Stochastic models and controllers for individual robots - Lecture 3

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MAE 598

- N robots moving in continuous state space $Y_r \in \mathbb{R}^d$, $d \in \{2, 3\}$
- At any given time, a robot's actions are determined by one of a set of controllers.
  - A controller causes a robot to fulfill a <u>task</u> or <u>subtask</u>.

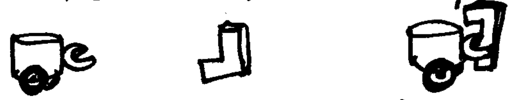
Analogue to <u>Chemical Reaction Network</u> (CRN):

X_i = species i = robot that is performing task i, object of type i, or robot-object complex of type i

$i \in \{1, 2, \ldots, S\}$

- Example reaction: $\overbrace{X_m + X_n}^{\text{reactants}} \xrightarrow{k_{ij}} \overbrace{X_p}^{\text{products}}$ — Represents a robot's task transition

k_{ij} = reaction rate constant



complex i complex j

To be able to abstract the microscopic model (CRN model of robot task transitions + set of controllers that accomplish the tasks) to the macroscopic model (set of differential equations), the task transitions must be executed in a way that conforms to the <u>fundamental hypothesis</u> of the stochastic formulation of chemical kinetics:

- A reaction that converts complex i into complex j is characterized by a stochastic reaction constant c_{ij}, defined such that:

$c_{ij}\,\delta t$ = average probability that a particular combination of elements of the species types in complex i will transition to the species types in complex j in the next infinitesimal time interval δt.

- k_{ij} is proportional to c_{ij}

- Reaction propensity a_{ij}:

$a_{ij}\,\delta t$ = probability that the reaction that converts complex i into complex j will occur in the next δt.

- h_{ij} = current number of distinct combinations of elements that can undergo the reaction

- $a_{ij} = c_{ij} h_{ij}$

I. Interaction-Dependent Switching $X_m + X_n \xrightarrow[\text{products}]{k_{ij}}$

- It is rare that 3 or more elements encounter each other simultaneously. Hence we will only consider bimolecular reactions in which the reactants consist of species m and n. [Note, however, that reactions with ≥ 3 reactants can approximate a sequence of 2-reactant encounters that occur very fast.

- Robots move in a bounded domain with volume V (area A in 2 dimensions).
- For the fundamental hypothesis to be true, the system must be "well-mixed."

How can this be implemented / checked in robotic systems?

- Can specify that robots perform a random walk with fixed speed and verify that they are uniformly randomly distributed over the domain
- Can check that diffusion rate associated with the random walk is much higher than the reaction rates.

[In molecular CRN's, "well-mixed" means that the species move randomly + quickly, giving every possible reactant set an equal chance to have a collision.]

$$c_{ij}\delta t = c_{ij}^{p} \cdot c_{ij}^{e}\,\delta t$$

$c_{ij}^{e}\,\delta t$ = prob. that a random pair of reactant elements of complex i will encounter each other in the next time interval δt

c_{ij}^{p} = prob. that these reactants will form complex j given that they are in close proximity. This is a parameter that we can design for macroscopic sys. objectives.

- To satisfy the fundamental hyp, c_{ij}^e and c_{ij}^p must both be independent of δt.

- N_i = number of elements of species i

$c_{ij} = \frac{k_{ij}}{V}$, $h_{ij} = N_m N_n$ if $m \neq n$

$c_{ij} = \frac{2k_{ij}}{V}$, $h_{ij} = N_m \frac{(N_m - 1)}{2}$ if $m = n$

Robot controller:

Robot in complex i that encounters another robot/element with which it can "react" to form complex j computes a uniformly distributed random number $u \in [0, 1]$ and follows through with the transition if $u < c_{ij}^p$.

II. Spontaneous Task Switching

$X_i \xrightarrow{k_{ij}}$ products

- Unimolecular reactions - reactant consists of one species
- To satisfy the fund. hyp, a robot at task i performs the task transition at probability per unit time c_{ij}.

$c_{ij} = k_{ij}$, $h_{ij} = N_i$

- k_{ij} is also called a "transition rate" for these types of reactions: $X_i \xrightarrow{k_{ij}} X_j$

Robot Controller:

Robot doing task i computes a uniformly random number $u \in [0, 1]$ at each (very small) timestep Δt and executes the transition if $u < k_{ij} \Delta t$.

- The number of transitions governed by k_{ij} that occur in Δt has a Poisson distribution with parameter $k_{ij} \Delta t$.