MAE 598: Multi-Robot Systems
Fall 2016

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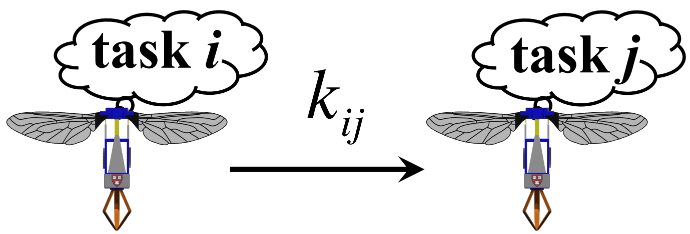
Lecture 5

Microscopic Model: Task Switching

$$k_{ij} = f(c_{ij})$$

c_{ij} = **Prob(**a particular combination of reactants in the reaction associated with k_{ij} will react**) per timestep Δ*t***

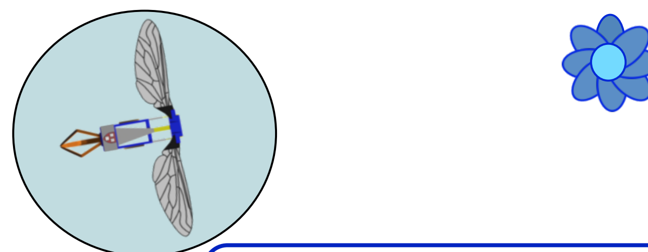
Spontaneous



c_{ij} Tunable

Robot executes transition with probability $c_{ij}\Delta t$ at each Δt

Interaction-dependent



$c_{ij} = c_{ij}^{enc} \cdot c_{ij}^{react}$ Tunable

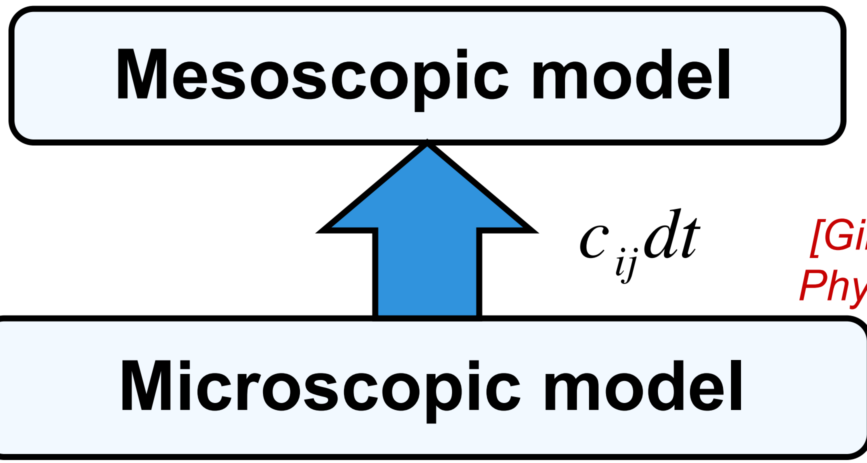
Robot encounters potential reactant in next Δt with probability $c_{ij}^{enc}\Delta t$, executes transition with probability c_{ij}^{react}

Modeling Approach

$\mathbf{N}(t) \in \mathbb{R}^S$ Species populations (integers)

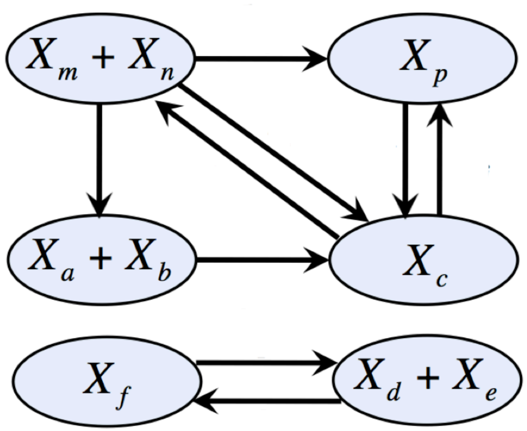
Chemical Master Equation

Time-evolution equation for $Prob\{\mathbf{N}(t) = \mathbf{n} \text{ given } \mathbf{N}(t_0) = \mathbf{n_0}\}$



[Gillespie, Annu. Rev. Phys. Chem. '07]

N elements,
S species



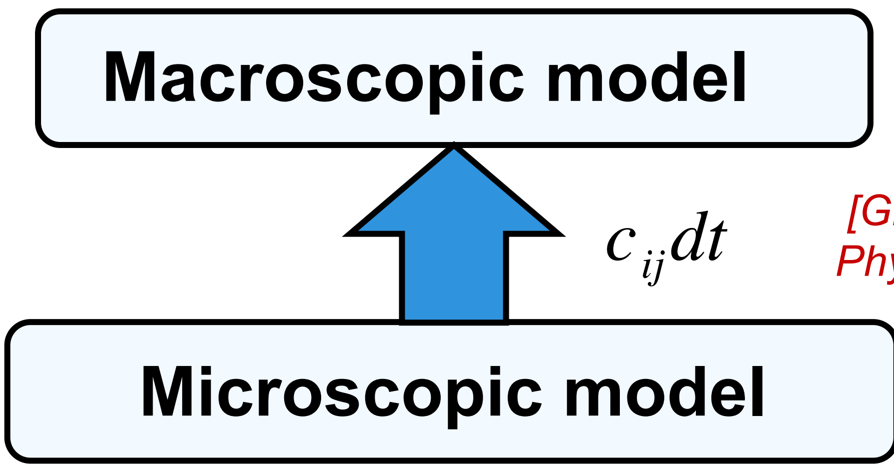
Directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$

Adjacent complexes: $(i, j) \in \mathcal{E}$

Modeling Approach

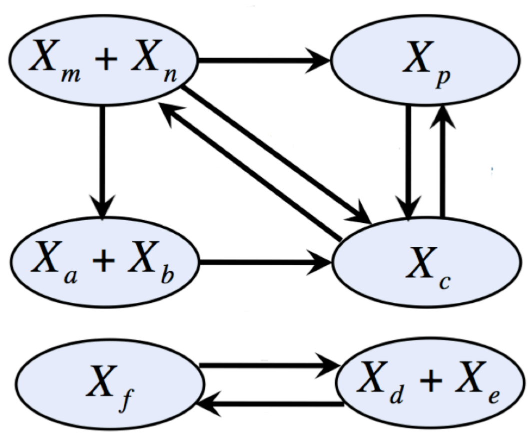
$\mathbf{N}(t) \in \mathbb{R}^S$ Species populations (integers)

Numerical realizations of $\mathbf{N}(t)$ using a Stochastic Simulation Algorithm *[Gillespie, J. Comp. Phys. 1976]*



[Gillespie, Annu. Rev. Phys. Chem. '07]

N elements,
S species



Directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$

Adjacent complexes: $(i, j) \in \mathcal{E}$

Modeling Approach

$\mathbf{x}(t) \in \mathbb{R}^S$ Species concentrations; $E(\mathbf{N}(t)/V)$

Linear model

$$\dot{\mathbf{x}} = -\mathbf{K}\mathbf{x}$$
$$\mathbf{1}^T\mathbf{x} = 1$$

$X_i \longrightarrow X_j$ only

Multi-affine model

$$\dot{\mathbf{x}} = -\mathbf{M}\mathbf{K}\mathbf{y}(\mathbf{x})$$
$$\mathbf{c}_i^T\mathbf{x} = c_i, \quad i = 1,..,S - rank(\mathbf{S})$$

$\mathbf{y}(\mathbf{x})$ = Vector of complexes

Macroscopic model

Thermodynamic limit

$N_i \rightarrow \infty, \quad V \rightarrow \infty,$

$N_i/V \quad finite$

$\mathbf{N}(t) \in \mathbb{R}^S$ **Mesoscopic model**

Top-Down Controller Synthesis

- **Analysis:** establish theoretical guarantees on performance

- **Controller synthesis:** Design rate constants k_{ij}

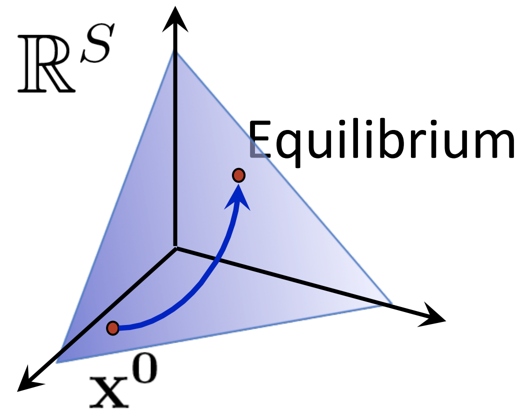
Macro-scopic model

Broadcast k_{ij}

Decentralized robot control policies based on c_{ij} that produce desired collective behavior

Microscopic model

Analysis of Macroscopic Model



Equilibria characterization

Model must have a unique, positive, asymptotically stable equilibrium
(= final swarm population distribution)

- Chemical Reaction Network Theory

• General network topology, mass action kinetics:
M. Feinberg, F. Horn, R. Jackson (1970's, 1980's)

• More restricted topology, monotone kinetics:
E. Sontag, D. Angeli, P. de Leenheer (2000's)

- Algebraic Graph Theory

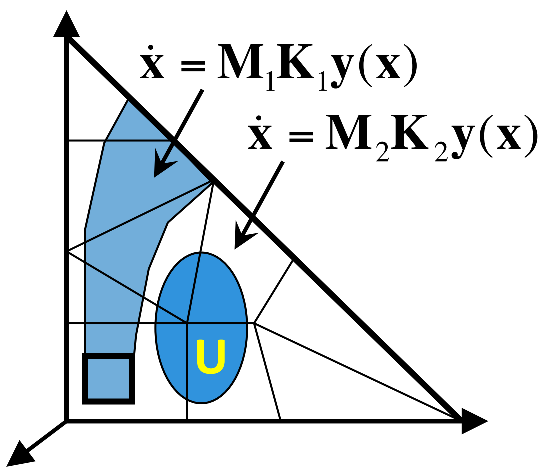
- Lyapunov Stability Theory

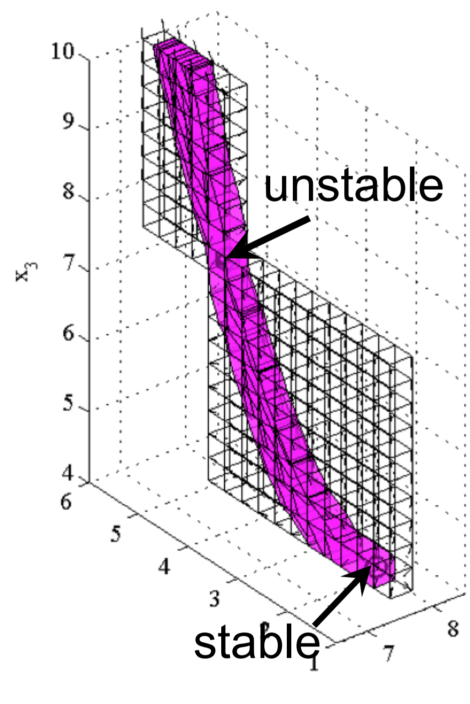
Hybrid System Macroscopic Models

- **Reachability analysis**

Algorithms for systems with multi-affine dynamics

[Berman, Halász, Kumar HSCC'07]





Reallocation of a Swarm among Multiple Sites

[Berman, Halász, Hsieh, Kumar, IEEE Trans. on Robotics 2009]

Develop a strategy for **redistributing a swarm of robots among multiple sites** in specified population fractions to perform tasks at each site

Applications:

- surveillance of multiple buildings
- search-and-rescue
- reconnaissance
- environmental monitoring
- construction

Required Robot Controller Properties

Synthesize robot controllers that:

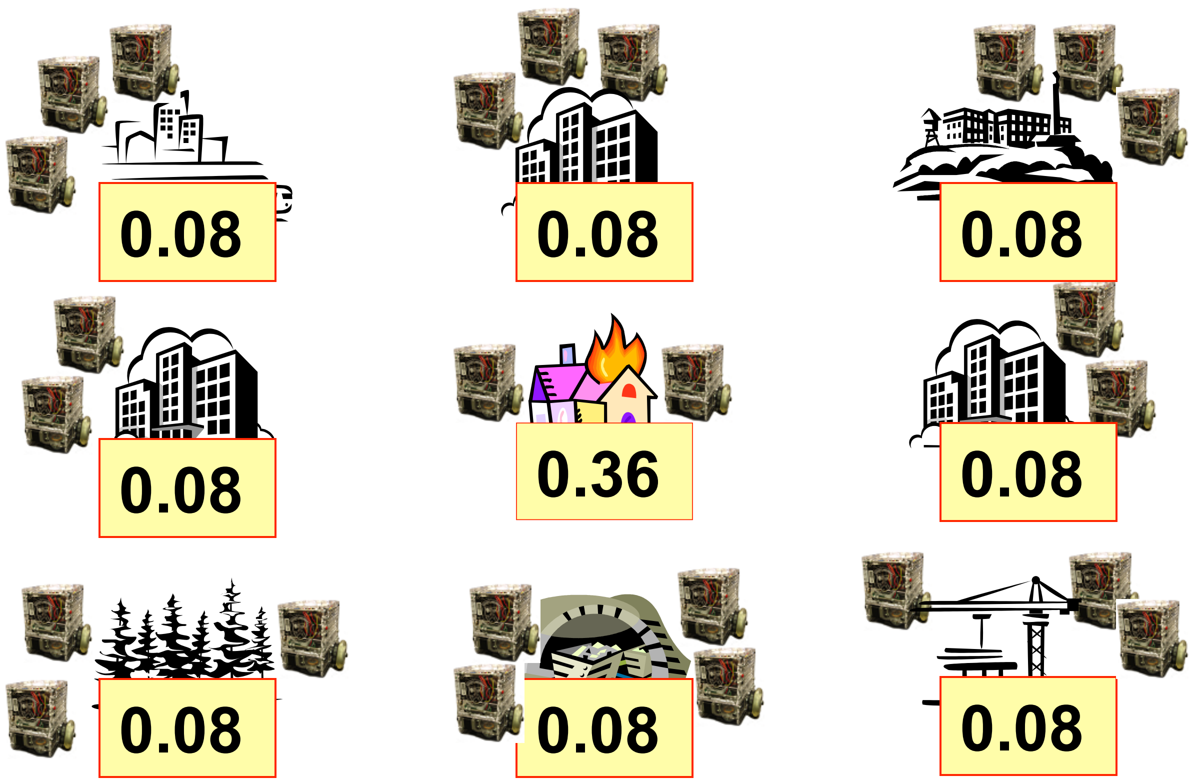
- can be computed *a priori* by an external supervisor
- are based on a set of parameters that are independent of swarm size
- do not require inter-robot communication
- have provable guarantees on performance
- can be optimized for fast convergence to the desired allocation among sites, with a constraint on robot traffic between sites
- require minimal adjustments when task demands change

Objective

- Develop a strategy for redistributing a swarm of robots among multiple sites in specified fractions

Objective

- Develop a strategy for redistributing a swarm of robots among multiple sites in specified fractions



Objective

- Develop a strategy for redistributing a swarm of robots among multiple sites in specified fractions

Approach

Challenges: Difficult to use centralized control, communication across sites may be risky or impossible

→ Decentralized decision-making, no communication for control

- Promotes *scalability*, *robustness* to changes in swarm size
- In contrast to coalition-formation algorithms such as market-based approaches

  Dias *et al.*, "Market-based Multirobot Coordination: A Survey and Analysis" *Proc. IEEE*, 2006

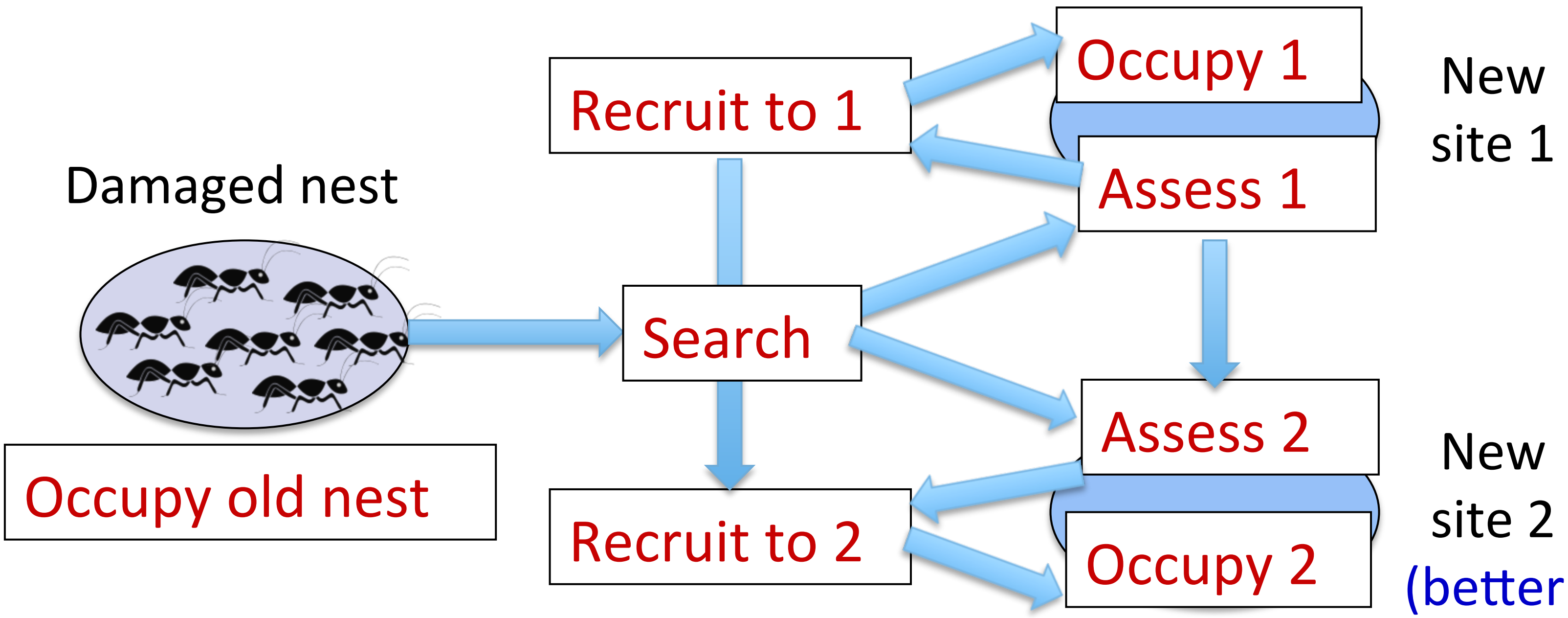
- Robots redistribute themselves autonomously by switching stochastically between sites

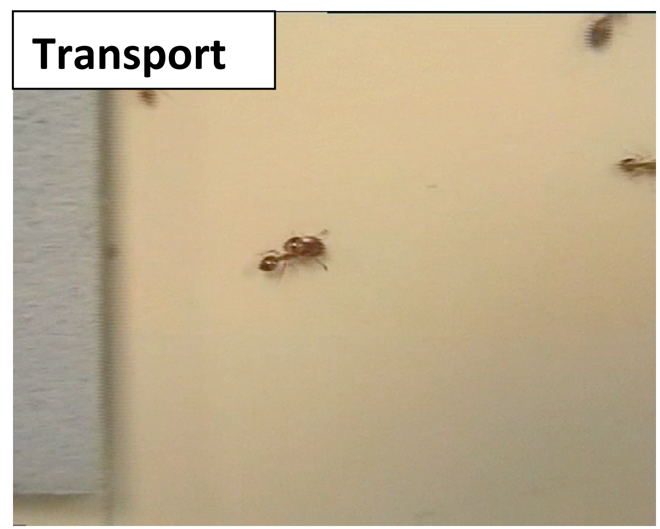
  Inspired by social insect behavior, particularly ant house-hunting (select a new nest and move the colony there)

  Franks *et al.*, "Information flow, opinion polling and collective intelligence in house-hunting social insects," *Phil. Trans. of the Royal Society B*, 2002

  Simple rules based on local sensing, physical contact

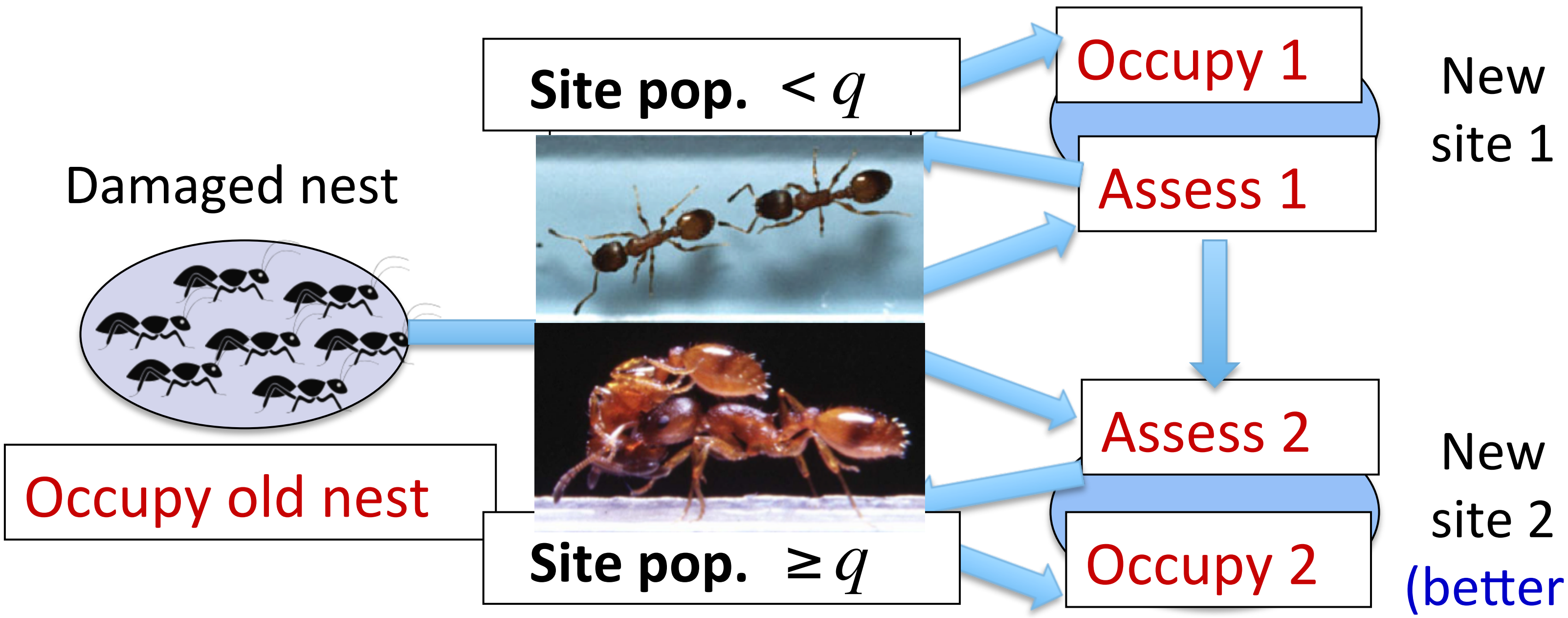
"House-Hunting" in *Temnothorax albipennis*

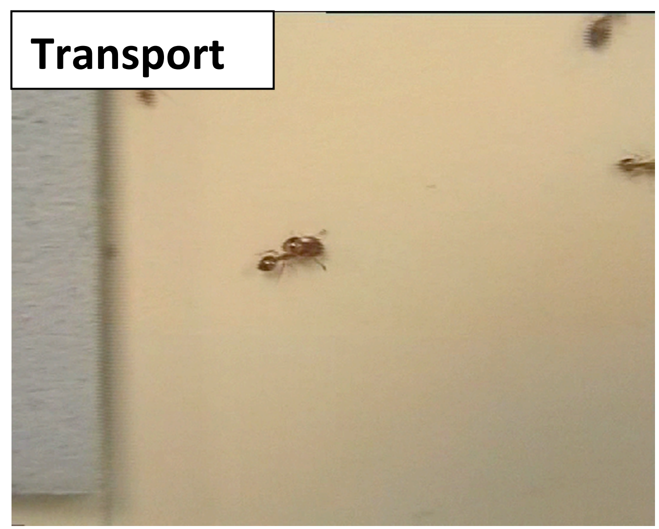




Courtesy of Prof. Stephen Pratt, ASU

“House-Hunting” in *Temnothorax albipennis*

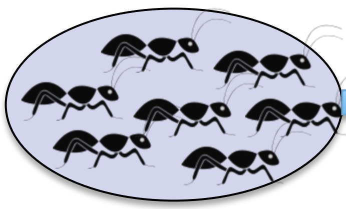




Courtesy of Prof. Stephen Pratt, ASU

"House-Hunting" in *Temnothorax albipennis*

Damaged nest



Occupy old nest

Rates of switching between tasks determine final allocation

$$X_i \xrightarrow{k_{ij}} X_j$$

$$k_{ij} = f(\text{site pop.}, \boldsymbol{q})$$

Occupy 1

Assess 1

New site 1

Assess 2

Occupy 2

New site 2 (better)

Tandem run

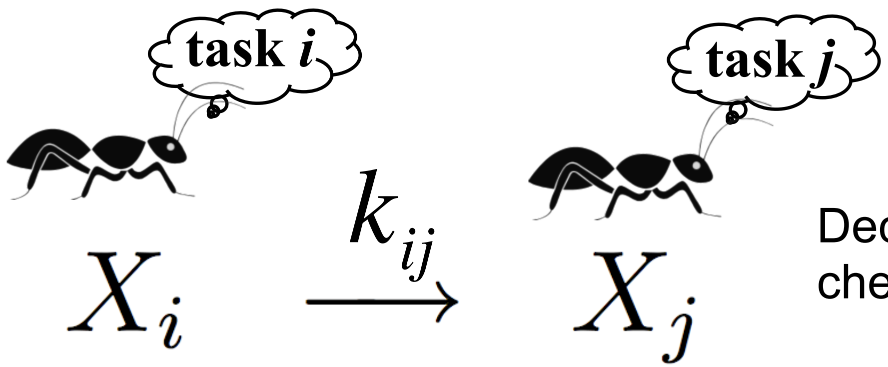
Site pop. $< q$

Transport

Site pop. $\geq q$

Microscopic Model

Unimolecular (spontaneous)

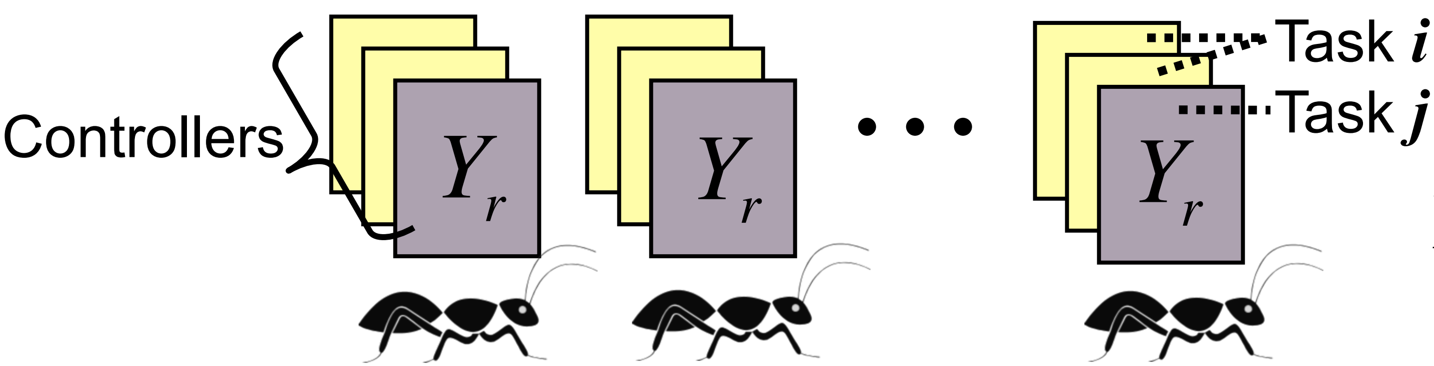


$$X_i \xrightarrow{k_{ij}} X_j$$

Decisions modeled as chemical reactions

X_i ~ chemical species *i*

Rate constant k_{ij}



$$Y_r \subset \mathbf{R}^n$$

Macroscopic Model [Franks 2002]

Site 0 (home) is destroyed; Site 2 is better than Site 1

Active Ants *pN*

$$\begin{cases} \dot{X} &= -(\mu_1 + \mu_2)X - \lambda_1 Y_1 \theta(X)\theta(T - Y_1) \\ & \quad -\lambda_2 Y_2 \theta(X)\theta(T - Y_2) \\ \dot{Y}_1 &= k_1 Z_1 - \rho_{12} Y_1 \\ \dot{Y}_2 &= k_2 Z_2 + \rho_{12} Y_1 \\ \dot{Z}_1 &= \mu_1 X + \lambda_1 Y_1 \theta(X)\theta(T - Y_1) - \rho_{12} Z_1 - k_1 Z_1 \\ \dot{Z}_2 &= \mu_2 X + \lambda_2 Y_2 \theta(X)\theta(T - Y_2) + \rho_{12} Z_1 - k_2 Z_2 \end{cases}$$

Passive Ants *(1 – p)N*

$$\begin{cases} \dot{B}_0 &= -\phi_1 Y_1 \theta(B_0)\theta(Y_1 - T) - \phi_2 Y_2 \theta(B_0)\theta(Y_2 - T) \\ \dot{B}_1 &= \phi_1 Y_1 \theta(B_0)\theta(Y_1 - T) \\ \dot{B}_2 &= \phi_2 Y_2 \theta(B_0)\theta(Y_2 - T) \end{cases}$$

$\theta(X) = 1$ when $X > 0$, 0 otherwise

Macroscopic Model: Active Ants

Naive

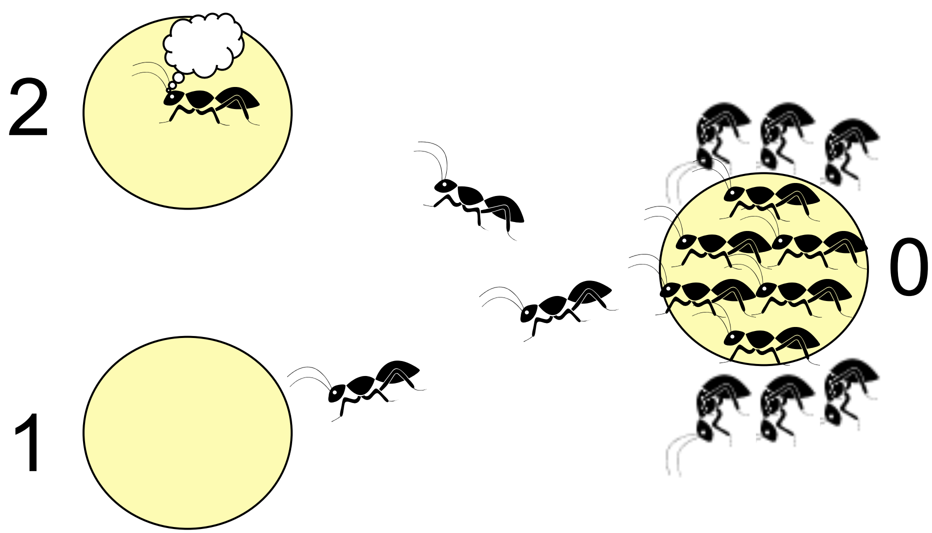
$$\dot{X} = -(\mu_1 + \mu_2)X - \lambda_1 Y_1 \theta(X)\theta(T - Y_1) - \lambda_2 Y_2 \theta(X)\theta(T - Y_2)$$

Recruiters

$$\begin{aligned} \dot{Y}_1 &= k_1 Z_1 - \rho_{12} Y_1 \\ \dot{Y}_2 &= k_2 Z_2 + \rho_{12} Y_1 \end{aligned}$$

Assessors

$$\begin{aligned} \dot{Z}_1 &= \mu_1 X + \lambda_1 Y_1 \theta(X)\theta(T - Y_1) - \rho_{12} Z_1 - k_1 Z_1 \\ \dot{Z}_2 &= \mu_2 X + \lambda_2 Y_2 \theta(X)\theta(T - Y_2) + \rho_{12} Z_1 - k_2 Z_2 \end{aligned}$$



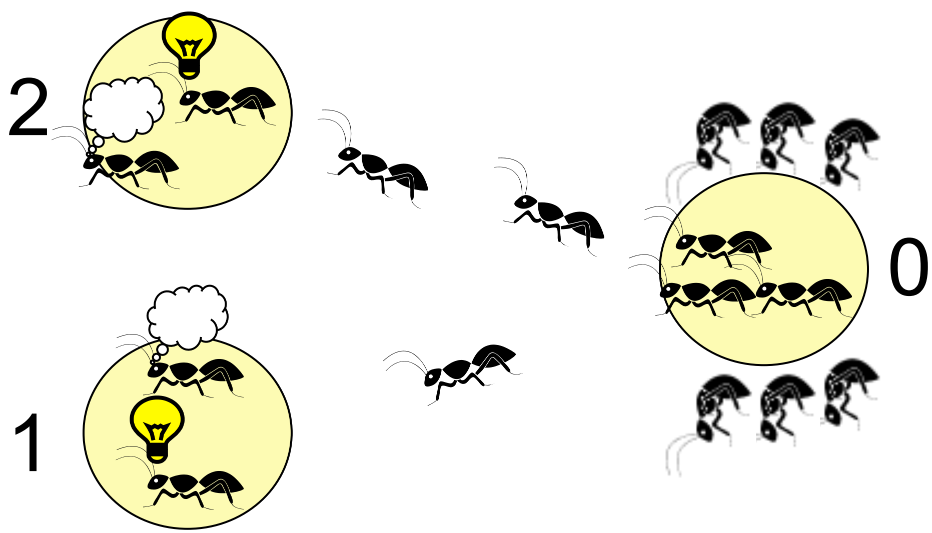
$\boldsymbol{\mu_i}$ = rate of discovery of site *i*

Macroscopic Model: Active Ants

Naive
$$\dot{X} = -(\mu_1 + \mu_2)X - \lambda_1 Y_1 \theta(X)\theta(T - Y_1) - \lambda_2 Y_2 \theta(X)\theta(T - Y_2)$$

Recruiters
$$\begin{aligned} \dot{Y}_1 &= \boxed{k_1 Z_1} - \rho_{12} Y_1 \\ \dot{Y}_2 &= \boxed{k_2 Z_2} + \rho_{12} Y_1 \end{aligned}$$

Assessors
$$\begin{aligned} \dot{Z}_1 &= \mu_1 X + \lambda_1 Y_1 \theta(X)\theta(T - Y_1) - \rho_{12} Z_1 \boxed{- k_1 Z_1} \\ \dot{Z}_2 &= \mu_2 X + \lambda_2 Y_2 \theta(X)\theta(T - Y_2) + \rho_{12} Z_1 \boxed{- k_2 Z_2} \end{aligned}$$



k_i = rate at which assessors of site *i* become recruiters to *i*

Macroscopic Model: Active Ants

Naive

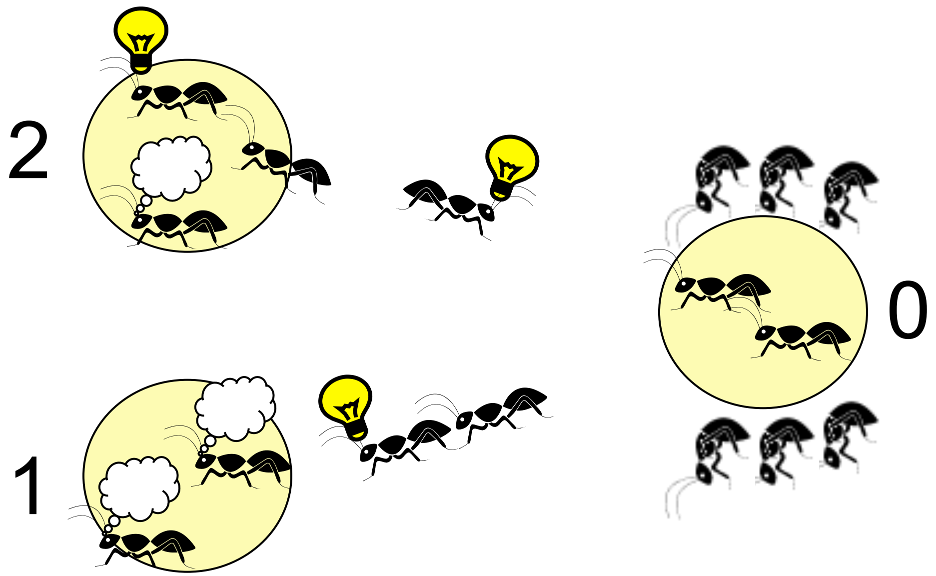
$$\dot{X} = -(\mu_1 + \mu_2)X \boxed{- \lambda_1 Y_1 \theta(X)\theta(T - Y_1)} \; \boxed{-\lambda_2 Y_2 \theta(X)\theta(T - Y_2)}$$

Recruiters

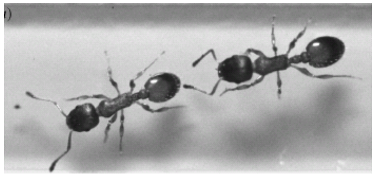
$$\begin{aligned} \dot{Y}_1 &= k_1 Z_1 - \rho_{12} Y_1 \\ \dot{Y}_2 &= k_2 Z_2 + \rho_{12} Y_1 \end{aligned}$$

Assessors

$$\begin{aligned} \dot{Z}_1 &= \mu_1 X \boxed{+ \lambda_1 Y_1 \theta(X)\theta(T - Y_1)} - \rho_{12} Z_1 - k_1 Z_1 \\ \dot{Z}_2 &= \mu_2 X \boxed{+ \lambda_2 Y_2 \theta(X)\theta(T - Y_2)} + \rho_{12} Z_1 - k_2 Z_2 \end{aligned}$$



λ_i = rate at which recruiters lead ***tandem runs*** to site *i*



[Franks 2002]

T = **Quorum**

Macroscopic Model: Active Ants

Naive

$$\dot{X} = -(\mu_1 + \mu_2)X - \lambda_1 Y_1 \theta(X)\theta(T - Y_1) - \lambda_2 Y_2 \theta(X)\theta(T - Y_2)$$

Recruiters

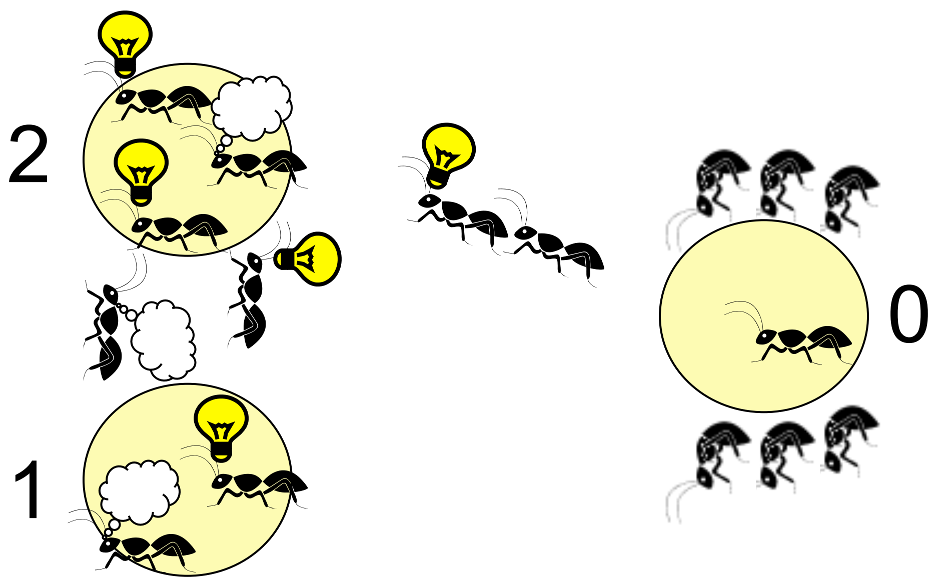
$$\dot{Y}_1 = k_1 Z_1 \boxed{- \rho_{12} Y_1}$$

$$\dot{Y}_2 = k_2 Z_2 \boxed{+ \rho_{12} Y_1}$$

Assessors

$$\dot{Z}_1 = \mu_1 X + \lambda_1 Y_1 \theta(X)\theta(T - Y_1) \boxed{- \rho_{12} Z_1} - k_1 Z_1$$

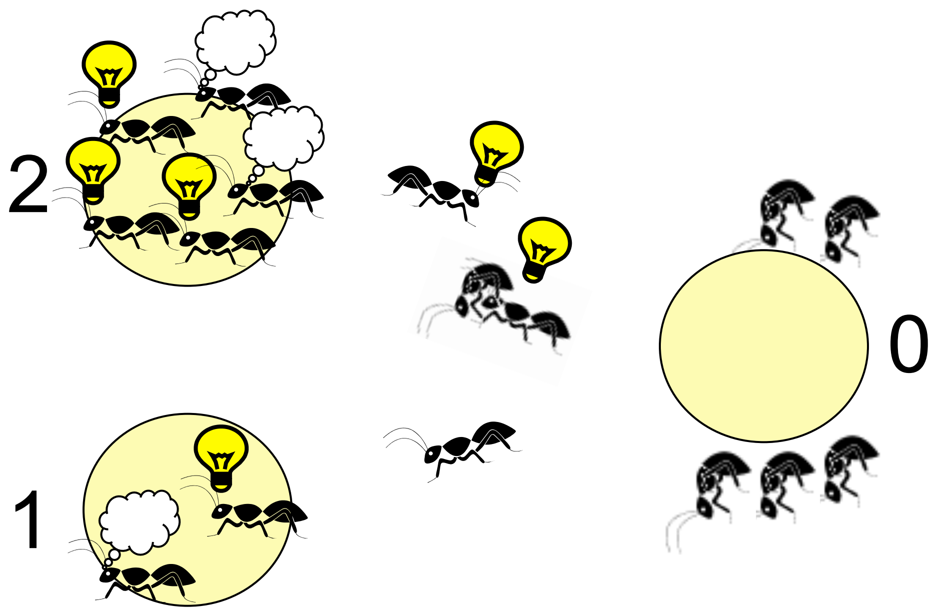
$$\dot{Z}_2 = \mu_2 X + \lambda_2 Y_2 \theta(X)\theta(T - Y_2) \boxed{+ \rho_{12} Z_1} - k_2 Z_2$$



$\boldsymbol{\rho_{ij}}$ = rate of switching allegiance from site *i* to site *j*

Macroscopic Model: Passive Ants

$$
\begin{aligned}
\dot{B}_0 &= -\phi_1 Y_1 \theta(B_0)\theta(Y_1 - T) - \phi_2 Y_2 \theta(B_0)\theta(Y_2 - T) \\
\dot{B}_1 &= \phi_1 Y_1 \theta(B_0)\theta(Y_1 - T) \\
\dot{B}_2 &= \phi_2 Y_2 \theta(B_0)\theta(Y_2 - T)
\end{aligned}
$$

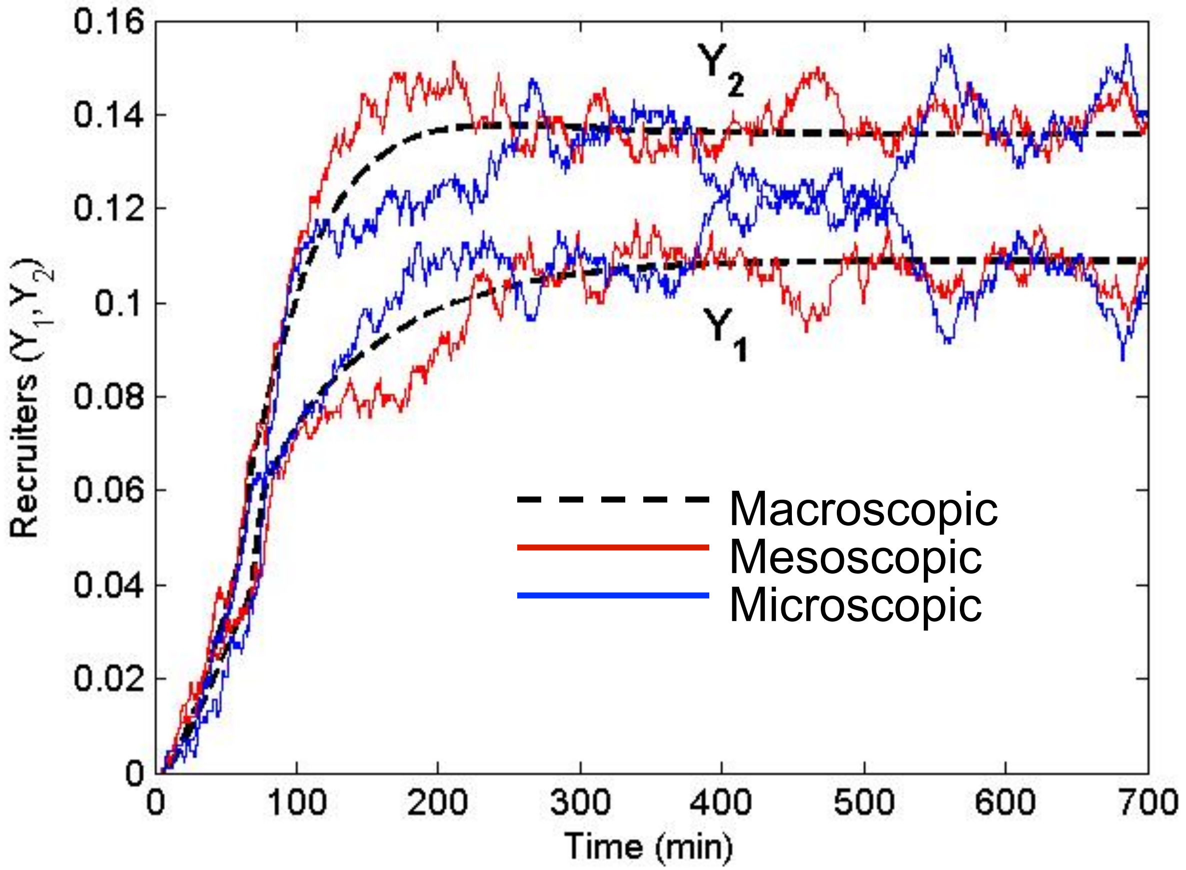


$\boldsymbol{\phi_i}$ = rate at which recruiters perform ***transports*** to site *i*

[Franks 2002]

T = **Quorum**

Agreement between macroscopic , mesoscopic, and microscopic models (modified ant house-hunting model)



208 ants

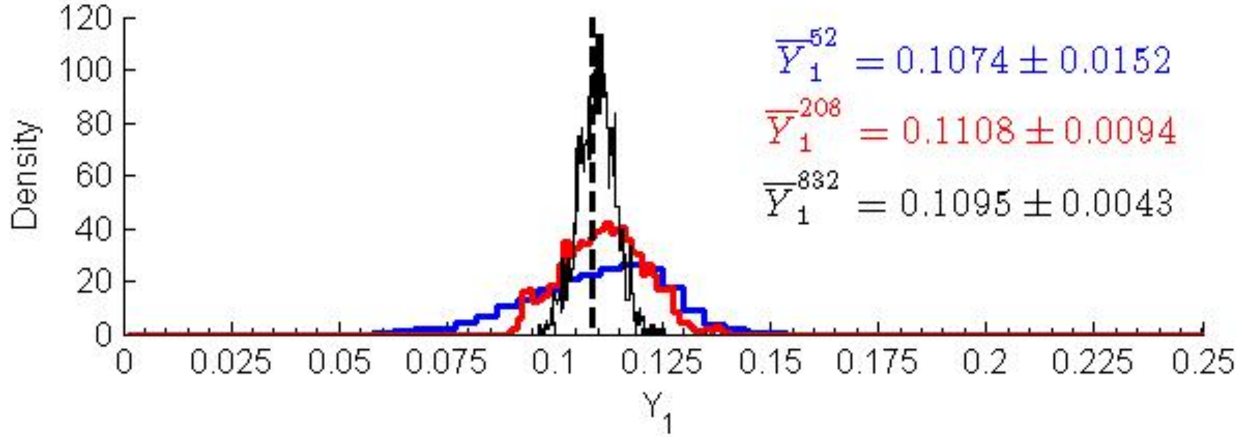
Spring Berman, Adam Halasz, Vijay Kumar, and Stephen Pratt, “Bio-Inspired Group Behaviors for the Deployment of a Swarm of Robots to Multiple Destinations” ICRA 2007.

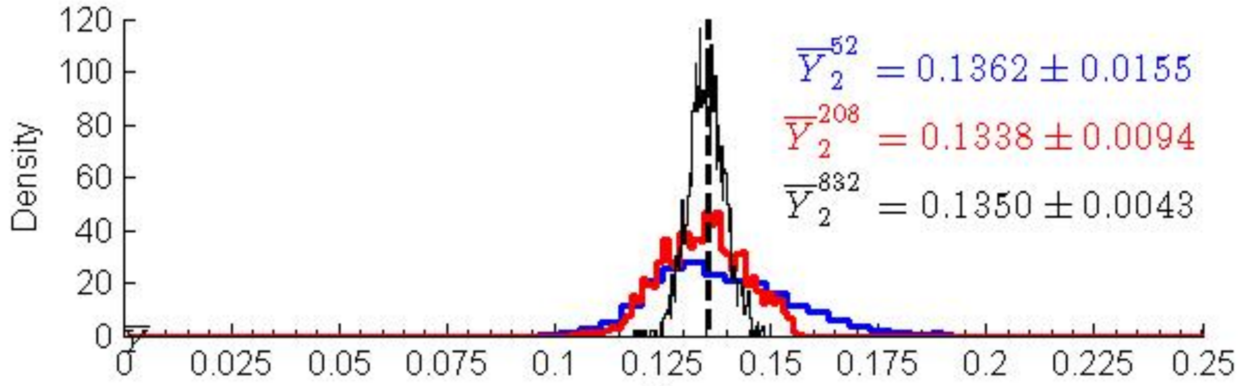
Mesoscopic Model Fluctuations in Recruiter Populations

- Effect of population size on steady-state Y_1, Y_2: N = 52, 208, 832

Dashed lines are macroscopic steady-state values

N = 208: Std dev is < 9% of mean





Approach to Swarm Multi-Site Deployment

- Model interconnection topology of sites as a directed graph

$\mathcal{G} = (\mathcal{V}, \mathcal{E})$ $\mathcal{V}$ = set of sites $\mathcal{E} = \{(i,j) \in \mathcal{V} \times \mathcal{V} \mid \underline{i \sim j} \}$

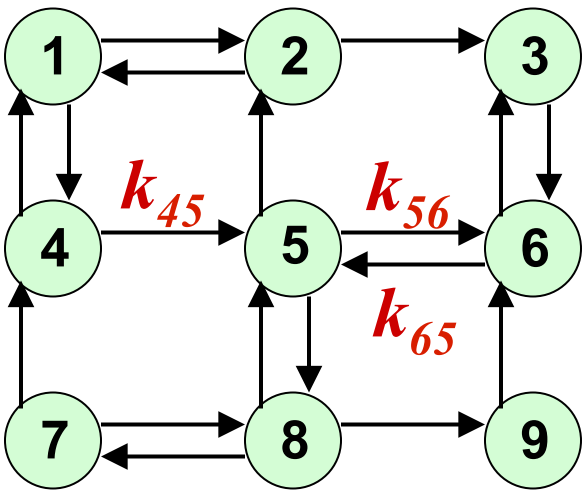
can travel from *i* to *j*

- Assume that $\mathcal{G}$ is strongly connected (directed path btwn. each pair of sites)

k_{ij} = Transition probability per unit time for one robot at site *i* to travel to site *j*

- Choose for rapid, efficient redistribution

M Sites



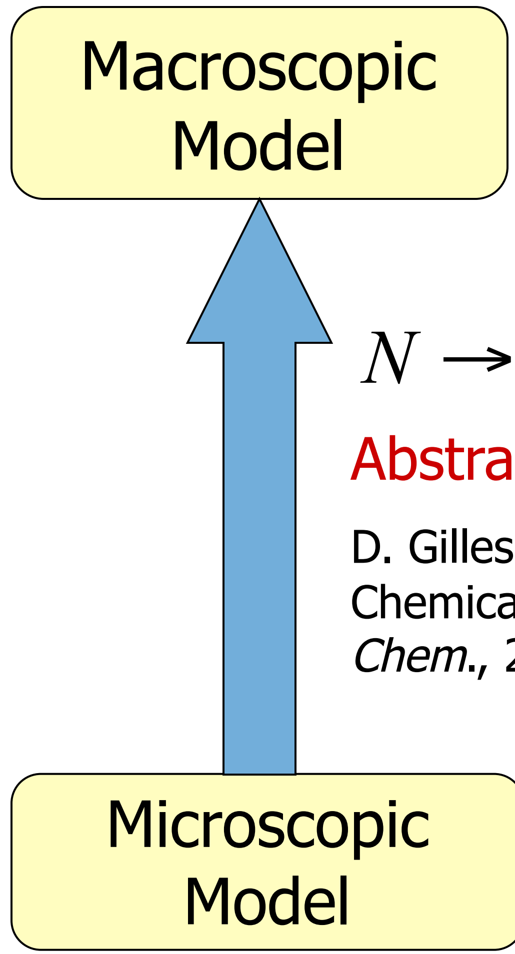
- Assume that each robot:
  - knows $\mathcal{G}$, all k_{ij}, task at each site
  - can navigate between sites
  - can sense neighboring robots

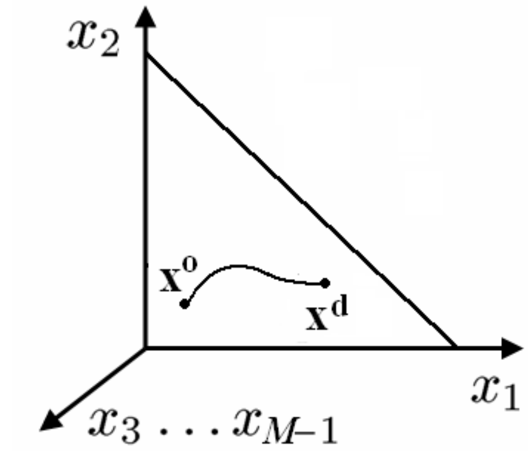
Approach

- Ordinary differential equations in terms of k_{ij} and the fraction of robots x_i at each site *i*

- *N* robots, *M* behavior states: **{Doing task at site 1, Doing task at site 2, …, Doing task at site *M*}**

- Could also include states that represent travel between pairs of sites

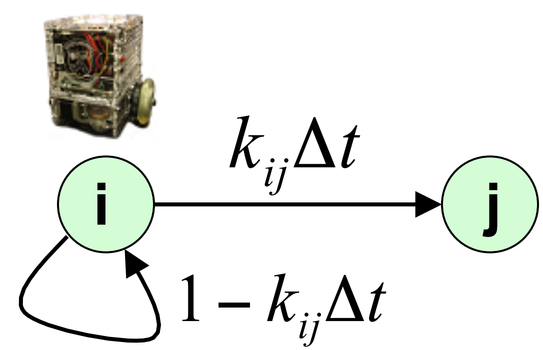




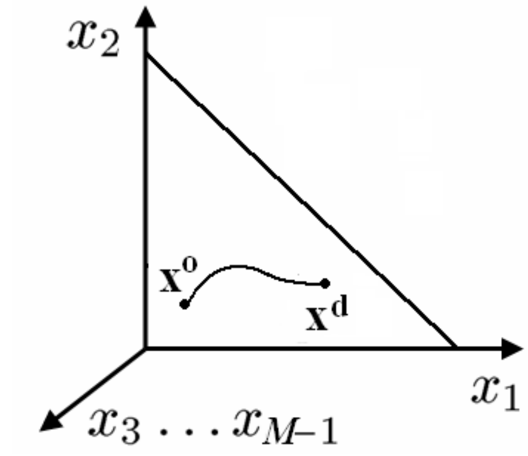
$N \rightarrow \infty$

Abstraction

D. Gillespie, "Stochastic Simulation of Chemical Kinetics," *Annu. Rev. Phys. Chem.*, 2007



Approach



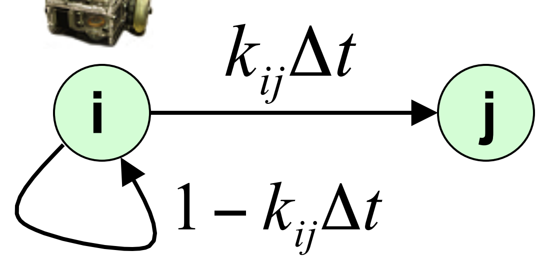
- Analysis and optimization tools to choose k_{ij}

Macroscopic Model

"Top-down" controller synthesis approach is computationally inexpensive and gives guarantees on performance

- Switch according to k_{ij} ; motion control for tasks at sites, navigation

Microscopic Model



Macroscopic Model

$x_i(t)$ = Fraction of robots at site *i* at time *t* $\quad \mathbf{x} = [x_1 \ \ldots \ x_M]^T$

$$\dot{x}_i(t) = \sum_{j \sim i} k_{ji} x_j(t) - \sum_{i \sim j} k_{ij} x_i(t)$$

$$\boxed{\dot{\mathbf{x}} = -\mathbf{K}\mathbf{x}}$$

(a) $\mathbf{K}^T \mathbf{1} = \mathbf{0}$,

(b) $\mathbf{K}_{ij} \leq 0 \quad \forall (i,j) \in \mathcal{E}$

Conservation constraint: $\mathbf{1}^T \mathbf{x} = 1$

Base Continuous Model

$$\boxed{\dot{\mathbf{x}} = -\mathbf{K}\mathbf{x}} \qquad \mathbf{1}^T\mathbf{x} = 1$$

(a) $\mathbf{K}^T\mathbf{1} = \mathbf{0}$, (b) $\mathbf{K}_{ij} \leq 0 \quad \forall (i,j) \in \mathcal{E}$

- There is a unique, stable equilibrium [Halász *et al.*, IROS' 07]

x_i^d = Target fraction of robots at site *i* $\qquad \mathbf{x^d} = [x_1^d \ldots x_M^d]^T$

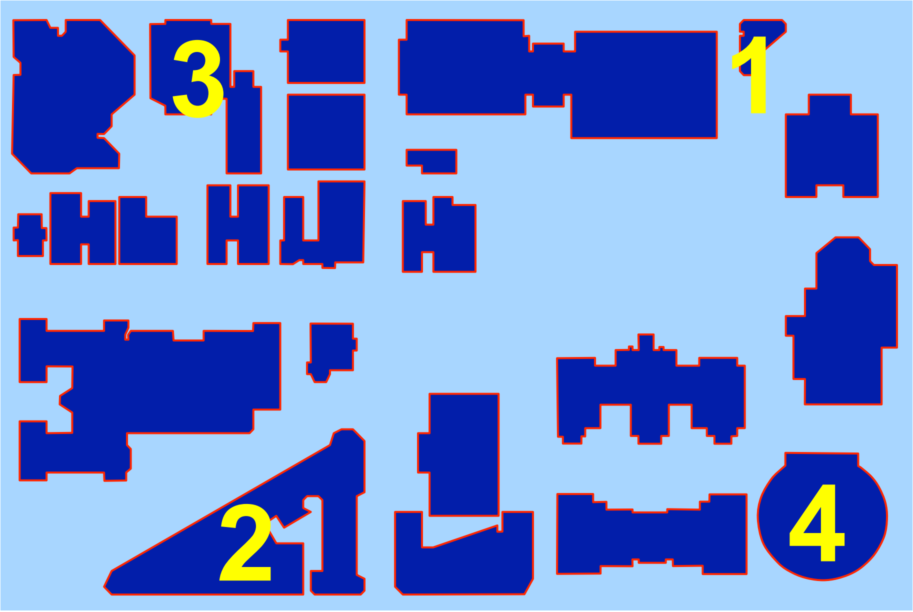
→ If k_{ij} are chosen so that (c) $\mathbf{K}\mathbf{x^d} = 0$,
the system always converges to the target distribution

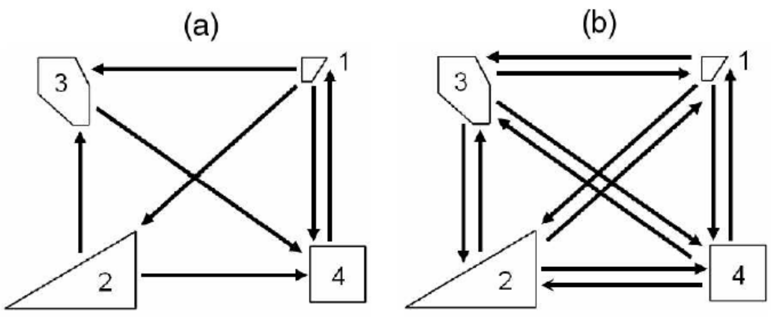
Simulation Methodology

- Swarm of 250 robots monitors the perimeters of 4 buildings on UPenn campus while redistributing to the desired allocation $\mathbf{x}^{\mathbf{d}}$

Swarm initially split between sites 3 and 4

$$x_1^d = 0.1,\ x_2^d = 0.4,\ x_3^d = 0.2,\ x_4^d = 0.3$$





Two possible site interconnection graphs

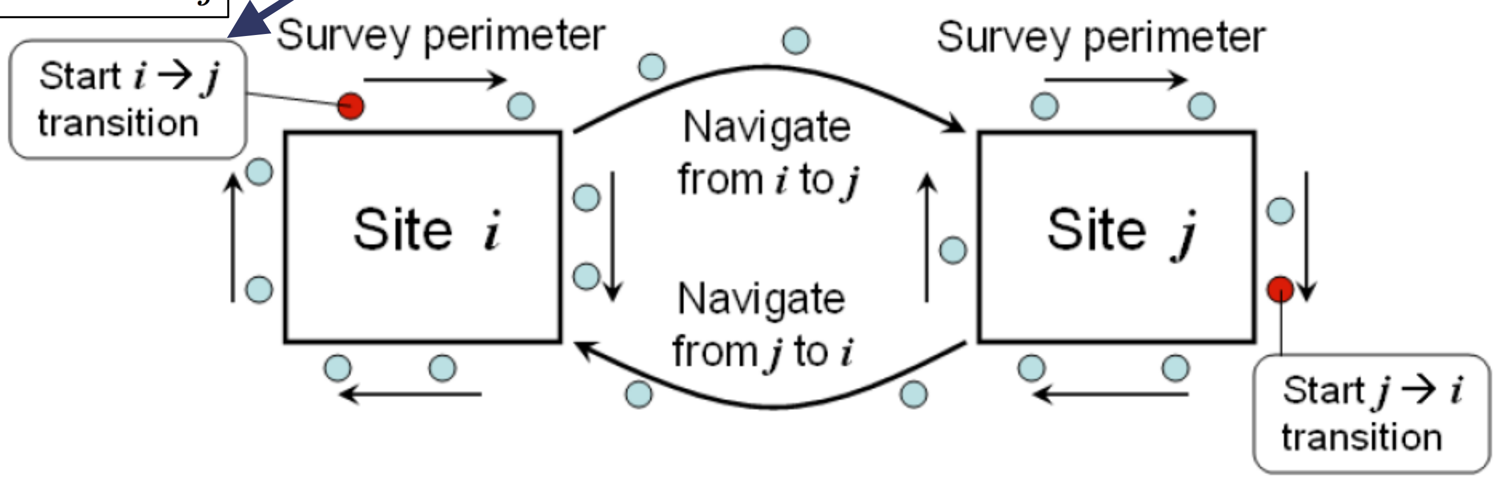
Simulation Methodology

Compare the sets of optimized $\boldsymbol{k_{ij}}$

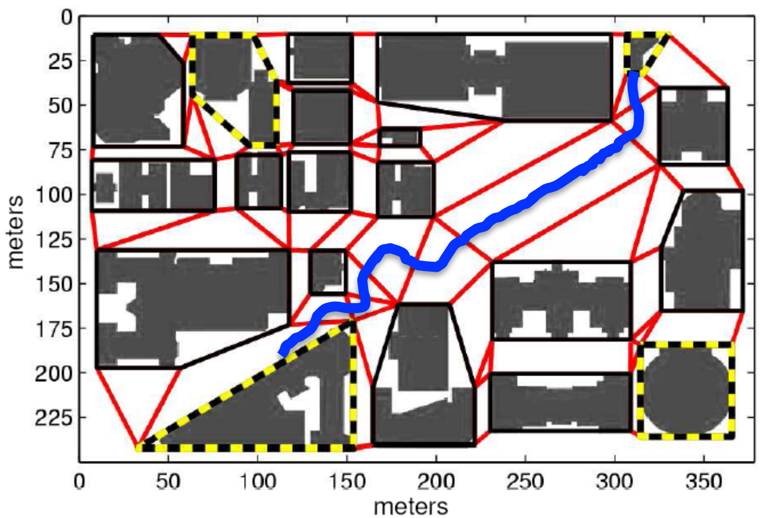
$$X_i \xrightarrow{k_{ij}} X_j$$

Simulate sequence of stochastic transitions using Gillespie's Direct Method

D. Gillespie, "A General Method for Numerically Simulating the Stochastic Time Evolution of Coupled Chemical Reactions," *J. Comp. Physics,* 1976



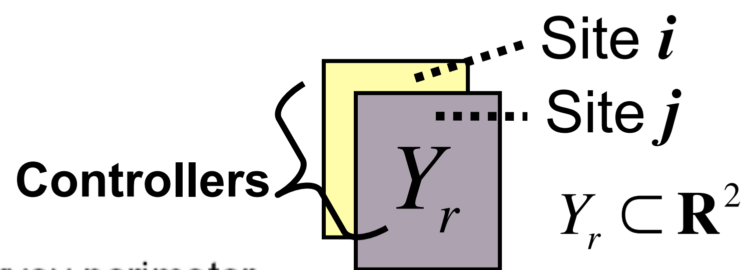
Simulation Methodology

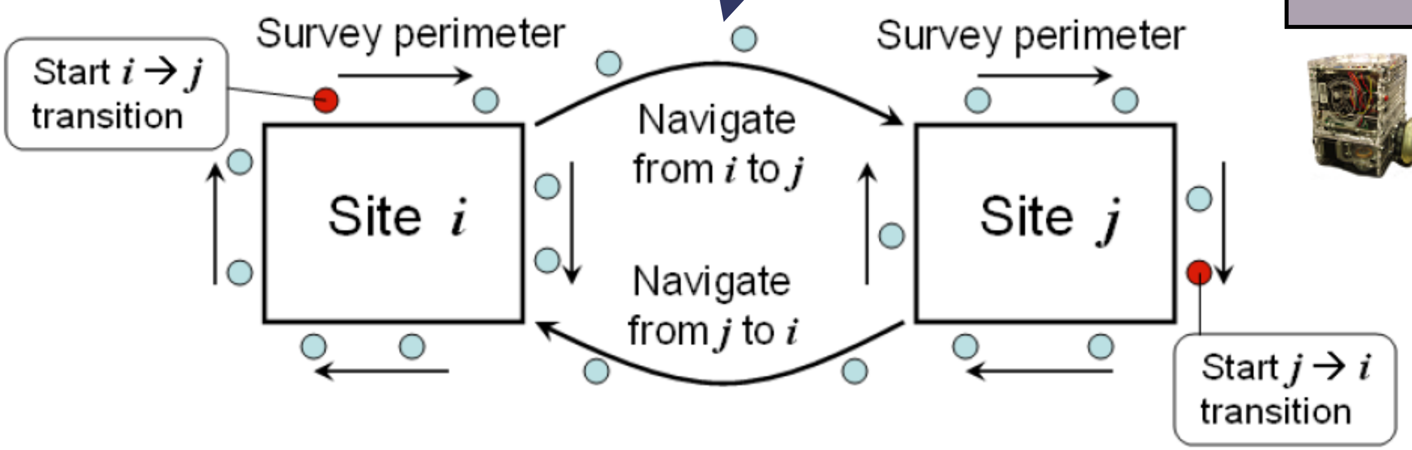


Local potentials to ensure arrival at goal cell **[1]** + repulsive potentials for inter-robot collision avoidance **[2]**

[1] H. G. Tanner, et al., "Flocking in fixed and switching networks," *IEEE Trans. Autom. Control*, 2007.

[2] D. C. Conner et al., "Composition of local potential functions for global robot control and navigation," *IROS* 2003





Simulation of Swarm Reallocation

Agreement between macroscopic and microscopic models

- Verifies the validity of our controller synthesis approach