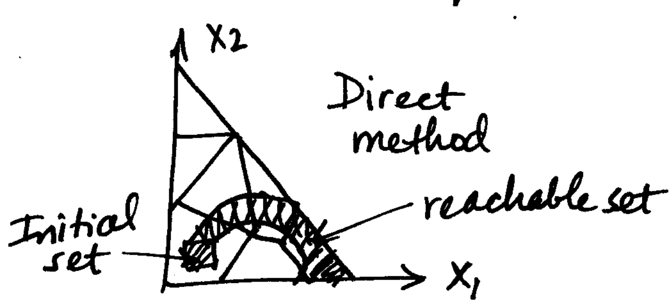
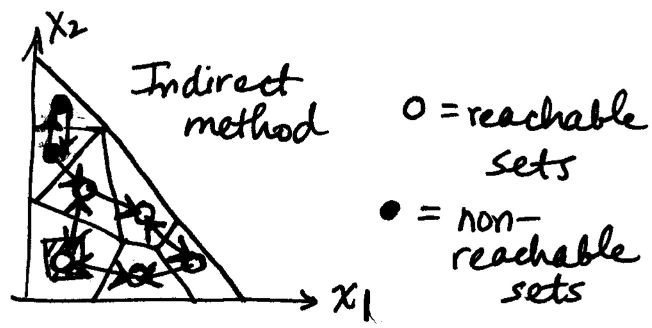
Hybrid Systems

- A set of discrete modes, each with different continuous dynamics; extends finite state machines to include dynamics.
  - Can use reachability analysis to approximate the sys. behavior over time and address the <u>reachability problem</u>:
    "Is there a trajectory from some initial state $(\underline{x}_0)$ to some target state $(\underline{x}_f)$?"
    This problem can be solved algorithmically in a finite # of steps (that is, the reachability problem is <u>decidable</u>) when the continuous dynamics:
    a) are constant (timed + multirate automata)
    b) take values in a constant interval (rectangular automata)
    c) are a certain class of linear systems.
    ("A New Class of Decidable Hybrid Systems," HSCC '99)

- If the reachability problem is <u>not</u> decidable for a system, then you can compute an overapproximation of the reachable set:

(1) Indirect method: develop a discrete abstraction of the hybrid system (can partition the continuous state space into finite # of sets)
(2) Direct method: directly calculate the reachable set on the state space





Notation for a hybrid system

"An introduction to hybrid dynamical systems", Van der Schaft and Schumacher, 2000

$$\boxed{H = (L, X, A, W, E, Inv, Act)}$$

L: set of discrete locations ("modes"); the vertices of a graph

X: continuous state space in which the continuous state variables take values

A: set of symbols that label the transitions

W: continuous space of external variables w

E: finite set of transitions (or "events") between locations; the edges of a graph

- Each transition is defined by $(l, a, Guard_{ll'}, Jump_{ll'}, l')$

  $l, l' \in L$, $a \in A$, $Guard_{ll'} \subset X$

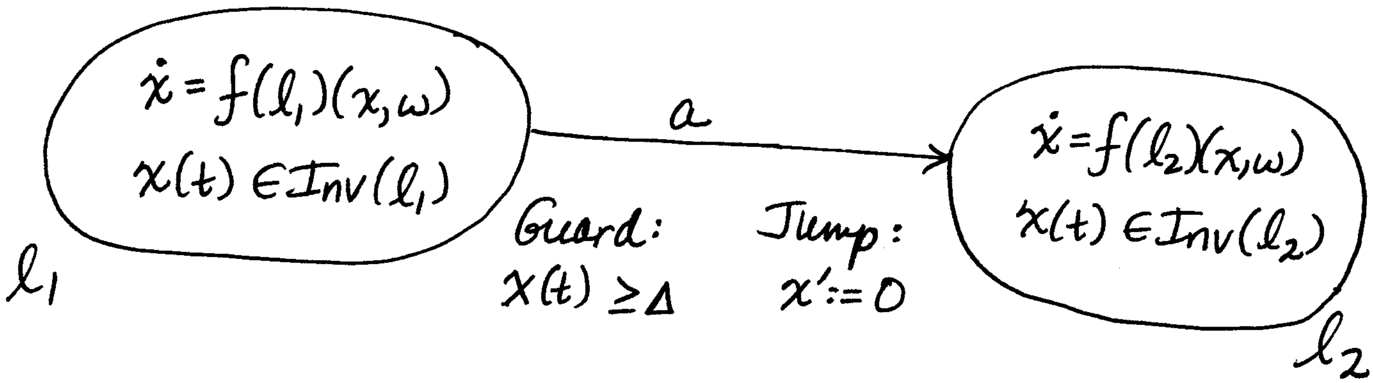
  The transition from l to l' is enabled when the state x is in $Guard_{ll'}$, and x is reset to a new value given by $Jump_{ll'} \subset X \times X$.

Inv: maps the locations to subsets of X such that when the system is at location l, $x \in Inv(l)$, the "location invariant" of l.

Act: a map that assigns a continuous vector field to each location, $\dot{x} = f(l)(x, w)$.

The solutions of the eq. are called the "activities" of the location l.

Ex



- Trajectory of a hybrid system:

An infinite sequence of continuous trajectories

$$(l_0, \delta_0, x_0, w_0) \xrightarrow{a_0} (l_1, \delta_1, x_1, w_1) \xrightarrow{a_1} (l_2, \delta_2, x_2, w_2) \xrightarrow{a_2} \cdots$$

such that at the event times

$$t_0 = \delta_0, \quad t_1 = \delta_0 + \delta_1, \quad t_2 = \delta_0 + \delta_1 + \delta_2, \ldots$$

we have that: $\begin{cases} x_j(t_j) \in Guard_{l_j l_{j+1}} & \forall j = 0, 1, 2, \ldots \\ (x_j(t_j), x_{j+1}(t_j)) \in Jump_{l_j l_{j+1}}. \end{cases}$

Here, (l, δ, x, w) is a continuous trajectory associated with location l:

δ = nonnegative time

$w : [0, \delta] \to W$ is a piecewise cont. function

$x : [0, \delta] \to X$ is a continuous, piecewise differentiable function such that:

$x(t) \in Inv(l) \quad \forall t \in (0, \delta)$,

$\dot{x}(t) = f(l)(x, w) \quad \forall t \in (0, \delta)$ except for points of discontinuity of w.