = k \left(v_{des} - \dot{x}_o + \dot{\theta}_o ||\mathbf{r}_i|| \sin(\theta_o + \theta_i) \right),$$

$$u_{i,y} = k \left(-\dot{y}_o - \dot{\theta}_o ||\mathbf{r}_i|| \cos(\theta_o + \theta_i) \right),$$
(4)

where θ_i is the angle of vector \mathbf{r}_i with respect to a local coordinate frame fixed to the load, as shown in Figure 2. Substituting the controller (4) for each robot into Equation (2), we obtain the following equations for the closed-loop system:

$$\begin{split} m\ddot{x}_{o} &= k \left(N(v_{des} - \dot{x}_{o}) + \dot{\theta}_{o} \sum_{i=1}^{N} ||\mathbf{r}_{i}|| \sin(\theta_{o} + \theta_{i}) \right), \\ m\ddot{y}_{o} &= k \left(-N\dot{y}_{o} - \dot{\theta}_{o} \sum_{i=1}^{N} ||\mathbf{r}_{i}|| \cos(\theta_{o} + \theta_{i}) \right), \\ I\ddot{\theta}_{o} &= \\ -k \sum_{i=1}^{N} \left(\dot{y}_{o} ||\mathbf{r}_{i}|| \cos(\theta_{o} + \theta_{i}) + \dot{\theta}_{o} ||\mathbf{r}_{i}||^{2} \cos^{2}(\theta_{o} + \theta_{i}) \right) \\ +k \sum_{i=1}^{N} (\dot{x}_{o} - v_{des}) ||\mathbf{r}_{i}|| \sin(\theta_{o} + \theta_{i}) \\ -k \sum_{i=1}^{N} \dot{\theta}_{o} ||\mathbf{r}_{i}||^{2} \sin^{2}(\theta_{o} + \theta_{i}). \end{split}$$
(5)

Defining $s_x \equiv \dot{x}_o - v_{des}$ and $s_y \equiv \dot{y}_o$, the closed-loop dynamics can be rewritten in the following compact form:

$$\begin{split} m\dot{s}_x + c_t s_x - kf_s(\theta_o)\theta_o &= 0, \\ m\dot{s}_y + c_t s_y + kf_c(\theta_o)\dot{\theta}_o &= 0, \\ I\ddot{\theta}_o + c_r\dot{\theta}_o - kf_s(\theta_o)s_x + kf_c(\theta_o)s_y &= 0, \end{split}$$
(6)



Fig. 2. Illustration of the geometric parameters that express the position of a robot in the local coordinate frame of the load.

where
$$c_t = kN$$
, $c_r = k \sum_{i=1}^{N} ||\mathbf{r}_i||^2$, and:

$$f_s(\theta_o) = \sum_{i=1}^{N} ||\mathbf{r}_i|| \sin(\theta_o + \theta_i),$$

$$f_c(\theta_o) = \sum_{i=1}^{N} ||\mathbf{r}_i|| \cos(\theta_o + \theta_i).$$
(7)

III. STABILITY ANALYSIS

In this section, we characterize the stability of the equilibria of the closed-loop system (6). Defining $\mathbf{z} = [s_x \, s_y \, \theta_o \, \dot{\theta}_o]^T$ as the state vector, we find that the system has no isolated equilibrium point and that the set \mathcal{M} , defined as:

$$\mathcal{M} = \left\{ \mathbf{z} \in \mathbb{R}^4 | s_x, s_y, \dot{\theta}_o = 0 \right\},\tag{8}$$

is a continuum of equilibrium points, *i.e.* an invariant set. When the system state is in this set, the payload moves directly to the goal along a straight path $(s_y = 0)$ at a regulated velocity $(s_x = 0)$ with no rotational motion $(\dot{\theta}_o =$ 0). The following theorem characterizes the convergence of the system trajectories to \mathcal{M} .

Theorem 3.1: The trajectories of system (2) with the decentralized controllers in Equation (3) exponentially converge to the set \mathcal{M} .

Proof: We consider the following Lyapunov function:

$$V = \frac{1}{2}m(s_x^2 + s_y^2) + \frac{1}{2}I\dot{\theta}_o^2.$$
 (9)

The time derivative of this function is:

$$\dot{V} = -c_t s_x^2 - c_t s_y^2 - c_r \dot{\theta}_o^2 + 2k (f_s(\theta_o) s_x - f_c(\theta_o) s_y) \dot{\theta}_o.$$
(10)

Defining $\mathbf{z}_1 = [s_x \ s_y \ \dot{\theta}_o]^T$, Equation (10) can be written as:

$$\dot{V} = -\mathbf{z}_1^T \mathbf{Q} \mathbf{z}_1, \tag{11}$$

in which

$$\mathbf{Q} = \begin{bmatrix} c_t & 0 & kf_s \\ 0 & c_t & -kf_c \\ kf_s & -kf_c & c_r \end{bmatrix}.$$
 (12)

To prove the convergence of the system, we need to show that \mathbf{Q} is positive definite, or equivalently, that all its eigenvalues are positive. These eigenvalues are given by:

$$\lambda_{1} = c_{t},$$

$$\lambda_{2} = \frac{1}{2} \left((c_{t} + c_{r}) + \sqrt{4k^{2}(f_{s}^{2} + f_{c}^{2}) + (c_{t} - c_{r})^{2}} \right),$$

$$\lambda_{3} = \frac{1}{2} \left((c_{t} + c_{r}) - \sqrt{4k^{2}(f_{s}^{2} + f_{c}^{2}) + (c_{t} - c_{r})^{2}} \right).$$
(13)

Since c_t and c_r are strictly positive numbers, we can conclude that λ_1 and λ_2 are strictly positive as well. Therefore, we only have to determine the sign of λ_3 . We first investigate the term $\xi \equiv \sqrt{f_s^2 + f_c^2}$.

Proposition 3.2: For the system described by Equation (2) with the robot controllers (3), ξ is a constant that is equal to $||\sum_{i=1}^{N} \mathbf{r}_{i}||$.

Proof: Using Equation (7), we can write:

$$\begin{aligned} \xi^2 &= \sum_{i=1}^{N} ||\mathbf{r}_i||^2 \\ &+ \sum_{i=1}^{N} \sum_{j \neq i}^{N} ||\mathbf{r}_i|| ||\mathbf{r}_j|| \cos(\theta_o + \theta_i) \cos(\theta_o + \theta_j) \\ &+ \sum_{i=1}^{N} \sum_{j \neq i}^{N} ||\mathbf{r}_i|| ||\mathbf{r}_j|| \sin(\theta_o + \theta_i) \sin(\theta_o + \theta_j). \end{aligned}$$
(14)

Combining the second two terms on the right-hand side, we obtain:

$$\xi^{2} = \sum_{i=1}^{N} ||\mathbf{r}_{i}||^{2} + \sum_{i=1}^{N} \sum_{j \neq i}^{N} ||\mathbf{r}_{i}|| ||\mathbf{r}_{j}|| \cos(\theta_{i} - \theta_{j}).$$
(15)

Denoting the components of \mathbf{r}_i in the local coordinate frame as \bar{x}_i and \bar{y}_i , we have that $\bar{x}_i = ||\mathbf{r}_i||\cos(\theta_i)$ and $\bar{y}_i =$ $||\mathbf{r}_i||\sin(\theta_i)$. We can then rewrite Equation (15) as:

$$\xi^{2} = \left(\sum_{i=1}^{N} (\bar{x}_{i}^{2} + \bar{y}_{i}^{2}) + \sum_{i=1}^{N} \sum_{j \neq i}^{N} (\bar{x}_{i}\bar{x}_{j} + \bar{y}_{i}\bar{y}_{j})\right).$$
(16)

Finally, by separating the x and y components, we can write:

$$\xi^{2} = \left((\bar{x}_{1} + \dots + \bar{x}_{N})^{2} + (\bar{y}_{1} + \dots + \bar{y}_{N})^{2} \right), \qquad (17)$$

which implies that $\xi = ||\sum_{i=1}^{N} \mathbf{r}_i||$.

Now, we can analyze the sign of λ_3 .

Proposition 3.3: For a transport team with a fixed number of robots and a fixed configuration on the load, λ_3 in Equation (13) is strictly positive.

Proof: First, we calculate the critical value of ξ , defined as ξ_{cr} , at which λ_3 becomes zero. If we can show that ξ is always less than this value, then we can conclude that λ_3 is always positive. From Equation (13), we calculate ξ_{cr} as:

$$\xi_{cr}^2 = N \sum_{i=1}^{N} ||\mathbf{r}_i||^2.$$
(18)

From the triangle inequality, we know that $||\sum_{i=1}^{N} \mathbf{r}_i|| \leq \sum_{i=1}^{N} ||\mathbf{r}_i||$, and by squaring both sides of this inequality, we have:

$$||\sum_{i=1}^{N} \mathbf{r}_{i}||^{2} \leq \left(\sum_{i=1}^{N} ||\mathbf{r}_{i}||\right)^{2}.$$
 (19)

From the Cauchy-Schwarz inequality [21], we know that

$$\left(\sum_{i=1}^{N} ||\mathbf{r}_{i}||\right)^{2} \leq N \sum_{i=1}^{N} ||\mathbf{r}_{i}||^{2}.$$
 (20)

which means that $\xi^2 \leq \xi_{cr}^2$ and consequently, $\xi \leq \xi_{cr}$. Excluding physically impossible configurations in which all robots occupy a single point on the perimeter of the load, which results in $\xi = \xi_{cr}$, λ_3 is strictly positive.

Since all the eigenvalues of **Q** are positive, **Q** is positive definite. Furthermore, the Lyapunov function (9) can be written in the quadratic form $V = \mathbf{z}_1^T \mathbf{P} \mathbf{z}_1$, where:

$$\mathbf{P} = \begin{bmatrix} m & 0 & 0\\ 0 & m & 0\\ 0 & 0 & I \end{bmatrix}.$$
 (21)

Then, we have the following inequalities, [22]:

$$\lambda_{min}(\mathbf{P})||\mathbf{z}_1||^2 \leq V(\mathbf{z}_1) \leq \lambda_{max}(\mathbf{P})||\mathbf{z}_1||^2$$
 (22)

In addition, using Equation (11), the following upper bound can be established for \dot{V} :

$$\dot{V}(\mathbf{z}_1) \leq -\lambda_{min}(\mathbf{Q}) ||\mathbf{z}_1||^2$$
 (23)

Therefore, we can write:

$$\dot{V} \leq -\frac{\lambda_{min}(\mathbf{Q})}{\lambda_{max}(\mathbf{P})}V$$
 (24)

and by Theorem 4.10 in [22], we can conclude that trajectories of the system (6) exponentially converge to the invariant set \mathcal{M} . This completes the proof of Theorem 3.1.

IV. CONVERGENCE ANALYSIS

Given the exponential stability of the closed-loop system, we can describe the convergence of its trajectories in a qualitative fashion using an exponential function that gives the lowest possible rate of convergence to \mathcal{M} . According to Theorem 4.10 in [22], the following inequality holds:

$$||\mathbf{z}_1(t)|| \leq b||\mathbf{z}_1(t_0)|| e^{-\epsilon t}, \quad t \geq t_0,$$
 (25)

where $b = \sqrt{\frac{\lambda_{max}(\mathbf{P})}{\lambda_{min}(\mathbf{P})}}$ and

$$\epsilon = \frac{\lambda_{min}(\mathbf{Q})}{\lambda_{max}(\mathbf{P})}.$$
(26)

Thus, ϵ bounds the convergence rate of the system trajectories. We now show how ϵ can be characterized in terms of the distribution of the robots around the load. Toward this end, we first determine $\lambda_{min}(\mathbf{Q})$.

Proposition 4.1: For a transport team with a fixed number of robots and a fixed configuration on the load, $\lambda_{min}(\mathbf{Q}) = \lambda_3$, defined in Equation (13). *Proof:* From Equation (13), we see that $\lambda_3 < \lambda_2$. Therefore, we only need to compare λ_1 and λ_3 . We rewrite λ_3 as:

$$\lambda_3 = \frac{1}{2}(c_t + c_r) - \frac{1}{2}(|c_t - c_r| + 2\varrho), \qquad (27)$$

where ρ is a positive number that monotonically increases with ξ . In this expression, if $c_r \leq c_t$, then $\lambda_3 = c_r - \rho$, which is clearly smaller than c_r . Hence, in this case, $\lambda_3 \leq c_r \leq c_t = \lambda_1$. If $c_t \leq c_r$, then $\lambda_3 = c_t - \rho$, which is clearly smaller than c_t . Thus, in this case, $\lambda_3 \leq c_t = \lambda_1$ as well. Therefore, we can conclude that $\lambda_{min}(\mathbf{Q}) = \lambda_3$.

To simplify our subsequent calculations, we replace the vector \mathbf{z}_1 in our stability analysis with $\mathbf{z}_2 = [s_x \ s_y \ r_g \dot{\theta}_o]^T$, where r_g is the radius of gyration of the system,

$$r_g = \left(\frac{I_o + m_o \|\mathbf{r}_c\|^2 + m_r \sum_{i=1}^N \|\mathbf{r}_i\|^2}{m_o + Nm_r}\right)^{\frac{1}{2}}.$$
 (28)

With this replacement, we can write the Lyapunov function (9) as $V = \mathbf{z}_2^T \mathbf{P}_2 \mathbf{z}_2$, where $\mathbf{P}_2 = m\mathbf{I} \in \mathbb{R}^{3\times3}$, and $\lambda_{max}(\mathbf{P}_2) = m$. Note that m is independent of the robot configuration on the load. We can also write Equation (11) as $\dot{V} = \mathbf{z}_2^T \mathbf{Q}_2 \mathbf{z}_2$. Setting $\lambda_{max}(\mathbf{P}) = m$, we can calculate ϵ as:

$$\epsilon = \frac{1}{2mr_g^2} \left((r_g^2 c_t + c_r) - \sqrt{4r_g^2 \xi^2 + (r_g^2 c_t - c_r)^2} \right).$$
(29)

We see that for a fixed number of robots, the distribution of the robots around the load (*i.e.*, the set of vectors \mathbf{r}_i) affects σ through the parameters r_g , c_r , and ξ . It is difficult to analyze the effect on all three parameters simultaneously. However, by following the procedure in the proof of Proposition 4.1, we can write ϵ in the following form:

$$\epsilon = \frac{k}{2mr_g^2} \left(N(r_g^2 + \rho^2 - |r_g^2 - \rho^2|) - 2\delta \right), \quad (30)$$

where $\rho^2 = \frac{1}{N} \sum_{i=1}^{N} ||\mathbf{r}_i||^2$ and δ is a positive number that monotonically increases with $r_g \xi$.

We now show that we can derive simplified expressions for ϵ based on the relative magnitudes of r_g and ρ . From Equation (28), the term $(r_g^2 - \rho^2)$ in Equation (30) can be written as:

$$(r_g^2 - \rho^2) = \frac{a}{a+N}(r_{g,o}^2 + ||\mathbf{r}_c||^2 - \rho^2), \qquad (31)$$

where $a = m_o/m_r$ and $r_{g,o}$ is the load's radius of gyration. Furthermore, we can write the vector \mathbf{r}_c as:

$$\mathbf{r}_c = -\frac{m_r}{m_o + Nm_r} \sum_{i=1}^N \mathbf{r}_{i,c}$$
(32)

where $\mathbf{r}_{i,c}$ is the vector from the load's center of mass to robot *i* (see Figure 2). For each robot *i*, we have the relation $\mathbf{r}_i = \mathbf{r}_c + \mathbf{r}_{i,c}$, and by applying the triangle inequality, we have that $||\mathbf{r}_i|| \leq ||\mathbf{r}_c|| + ||\mathbf{r}_{i,c}||$. Squaring both sides of this inequality and summing the resulting terms over i = 1, ..., N, we obtain:

$$N\rho^2 \leq N||\mathbf{r}_c||^2 + N\rho_c^2 + 2||\mathbf{r}_c||\sum_{i=1}^N ||\mathbf{r}_{i,c}||,$$
 (33)

where $\rho_c^2 = \frac{1}{N} \sum_{i=1}^{N} ||\mathbf{r}_{i,c}||^2$. Hence, we can write:

$$-\rho_c^2 - \frac{2}{N} ||\mathbf{r}_c|| \sum_{i=1}^N ||\mathbf{r}_{i,c}|| \leq ||\mathbf{r}_c||^2 - \rho^2.$$
(34)

Moreover, using the expression for \mathbf{r}_c in Equation (32) and applying Equation (19) and Equation (20) to the vectors $\mathbf{r}_{i,c}$ we can write:

$$-\rho_c^2 - \frac{2N}{a+N}\rho_c^2 \leq ||\mathbf{r}_c||^2 - \rho^2,$$
(35)

which yields the following lower bound for $(r_q^2 - \rho^2)$:

$$\frac{a}{a+N}\left(r_{g,o}^2 - (1+\frac{2N}{a+N})\rho_c^2\right) \leq (r_g^2 - \rho^2).$$
(36)

Note that this lower bound is a function of a single parameter, ρ_c , that depends on the distribution of robots around the load. We can now determine the sign of $(r_g^2 - \rho^2)$ for the following two cases:

(a) $r_g < \rho$. This case happens when ρ_c is sufficiently large to make the lower bound in (36) a negative large number. This occurs when the robots are mostly located at positions that are far from the load's center of mass. In this scenario,

$$\epsilon = \frac{k}{m} \left(N - \frac{\delta}{r_g^2} \right). \tag{37}$$

This means that for a fixed number of robots, ϵ mainly depends on the value of ξ through δ .

(b) $r_g > \rho$. This case happens when ρ_c is a small number that makes the lower bound in (36) positive. This occurs when the robots have a uniform and close-to-symmetric distribution around the load. Under this condition,

$$\epsilon = \frac{k}{mr_q^2} (N\rho^2 - \delta). \tag{38}$$

Here, ϵ is not as sensitive to changes in δ (and hence ξ) as it is in the first case, since such changes could be compensated by the value of ρ^2 .

V. DRIFT COMPENSATION BY INTEGRAL CONTROL

When the proposed proportional controller is used, the load will inevitably drift away from the line between its initial position and its target position, which we will refer to as the *desired path*. To eliminate this drift, we add an integral term to Equation (3) and modify the control law as follows:

$$u_{i,x} = k(v_{des} - \dot{x}_i) + k_I \int_o^t (v_{des} - \dot{x}_i) d\tau,$$

$$u_{i,y} = k(-\dot{y}_i) + k_I \int_o^t (-\dot{y}_i) d\tau.$$
 (39)

With this new controller, the closed-loop dynamics in Equation (6) can be rewritten as:

$$\begin{split} & m\ddot{\sigma}_x + c_t\dot{\sigma}_x + k_IN\sigma_x - kf_s(\theta_o)\theta_o - k_I\eta_s = 0, \\ & m\ddot{\sigma}_y + c_t\dot{\sigma}_y + k_IN\sigma_y + kf_c(\theta_o)\dot{\theta}_o + k_I\eta_c = 0, \\ & I\ddot{\theta}_o + c_r\dot{\theta}_o - kf_s(\theta_o)\dot{\sigma}_x + kf_c(\theta_o)\dot{\sigma}_y \\ & - k_I\left(f_s(\theta_o)\sigma_x - f_c(\theta_o)\sigma_y - \sum_{i=1}^N(\eta_{i,s}\dot{\eta}_{i,s} + \eta_{i,c}\dot{\eta}_{i,c})\right) = 0, \end{split}$$

$$(40)$$

in which

$$\sigma_x = \int_o^t s_x d\tau, \quad \sigma_y = \int_o^t s_y d\tau, \quad (41)$$

and

$$\eta_s = \sum_{i=1}^N \eta_{i,s}, \quad \eta_c = \sum_{i=1}^N \eta_{i,c},$$
(42)

where

$$\eta_{i,s} = ||\mathbf{r}_i|| \int_o^t \sin(\theta_o + \theta_i) \dot{\theta}_o d\tau,$$

$$\eta_{i,c} = ||\mathbf{r}_i|| \int_o^t \cos(\theta_o + \theta_i) \dot{\theta}_o d\tau.$$
 (43)

This model has two more state variables than Equation (6): σ_x , which represents the accumulation of error from the desired velocity, and σ_y , which represents the drift from the desired path. Defining the new state vector as $\zeta = [\sigma_x \ \sigma_y \ \dot{\sigma}_x \ \dot{\sigma}_y \ \theta_o \ \dot{\theta}_o]^T$, we see that the set $\mathcal{B} = \{\zeta \in \mathbb{R}^6 | \sigma_x, \sigma_y, \dot{\sigma}_x, \dot{\sigma}_y, \dot{\theta}_o = 0\}$ is a continuum of equilibrium points, *i.e.* an invariant set, and when the system state is in this set, then the accumulation of velocity error σ_x , and more importantly, the drift from the desired path σ_y , are driven to zero while $\dot{\sigma}_x(=s_x), \ \dot{\sigma}_y(=s_y)$, and the angular velocity $\dot{\theta}_o$ still converge to zero. The following theorem characterizes the convergence of the trajectories of the closed-loop system (40) to \mathcal{B} .

Theorem 5.1: The trajectories of system (2) with the decentralized controllers in Equation (39) asymptotically converge to the set \mathcal{B} .

Proof: We modify the Lyapunov function V in Equation (9) as:

$$W = V + \frac{1}{2}k_I \sum_{i=1}^{N} \left((\sigma_x - \eta_{i,s})^2 + (\sigma_y + \eta_{i,c})^2 \right). \quad (44)$$

The time derivative of this function is calculated as:

$$\begin{split} \dot{W} &= -c_t (\dot{\sigma}_x^2 + \dot{\sigma}_y^2) - c_r \dot{\theta}_o^2 + 2k (f_s(\theta_o) \dot{\sigma}_x - f_c(\theta_o) \dot{\sigma}_y) \dot{\theta}_o \\ &- k_I N (\sigma_x \dot{\sigma}_x + \sigma_y \dot{\sigma}_y) \\ &+ k_I (\dot{\sigma}_x \eta_s + \sigma_x f_s(\theta_o) \dot{\theta}_o - \dot{\sigma}_y \eta_c - \sigma_y f_c(\theta_o) \dot{\theta}_o) \\ &- k_I \sum_{i=1}^N (\eta_{i,s} \dot{\eta}_{i,s} + \eta_{i,c} \dot{\eta}_{i,c}) \\ &+ k_I \sum_{i=1}^N ((\sigma_x - \eta_{i,s}) (\dot{\sigma}_x - \dot{\eta}_{i,s}) + (\sigma_y + \eta_{i,c}) (\dot{\sigma}_y + \dot{\eta}_{i,c})) \,. \end{split}$$

$$(45)$$

Using Equation (42) and the fact that $\dot{\eta}_s = f_s(\theta_o)\dot{\theta}_o$ and $\dot{\eta}_c = f_c(\theta_o)\dot{\theta}_o$, we can cancel many terms in the expression for \dot{W} and simplify it to:

$$\dot{W} = -\mathbf{z}_1^T \mathbf{Q} \mathbf{z}_1, \tag{46}$$

where \mathbf{z}_1 is the same vector as in Equation (11), *i.e.* $\mathbf{z}_1 = [\dot{\sigma}_x \ \dot{\sigma}_y \ \dot{\theta}_o]^T$. As we see, the time derivative is negative semidefinite. Thus, we can conclude that W is bounded. Also, we see that when \dot{W} is identically zero, *i.e.* $\dot{W} \equiv 0$, we have $\dot{\sigma}_x, \dot{\sigma}_y, \dot{\theta}_o \equiv 0$, and from Equation (40), we can obtain $\sigma_x, \sigma_y \equiv 0$. Thus, by LaSalle's invariance principle [22], we conclude that the system trajectories asymptotically converge to the aforementioned invariant set. In other words, while the objectives with the proportional controller are still achieved, the drift from the desired path is driven to zero.

VI. SIMULATION RESULTS

A. Proportional control

In this section, we validate our analysis with simulation results for collective transport by a team of robots that are arranged in three different distributions around a payload. We study the effect of the robot distribution on the convergence rate of the system to the target transport velocity, the amount of rotation exhibited by the load, and the translational drift of the load from the desired path. The load is modeled as a homogeneous circular ring with mass $m_o = 1$ kg and moment of inertia $I_o = 0.33$ kg·m². Six point-mass robots, each with mass $m_r = 0.05$ kg, are rigidly attached to the load. The controller gain is k = 0.08 and the target transport speed is $v_{des} = 0.1$ m/s. The simulations were each run for 200 s.

Figure 3-Figure 5 show snapshots of the load over time for each robot distribution. The robot locations are marked as colored points on the perimeter of the load in its initial and final configurations. The target path for the load's center of mass is shown as a red dotted line, and its actual trajectory is plotted in blue. The red line on the load indicates its orientation. In addition, Figure 6 plots the corresponding time evolution of the load's rotation and angular velocity, along with the drift d of the system's center of mass from the target path for all three distributions.

In the first simulation (Figure 3), the robots have an equally-spaced distribution, and the load is transported to the goal with no change in θ_o and no drift d from the target path, as shown in Figure 6. This is because both $||\mathbf{r}_c||$ and ξ are zero. For this case, $\epsilon = 0.1584$. In the second simulation (Figure 4), the robots have a nonuniform distribution for which $\xi = 0.179$ and $||\mathbf{r}_c|| = 0.02$. The load undergoes a total rotation of about $\theta_o = 30^\circ$, and its drift from the target path increases to about d = 15 cm. For this case, ϵ has decreased slightly to 0.1582. In the third simulation (Figure 5), the robots are clustered within a quarter of the load's perimeter. The load undergoes a large rotation of about $\theta_o = 140^\circ$, and its drift from the target path reaches a maximum of about d = 1.4 m. For this case, $||\mathbf{r}_c|| = 0.05$ and ξ has increased to 0.3875, which has lowered ϵ to 0.1577.



Fig. 3. Snapshots of the payload over time with an equally-spaced distribution of robots around its perimeter (*Distribution 1*).



Fig. 4. Snapshots of the payload over time with a nonuniform distribution of robots around its perimeter (*Distribution 2*).

Finally, Figure 7 shows the time evolution of the variables $s_x \equiv \dot{x}_o - v_{des}$ and $s_y \equiv \dot{y}_o$, the discrepancies between the actual and target velocity components of the system's center of mass, for all three distributions. In all cases, s_x converges to zero at an exponential rate, which is slowest for the third distribution. For the second and third distributions, s_y displays an overshoot before converging to zero, with a much higher overshoot for the third distribution because of its relatively large value of ξ compared to the other two cases. While the second distribution results in convergence to the desired velocity within about 150 s, the third distribution requires more than 200 s to converge.

B. Proportional-Integral control

The effect of adding the integral control for the third distribution, which had the highest drift, is shown in Figure 8. The system parameters are the same as in the case with proportional control only, and the controller gains are chosen



Fig. 5. Snapshots of the payload over time with a highly nonuniform distribution of robots around its perimeter (*Distribution 3*).



Fig. 6. Time evolution of the load's rotation θ_o and angular velocity $\omega_o = \dot{\theta}_o$ and the drift *d* of the system's center of mass from the desired path for all three distributions.

as k = 0.1 and $k_I = 0.005$. Figure 8 confirms that the large drift in Figure 5 is driven to zero, and the payload motion converges to the desired path after a transient phase. The convergence of the states of system (40) is shown in Figure 9.

VII. CONCLUSIONS

We have presented decentralized proportional control (Pcontrol) and proportional-integral control (PI-control) strategies for collective transport by a team of point-mass robots. The robot controllers require only local velocity measurements, and the only information provided to the robots is the target direction and speed of transport. We proved that the closed-loop system comprised of the payload and robots is exponentially stable with P-control and asymptotically stable with PI-control. We also analyzed the system's rate of convergence to the target velocity in the case of P-control, finding that it is mainly affected by the robots' distribution



Fig. 7. Time evolution of $s_x \equiv \dot{x}_o - v_{des}$ and $s_y \equiv \dot{y}_o$ for all three distributions.



Fig. 8. Snapshots of the payload over time with proportional-integral control for *Distribution 3*.

around the load and that it influences the load's total rotation and drift from the target path during the transient phase of motion. Our simulations verified the correctness of our analysis for different robot distributions around the load, as well as the effectiveness of the PI-control in driving the payload's motion to the desired path.

In future work, we will modify the proposed PI-controller to incorporate robustness against external disturbances such as friction. In addition, we will design decentralized controllers for collective transport that implement autonomous obstacle avoidance for scenarios where the robots have no prior knowledge about the environment and must compute appropriate control commands based only on their sensed distance from the obstacles. In contrast to [23], which proposes decentralized controllers for this scenario, our controllers will not require feedback about the payload's motion. We will investigate the correctness and stability of a controller that combines the proposed PI-controller with a repulsive component that is computed from local potential functions constructed by each robot. As illustrated by the



Fig. 9. Time evolution of the integral of velocity error (σ_x) and $(\sigma_y = d)$, velocity error $(\dot{\sigma}_x = s_x)$ and $(\dot{\sigma}_y = s_y)$, and the load's rotation θ_o and angular velocity $\omega_o = \dot{\theta}_o$ for *Distribution 3*.



Fig. 10. Motion of the payload over time with a controller that combines a PI-controller for convergence to the desired path and a repulsive force for obstacle avoidance.

preliminary simulation shown in Figure 10, this controller seems to yield promising results. We plan to establish theoretical guarantees on convergence and safety certificates for this controller, and we will experimentally validate the controller with multi-robot experiments.

REFERENCES

- T. J. Czaczkes and F. L. Ratnieks, "Cooperative transport in ants (hymenoptera: Formicidae) and elsewhere," *Myrmecological News*, vol. 18, pp. 1–11, 2013.
- [2] H. F. McCreery and M. Breed, "Cooperative transport in ants: a review of proximate mechanisms," *Insectes Sociaux*, vol. 61, no. 2, pp. 99– 110, 2014.
- [3] A. Gelblum, I. Pinkoviezky, E. Fonio, N. S. Gov, and O. Feinerman, "Emergent oscillations assist obstacle negotiation during ant coopera-

tive transport," *Proceedings of the National Academy of Sciences*, vol. 113, no. 51, pp. 14615–14620, 2016.

- [4] S. Shahrokhi and A. T. Becker, "Object manipulation and position control using a swarm with global inputs," in 2016 IEEE International Conference on Automation Science and Engineering (CASE), Aug 2016, pp. 561–566.
- [5] Z. Wang and M. Schwager, "Force-amplifying n-robot transport system (force-ants) for cooperative planar manipulation without communication," *The International Journal of Robotics Research*, vol. 35, no. 13, pp. 1564–1586, 2016.
- [6] —, "Kinematic multi-robot manipulation with no communication using force feedback," in 2016 IEEE International Conference on Robotics and Automation (ICRA), May 2016, pp. 427–432.
- [7] J. Chen, M. Gauci, W. Li, A. Kolling, and R. Groß, "Occlusion-based cooperative transport with a swarm of miniature mobile robots," *IEEE Transactions on Robotics*, vol. 31, no. 2, pp. 307–321, 2015.
- [8] A. Franchi, A. Petitti, and A. Rizzo, "Distributed estimation of the inertial parameters of an unknown load via multi-robot manipulation," in 53rd IEEE Conference on Decision and Control, Dec 2014, pp. 6111–6116.
- [9] A. Marino, G. Muscio, and F. Pierri, "Distributed cooperative object parameter estimation and manipulation without explicit communication," in 2017 IEEE International Conference on Robotics and Automation (ICRA), May 2017, pp. 2110–21116.
- [10] A. Tsiamis, C. K. Verginis, C. P. Bechlioulis, and K. J. Kyriakopoulos, "Cooperative manipulation exploiting only implicit communication," in 2015 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS), Sept 2015, pp. 864–869.
- [11] W. Gueaieb, F. Karray, and S. Al-Sharhan, "A robust adaptive fuzzy position/force control scheme for cooperative manipulators," *IEEE Transactions on Control Systems Technology*, vol. 11, no. 4, pp. 516– 528, July 2003.
- [12] Z. Li, C. Yang, C. Y. Su, S. Deng, F. Sun, and W. Zhang, "Decentralized fuzzy control of multiple cooperating robotic manipulators with impedance interaction," *IEEE Transactions on Fuzzy Systems*, vol. 23, no. 4, pp. 1044–1056, Aug 2015.
- [13] H. Bai and J. T. Wen, "Cooperative load transport: A formation-control perspective," *IEEE Transactions on Robotics*, vol. 26, no. 4, pp. 742– 750, Aug 2010.
- [14] N. Sadati and A. Ghaffarkhah, "Decentralized position and force control of nonredundant multi-manipulator systems," in 2007 International Conference on Control, Automation and Systems, Oct 2007, pp. 2223–2229.
- [15] G. B. Dai and Y. C. Liu, "Distributed coordination and cooperation control for networked mobile manipulators," *IEEE Transactions on Industrial Electronics*, vol. 64, no. 6, pp. 5065–5074, June 2017.
- [16] H. Lee, H. Kim, and H. J. Kim, "Planning and control for collision-free cooperative aerial transportation," *IEEE Transactions on Automation Science and Engineering*, vol. PP, no. 99, pp. 1–13, 2017.
- [17] A. Marino, "Distributed adaptive control of networked cooperative mobile manipulators," *IEEE Transactions on Control Systems Technology*, vol. PP, no. 99, pp. 1–15, 2017.
- [18] J. Pliego-Jimenez and M. Arteaga-Perez, "On the adaptive control of cooperative robots with time-variant holonomic constraints," *International Journal of Adaptive Control and Signal Processing*, vol. 31, no. 8, pp. 1217–1231, 2017, acs.2758. [Online]. Available: http://dx.doi.org/10.1002/acs.2758
- [19] M. Rubenstein, A. Cabrera, J. Werfel, G. Habibi, J. McLurkin, and R. Nagpal, "Collective transport of complex objects by simple robots: Theory and experiments," in *Proc. Int'l. Conf. on Autonomous Agents* and Multi-Agent Systems (AAMAS), 2013, pp. 47–54.
- [20] H. Farivarnejad, S. Wilson, and S. Berman, "Decentralized sliding mode control for autonomous collective transport by multi-robot systems," in 2016 IEEE 55th Conference on Decision and Control (CDC), Dec 2016, pp. 1826–1833.
- [21] R. Horn and C. Johnson, *Matrix Analysis*, ser. Matrix Analysis. Cambridge University Press, 2012. [Online]. Available: https: //books.google.com/books?id=515AYeeh0JUC
- [22] H. K. Khalil, *Nonlinear Systems*. Upper Saddle River, N.J.: Prentice Hall, 1996.
- [23] S. G. Faal, S. T. Kalat, and C. D. Onal, "Decentralized obstacle avoidance in collective object manipulation," in 2017 NASA/ESA Conference on Adaptive Hardware and Systems (AHS), July 2017, pp. 133–138.