# An enzyme-inspired approach to stochastic allocation of robotic swarms around boundaries

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**Abstract** This work presents a novel control approach for allocating a robotic swarm among boundaries. It represents the first step toward developing a methodology for encounter-based swarm allocation that incorporates rigorously characterized spatial effects in the system without requiring analytical expressions for encounter rates. Our approach utilizes a macroscopic model of the swarm population dynamics to design stochastic robot control policies that result in target allocations of robots to the boundaries of regions of different types. The control policies use only local information and have provable guarantees on the collective swarm behavior. We analytically derive the relationship between the stochastic control policies and target allocations for a scenario in which circular robots avoid collisions with each other, bind to boundaries of disk-shaped regions, and command bound robots to unbind. We validate this relationship in simulation and show that it is robust to environmental changes, such as a change in the number or size of robots and disks.

**Key words:** distributed robotic systems; stochastic robotics; bio-inspiration; chemical reaction networks; attachment–detachment

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# **1** Introduction

In recent years, there has been an increasing focus on the development of *robotic swarms* [5] for performing tasks that require high degrees of parallelism, redundancy in system components and behaviors, and adaptability to changes in environmental conditions and failures. These systems would be composed of hundreds or thousands of relatively expendable, resource-constrained robots that operate with little-to-no human supervision. Advances in computing, sensing, actuation, power, communication, and control technologies are currently enabling the production of affordable robots that are designed to act in collectives, both in research and education [4, 20]. In the past few years, the miniaturization of these technologies has led to a plethora of novel platforms for swarm applications, including micro quadrotors [15] and flapping-wing micro aerial vehicles (MAVs) [27]. At even smaller scales, advances in MEMS, low-power VLSI, and nanotechnology are facilitating the development of sub-millimeter self-powered robots [25].

Many potential swarm applications will require the self-organization of robots into groups of different sizes around various regions or objects in their environment (see Figure 1). For instance, a swarm may be tasked to transport multiple payloads that are each too heavy for a single robot to retrieve, which would necessitate that enough robots aggregate around each load to move it to a target destination, possibly at a desired speed. However, simply allocating robots to saturation on each payload may be inefficient. Similarly, surveillance tasks may require



**Figure 1** Example scenario with two types of disk-shaped regions, labeled 1 and 2. The unlabeled circles are robots that are allocating themselves to the region boundaries.

a swarm to surround different types of regions, such as structure perimeters, to achieve particular degrees of sensor coverage. Other possible applications include environmental monitoring and mapping, automated construction and manufacturing, and disaster response tasks such as cordoning off a hazardous area or extinguishing a fire. At the micro- and nano-scale, applications include micro-object manipulation, microfactories and nanofactories, and medical monitoring, diagnosis, and treatment. For example,

nano-scale robots could collect in desired proportions around objects that are transparent to macroscopic sensing technologies. If the proportions of nano-scale robots were detectable, then the presence of the objects could be inferred.

In order for robotic swarms to reliably carry out these tasks, a rigorous methodology is needed for synthesizing individual robot behaviors that provably result in target robot allocations around boundaries. The swarm control framework must be *scalable* to arbitrary robot population sizes and accommodate possibly extreme *limitations* on each robot's sensing, communication, and computation abilities. It must also account for *stochasticity* arising from noise due to sensor and actuator errors; inherent randomness in robot encounters with each other and with environmental features; and, for nanorobots, the effects of Brownian motion and chemical interactions at scales below tens of micrometers [9].

In this work, we present our first efforts toward developing a control framework with the aforementioned properties for the problem of allocating a robotic swarm in target group sizes around the boundaries of disjoint, stationary regions of different types. For simplicity, we consider only disk-shaped regions here, which we refer to as *disks*, but this work can be extended to other region shapes. The robots have no prior information about the disks and they use only local sensing and local communication, encountering the disks during the course of random walks. Disk *types* may be categorized according to physical or subjective properties; for instance, size or weight if the disks are payloads to be transported, or relative surveillance value if they are areas to be monitored. Figure 1 depicts an example scenario in which the objective is to attain an average allocation of three robots per type-1 disk and one robot per type-2 disk. Stochastic binding and unbinding behaviors of the robots will result in fluctuations around these target allocations, as illustrated by the variation in number of robots bound to each disk type.

We employ a top-down approach to synthesizing robot control policies that produce target allocations among the disks with probabilistic guarantees on performance. We represent the stochastic robot interactions with disks and with each other as a well-mixed chemical reaction network (CRN). The robot-to-robot interactions consist of an enzyme-inspired behavior, implemented at the disk boundaries, that greatly reduces the dependence of the allocation strategy on the *encounter rates*, the difficult-to-characterize probabilities per unit time that a robot encounters an occupied or unoccupied section of a disk boundary. This behavior also decouples allocation tasks that may be occurring in parallel. The CRN formulation allows us to abstract the system to a *macroscopic* population model, a set of ordinary differential equations (ODEs) that is amenable to analysis and control. Our approach can be implemented by using a supervisory agent to design the model parameters for a particular global objective and broadcast them to the robots. The robots use these parameters to define their stochastic decision-making policies, and the resulting collective behavior follows the macroscopic prediction in expectation.

Through agent-based NetLogo [26] simulations<sup>1</sup> of a *microscopic* model of robots interacting with disks and other robots, we have validated that the simulated system retains all of the qualitative features of the predictions of the macroscopic model and is robust to environmental parameter variations. That is, a single control strategy can be implemented based on the geometric properties of a single robot and a single disk, and the equilibrium occupancy levels of robots around disks will be invariant to changes in the total numbers of robots, disks, and disk types, as well

<sup>&</sup>lt;sup>1</sup> To obtain the code for the simulations presented in this paper, contact Dr. Theodore Pavlic (e-mail: tpavlic@asu.edu).

as robust to changes in robot speed (e.g., due to battery decay). Hence, the control strategy need not be re-tuned if the environment around the robot changes over time.

# 1.1 Related Work

Similar to our swarm allocation strategy, much existing work on understanding and controlling robotic swarm behaviors relies on developing an accurate macroscopic model of the population dynamics. The model's dimensionality is independent of the swarm size, which facilitates quick simulation and a scalable control approach. Previous work has addressed stochastic approaches to swarm robotic task allocation, in which the robot task-switching rates are optimized using non-spatial macroscopic models that describe the time evolution of the robot population in each state [1, 6, 16, 18, 23]. Non-spatial swarm models have also been used to optimize stochastic robot behaviors in problems of robotic assembly of parts into products and self-assembly via binding through random collisions [10, 14, 19, 22]. Spatial macroscopic models of swarms that describe robot deterministic and random motion in addition to stochastic task switching have also been developed recently [8, 12, 24].

The utility of the macroscopic model in these works hinges on the ability to accurately determine the non-tunable components of the model parameters. Specifically, in applications where the model captures random interactions between entities in the system, these components are the corresponding encounter rates. However, encounter rates can often be determined only through simulation [11, 13] because the robot motion pattern and sensor footprint and the environment configuration can induce unpredictable spatially dependent effects, or simply *spatial effects*, on the frequency of robot encounters with other system entities. Encounter-rate formulas based on geometric parameters have been used in previous work on macroscopic swarm modeling of systems in which robots encounter objects that are small [17] or large (and elongated) [7] relative to them. However, these formulas can be applied only for environments with low densities of robots and objects in which robots are uniformly spatially distributed at all times, which is ensured when robots execute random walks and the objects do not bias the robots' movements. Our scenario violates these assumptions in that the encountered objects are adjacent to one another and thus not distributed at low density. The implausibility of deriving analytical solutions of encounter rates for our scenario motivated us to find a way of controlling the swarm without knowledge of these rates while still using a macroscopic model.

# 2 Robot Controller

We assume that each robot has a small sensing and communication radius. Even when communication is possible, it may be difficult for a robot to identify the lo-



**Figure 2** Diagram of control flow. Robots randomly cycle through searching and encountering robots and unbound zones. On encountering those disk regions, they probabilistically choose to bind to unbound regions or tell bound robots to unbind. On encountering unbound robots, they avoid collisions. Probabilities can be implemented with a pseudo-random number  $R \in \text{unif}(0, 1)$ .

cation of the source of a message. Consequently, we propose a control strategy that achieves a desired average allocation around each type of disk using only local robot–robot and robot–disk interactions. The controller for each robot, shown in Figure 2, incorporates:

- **Robot movement:** Robots move according to a *correlated random walk* (CRW) in order to achieve approximately uniform distributions throughout empty space. That is, each robot moves straight ahead in a short segment and then turns to a random angle before repeating. If this assumption of spatial homogeneity is violated, then different regions of space may approach equilibrium at faster rates than others. However, the limiting average allocations will be robust to inhomogeneity of robot density.
- **Robot–disk interactions:** Each robot can identify the type of disk that it encounters. The robot then chooses to bind to that disk with probability  $p_b$  that depends on disk type; otherwise, the robot ignores the encounter and continues its CRW.
- **Robot–robot interactions:** Upon encountering another robot, a robot can identify whether it is an unbound robot or a robot that is bound to a disk. If it encounters an unbound robot, the robot executes a collision avoidance maneuver. If it encounters a bound robot, the robot chooses to command that robot to unbind based on a probability  $p_u$  that depends on the disk type; otherwise, the robot ignores the encounter and continues its CRW.

We do not specify a behavior in which robots unbind spontaneously at a certain probability rate. Robots either probabilistically choose to bind to encountered disks or stochastically command other robots to unbind from disks. If a bound robot is never encountered by a free robot, it will never unbind from the disk.

## 3 Microscopic Model: Enzymatic Chemical Reaction Network

The robot–disk system described in Section 2 resembles a gas made up of species of *free robots* and disk zones that are either *bound* or *unbound*. For simplicity, we

only consider one disk type here; however, we will show how these results naturally extend to an arbitrary number of disk types. The corresponding well-mixed chemical reaction network (CRN) for the single-type case is:

$$r + U \xrightarrow{p_b e_u} B$$
 (1a)

$$r + B \xrightarrow{p_u e_b} U + 2r$$
 (1b)

where *r* represents the free-robot species, *B* represents bound zones, and *U* represents unbound zones. The mass-action rate constants  $p_b e_u$  and  $p_u e_b$  include:

- $e_u$ : The probability per unit time that a single free robot will encounter a single unbound zone. This encounter rate is an environmental parameter.
- $e_b$ : The probability per unit time that a single free robot will encounter a single bound zone. This encounter rate is an environmental parameter.
- $p_b$ : The probability that a free robot will bind to an unbound zone given that it has just encountered it. This parameter is under the control of the designer.
- $p_u$ : The probability that a free robot will command a bound robot to unbind given that the free robot has just encountered the bound zone. This parameter is under the control of the designer.

Reverse reactions are necessary to stabilize unique non-trivial equilibria. Without a reverse reaction, the reaction in Equation (1a) would cause the system to reach trivial saturation of bound zones. In other examples in stochastic robotics [e.g., 1– 3, 14, 19, 21], event-driven forward reactions like Equation (1a) are accompanied with delay-driven reverse reactions – robots have a tendency to *decay* back into earlier behavioral modes. Instead of implementing the reverse reactions as decay processes, we implement the reverse direction with the event-driven enzymatic reaction in Equation (1b). That is, the free robot that encounters the bound zone in Equation (1b) is not consumed by the reaction; it is analogous to an enzyme which rapidly increases the decay rate of bound zones. As we will show, because both reactions are event driven, the expected equilibrium distributions will vary with the ratio  $e_b/e_u$  as opposed to the absolute encounter rates. In general, the absolute encounter rates  $e_b$  and  $e_u$  will change with robot density, total number of zones, and robot speed (which itself can change over time with battery fatigue). However, the ratio  $e_b/e_u$ , and thus the equilibrium distribution, will be invariant to these changes.

## 4 Macroscopic Model: Concentration Fields of Zones and Robots

From the theory of mass-action kinetics in well-mixed gases, a smooth concentration-field approximation of the CRN in Equation (1) for large populations is the multi-affine system

$$\begin{cases} \dot{r} = p_u e_b r B - p_b e_u r U \\ \dot{U} = p_u e_b r B - p_b e_u r U \\ \dot{B} = p_b e_u r U - p_u e_b r B \end{cases}$$
(2)

where r, U, and B represent the number of free robots, unbound zones, and bound zones, respectively, for a given arena (i.e., concentrations in a fixed volume size). The system clearly has a continuum of trivial equilibria characterized by r = 0, which represents the total depletion of free robots. Moreover, because this system is continuous, the set  $\{r : r > 0\}$  is positively invariant; if the initial concentration of free robots is positive, then the concentration will remain positive indefinitely. Thus, assuming non-zero mass-action rate constants, there is an additional equilibrium  $(r, U, B) = (r^*, U^*, B^*)$  where  $r^* > 0$  and

$$\frac{B^*}{U^*} = \frac{p_b e_u}{p_u e_b} \quad \text{or, equivalently,} \quad \frac{B^*}{B^* + U^*} = \frac{p_b e_u}{p_b e_u + p_u e_b} = \frac{\frac{p_b}{p_u}}{\frac{p_b}{p_u} + \frac{e_b}{e_u}}.$$
 (3)

Let  $B_0$ ,  $U_0$ , and  $r_0$  represent the initial number of bound zones, unbound zones, and free robots, respectively. Noting that  $\dot{U} = \dot{r}$  and  $\dot{B} = -\dot{r}$ , it must be the case that  $B^* + U^* = B_0 + U_0$ . Additionally, the the third-order system in Equation (2) can be re-written as a first-order differential equation

$$\dot{r} = p_u e_b r(B_0 - (r - r_0)) - p_b e_u r(U_0 + (r - r_0))$$

$$= ((B_0 + r_0) p_u e_b - (U_0 - r_0) p_b e_u) r - (p_b e_u + p_u e_b) r^2$$
(4)

representing the dynamics of number of free robots. Under the assumption of nonzero mass-action rate constants, the non-trivial equilibrium at  $r = r^* > 0$  is such that

$$r^* = (B_0 + r_0) \frac{p_u e_b}{p_b e_u + p_u e_b} - (U_0 - r_0) \frac{p_b e_u}{p_b e_u + p_u e_b}.$$

So, by Equation (3) and because  $B^* + U^* = B_0 + U_0$ ,

$$r^* = (B_0 + r_0) \left( 1 - \frac{B^*}{B_0 + U_0} \right) - (U_0 - r_0) \frac{B^*}{B_0 + U_0}$$

which is positive and asymptotically stable so long as  $B_0 + r_0 > B^*$ . That is, so long as the total number of free and bound robots  $r_0 + B_0$  is larger than the predicted equilibrium number of bound robots  $B^*$ , the  $(r^*, U^*, B^*)$  equilibrium will be asymptotically stable with  $r^* > 0$ . In other words, from Equations (3) and the condition that  $B_0 + r_0 > B^*$ , the system in Equation (1) has an asymptotically stable equilibrium described by

$$(r, B, U) = \begin{cases} (r^*, B^*, U^*) & \text{if } r_0 + B_0 > B^*, \\ (0, U_0 - r_0, B_0 + r_0) & \text{otherwise.} \end{cases}$$
(5)

So the system is driven by the imbalance between fluxes to and from bound and unbound zones; it comes to rest when enough free robots are converted into bound zones to restore flux balance or when the pool of free robots is totally depleted.

## 4.1 High-Population Linear Approximation

Due to its quadratic structure, Equation (4) can be solved explicitly. For any t > 0,

$$r(t) = r^* \frac{r_0}{r_0 + (r^* - r_0)\exp(-r^*(p_b e_u + p_u e_b)t)}$$

which, for  $r_0 \gg 0$ , is essentially constant. That is,  $r(t) \approx r^* \approx r_0$  for  $r_0 \gg 0$ . Consequently, for  $r_0 \gg 0$ , the multi-affine system in Equation (2) that approximates the bimolecular CRN in Equation (1) can be viewed as the linear system

$$\begin{cases} \dot{U} = p_u e_b r_0 B - p_b e_u r_0 U\\ \dot{B} = p_b e_u r_0 U - p_u e_b r_0 B \end{cases} \quad \text{that models the unimolecular} \quad \begin{aligned} U \xrightarrow{p_b e_u r_0} B\\ B \xrightarrow{p_u e_b r_0} U \end{aligned}$$
(6)

In this  $r_0 \gg 0$  regime, the number of free robots scales the per-zone encounter rates. Moreover, although the linear system in Equation (6) has an equilibrium in Equation (3) that is independent of  $r_0$ , the time constant of the system is

$$\frac{1}{(p_b e_u + p_u e_b)r_0}.$$
(7)

Thus, increasing  $r_0$  increases the total speed of the system but has no impact on the equilibrium allocation of robots to disks.

# 4.2 Multiple Disk Types: Decoupled Analysis and Control

For any number of zones, there is some sufficiently large initial number of free robots  $r_0$  that satisfies the condition that  $B_0 + r_0 > B^*$  and thus guarantees the stability of a non-trivial equilibrium zone concentration described by Equation (3), which is invariant to changes in  $r_0$ . So if the pool of free robots is sufficiently large, the equilibrium analysis of a system with multiple disk types can be performed independently for each disk type.

For example, if there are *n* disk types and the number of free robots  $r_0$  is initially greater than the total number of unbound zones across all disk types, then the concentration of bound zones  $B^i$  and unbound zones  $U^i$  on disk type  $i \in \{1, 2, ..., n\}$  at equilibrium is such that  $B^i/U^i = (e_u^i/e_b^i)(p_b^i/p_u^i)$  where  $e_u^i, p_b^i, e_b^i$ , and  $p_u^i$  are the encounter rates and reaction probabilities specialized for type *i*. That is, the equilibrium analysis for any type is decoupled from the analysis of any other type.

Although the multiple types have a coupled effect on convergence rate and transient dynamics in general, the equilibrium allocations can be predicted in isolation. So for the remainder of this paper, we assume a sufficiently large pool of robots to meet subjective convergence time constraints for an arbitrary number of disk types. Moreover, we will only explicitly discuss design for a single disk type; it is implied that the process is identical for multiple coexisting types.

## 4.3 Corrections for Spatial Effects on Boundaries

In principle, robots can be organized around a disk to reach 100% allocation (i.e., B/(B+U) = 1). However, in practice, it is likely that two robots interacting stochastically with a disk will bind with non-zero inter-robot space between them that is nevertheless too small for another robot to encounter. So although the amount of unbound space on a disk may be large, the actual number of unbound zones available for additional binding may be small. Thus, the maximum value of B/(B+U) will be less than unity; even a  $(p_b, p_u) = (1,0)$  policy will saturate with free space remaining on disks. Moreover, even well before saturation, some amount of free space will be inaccessible for free robots to bind to because it will be too close to existing bound robots. Thus, a theory is needed to model the non-linear reduction in remaining unbound space as robots bind to disks.

In the following, assume that all linear distances are given in units of the arc length occupied by a robot when bound to a disk. That is, each robot binds to 1 unit of arc length, and so the theoretical maximum number of robots bound to a disk is equal to the disk's circumference. However, because robots are not equipped with the ability to cluster together, the actual maximum number of bound robots will be much lower. Consider:

- A disk with a single robot bound to it. An incoming unbound robot will not be able to discover disk space adjacent to a bound robot unless its center is at least 0.5 units away from the edge of the bound robot. So the bound robot effectively occupies both its own 1 unit of arc space plus an additional 1 unit of arc length adjacent to it. Additionally, if the incoming unbound robot maintains a distance *a* between itself and every other robot (e.g., to avoid collisions), then the additional space occupied by the bound robot increases to  $\delta_{\max} \triangleq 1 + 2a$ units because there are 0.5 + a units of additional occupation on both sides of the bound robot.
- A disk with many robots bound to it. If two robots have less than 1 + 2a unbound arc length between them, an incoming unbound robot will not be able to discover it. However, these small distances can be no smaller than the avoidance distance  $\delta_{\min} \triangleq a$ .

With this in mind, we partition the space between robots into sections no longer than  $\delta_{\max} \triangleq 1 + 2a$ , as shown in Figure 3. We then define the quantity  $\delta$  to be the mean size of the partitioned inter-robot spaces. Thus, although truncation of an inter-robot space that is only slightly larger than 1 + 2a can create a truncated space smaller than a, the mean  $\delta$  is bounded above and below such that

$$\delta_{\min} = a \le \delta \le 1 + 2a = \delta_{\max}.$$

The statistic  $\delta$  is actually a function  $\delta : [0,1] \rightarrow [a, 1+2a]$  that maps an allocation ratio B/(U+B) to the mean additional arc occupancy per bound zone  $\delta(B/(U+B))$ , which we abbreviate to  $\delta$  here for convenience. At low allocation ratios,  $\delta \approx 1+2a$  because the space between bound robots is large. Consequently, there will be



**Figure 3** Partitioning of space between robots. Here, four robots are shown connected to a single disk. Each robot with its corresponding disk sector, shown with a "B", constitutes a bound zone. The four spaces between each pair of robots have been partitioned into smaller spaces no larger than  $\delta_{\text{max}} = 1 + 2a$ , which represents the maximum additional arc length that a bound robot can interfere with due to spatial effects.

more encounters with bound zones and fewer encounters with unbound zones than otherwise expected. Similarly, at high allocation ratios,  $\delta \approx a$ . So although bound zones are still magnified, this magnification decreases with added allocation ratio.

To model the effective increase in *B* by  $\delta B$  and the corresponding effective decrease in *U* by  $\delta B$ , we apply the substitution  $B^* \mapsto (1+\delta)B^*$  and  $U^* \mapsto U^* - \delta B^*$  to the equilibrium condition in Equation (3). Consequently, the actual  $(B^*, U^*)$  equilibrium will be such that

$$\frac{\overbrace{(1+\delta)}^{B^*}}{1-\delta\frac{B^*}{U^*}}\frac{B^*}{U^*} = \frac{e_u p_b}{e_b p_u}$$
(8)

where the overbraced expression is a correction factor for the spatial effects in a physical robot scenario. For comparison, the corrected allocation can be related to the idealized allocation by

$$\frac{B^{*}}{U^{*}+B^{*}} = \frac{B^{*}}{(U^{*}-\delta B^{*}) + (1+\delta)B^{*}} = \frac{\frac{B^{*}}{(1+\delta)B^{*}}}{\frac{U^{*}-\delta B^{*}}{(1+\delta)B^{*}} + 1} = \frac{1}{1+\delta} \underbrace{\frac{p_{b}}{p_{u}}}_{p_{u}} \underbrace{\frac{p_{b}}{p_{u}} + \frac{e_{b}}{e_{u}}}_{p_{u}}$$
(9)

where the overbraced expression matches the idealized allocation ratio in Equation (3). So the ideal and actual allocations are predicted to be related by a  $1/(1+\delta)$  gain. For low allocations, this gain will be 1/(1+2a); for high allocations, this gain will be determined by the saturated value of  $\delta > \delta_{\min} = a$ .

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#### **4.3.1** Shape of $\delta$ Function

An important future direction is to develop theory to predict the precise shape of the  $\delta$  function. However, as we will discuss later, we have experimental evidence that  $\delta$  is only determined by the value of *a*. Moreover,  $\delta$  appears to be a cosine of the form  $\delta(r) = A \cos(2\pi(r/T) + c)$  that pierces  $\delta(0) \approx 1 + 2a$  and  $\delta(1/(1+a)) = a$  subject to the constraints  $A \ge (1+a)/2$ ,  $T \ge 2/(1+a)$ , and  $c \ge 0$ .

## 5 Control of Equilibrium Allocations

From the equilibrium described by Equation (8), a  $(p_b, p_u)$  control policy can be synthesized using the rule

$$\frac{p_b}{p_u} = \frac{e_b}{e_u} \frac{B^*}{U^*} \frac{(1+\delta)}{1-\delta \frac{B^*}{U^*}}$$
(10)

where  $B^*/U^*$  is the desired bound–unbound allocation ratio of zones at equilibrium. Equivalently, if the desired robot-to-boundary-space *allocation ratio* is  $B^*/(B^* + U^*)$ , then  $(p_b, p_u)$  should chosen according to Equation (9). Thus, for any given allocation ratio, there is a continuum of  $(p_b, p_u)$  pairs that will achieve the desired equilibrium.

The control policy in Equation (10) has one degree of freedom over which some feature of the system can be optimized. For example, by Equation (7), the convergence rate of the system can be maximized by making the sum  $p_b + p_u$  as large as possible. So, for fastest convergence for to a desired allocation  $(B^*, U^*)$ ,  $p_b$  and  $p_u$  can be chosen so that

$$(p_b, p_u) = \begin{cases} \left(\frac{e_b}{e_u} \frac{B^*}{U^*} \frac{(1+\delta)}{1-\delta B^*/U^*}, 1\right) & \text{if } e_b B^*(1+\delta) < e_u U^*(1-\delta B^*/U^*), \\ \left(1, \frac{e_u}{e_b} \frac{U^*}{B^*} \frac{1-\delta B^*/U^*}{(1+\delta)}\right) & \text{otherwise.} \end{cases}$$
(11)

However, optimization criteria other than maximal convergence rate may suggest other choices of  $(p_b, p_u)$ . For example, there will be fewer temporal variations in the number of robots bound to each disk if  $p_b + p_u$  is reduced. Similarly, the variance in allocation across disks may be reduced for certain  $(p_b, p_u)$  combinations. Furthermore, if the  $e_b/e_u$  ratio can be artificially shifted (e.g., by asymmetrically changing the relative distance that sensors react to bound and unbound zones) or the avoidance distance *a* changed, it is possible to shift the  $p_b/p_u$  control policy for a desired  $B^*/U^*$  allocation ratio. Thus, there are mechanisms that can further adjust the  $p_u + p_b$  sum without changing the equilibrium allocation ratio.

## 6 Model Validation in Simulation

To test our macroscopic model of stochastic allocation to circular boundaries, experimental trials were conducted using NetLogo [26]. The framework allowed for simulating hundreds of mobile robots randomly interacting with each other and with disks of different types.

# 6.1 Variations due to Encounter-rate Ratio

For many robot motion behaviors, the encounter-rate ratio  $e_b/e_u$  may be approximated, for example, by dividing the sum of the areas of a robot and an unbound zone sector by the area of an unbound zone sector alone, where zone sectors are slices of each disk with arcs that are the length of the interaction region with

the robot. In general, it can be estimated from equilibrium allocation data. If, for example, it is incorrectly assumed that the  $e_b/e_u$  ratio is unity, the equilibrium allocation will shift in a predictable way based on the correct  $e_b/e_u$  ratio, as shown in Figure 4. Consequently, if the  $e_b/e_u$  ratio is not well known, it can be estimated by measuring this curve during system testing. Inferring this encounter-rate ratio is empirically much simpler than inferring the actual encounter rates. Also shown in Figure 4 is the effect of the interrobot space  $\delta(1.0)$  being both non-zero and yet smaller than required to fit any additional robots. Thus, disks saturate at a level less than full occupancy.

To validate these predictions, experimental trials were conducted using 500 simulated mobile robots moving along correlated random walks in a space with 6 disks with circumference capacity for 28.27 robots per disk. By increasing the so-called *turning angle* of the CRW, the robot motion became



**Figure 4** Effect of encounter ratio. For each  $e_b/e_u$  ratio, a plot comparing the idealized allocation ratio to the actual allocation ratio is shown according to Equation (9). Here,  $\delta(r) = \cos(2\pi(r/4.6)+0.2)$ , which is consistent with an a = 0 case. Allocations saturate near an actual ratio of 0.75 because the mean slack space  $\delta(1)$  is both non-zero and too small to accommodate additional binding.

less directional and more Brownian. Consequently, robots with higher turning angle are more likely to re-encounter a disk and re-bind immediately after being told to unbind. This decrease in unbinding efficacy decreases the effective  $e_b/e_u$  ratio, as shown in Figure 5 which matches Figure 4 for different  $e_b/e_u$  ratios.



**Figure 5** Effect of encounter ratio in simulation. For each  $e_b/e_u$  ratio, a plot comparing the idealized allocation ratio to the actual allocation ratio is shown according to Equation (9). The particular  $e_b/e_u$  ratios corresponding to the two motion primitives (i.e., low and high turning angle) were fit to the observed data. Additionally,  $\delta(r) = \cos(2\pi r/4.6 + 0.2)$ , which is consistent with predictions from an a = 0 case with a disk circumference of 28.27 robot widths. Allocations saturate near an actual ratio of 0.75 because the mean slack space  $\delta(1)$  is both non-zero and too small to accommodate additional binding. The slight deviations from prediction in (b) can be improved with better understanding of the derivation of the  $\delta$  function. Small dots show outcomes of individual simulation runs. Open circles show means across ten trials of each allocation ratio. Error bars show  $\pm 1$  standard error of the mean (SEM). Each trial uses 500 simulated robots and 6 disks.

### **6.1.1** Estimation of $\delta$ Function

The  $\delta$  function used in Figures 4 and 5 is based on an avoidance range of a = 0 and a circumference of 28.27 robot widths. As shown in Figure 6, this  $\delta$  fits mean data from simulated scenarios regardless of CRW parameters and effective encounterrate ratio. The empty-occupancy  $\delta(0) < 1 + 2a$  because the circumference does not divide evenly by 1 + 2a. That is, the partitioned space of the empty disk includes a residual sub-unity partition, and so the mean across those partitions is less than 1.

## 6.2 Robustness to Environmental Variations

Empirical studies show that the relationship between idealized and actual allocation is not sensitive to environmental variations. For example, Figure 7 shows statistics taken from simulation runs with several combinations of robot population, robot size, number of disks, and disk size. As shown, varying the size and number of disks and robots does not change the actual allocation ratio. A single control strategy leads to the same equilibrium mean allocation ratio in every case.



**Figure 6** Simulated effect of encounter ratio on  $\delta$ . The single  $\delta$  function from Figure 4 accurately predicts mean inter-robot space across different encounter ratios and motion primitives. Each small dot shows a result from an individual simulation run. Open circles show means for different allocations. Error bars show SEM. Ten trials were run per allocation ratio.



Figure 7 Effect of varying environmental parameters. Ten trials were generated for each disk size, and the average across the trials are shown with error bars indicating  $\pm 1$  standard error of the mean. A dashed line of unity slope is shown for reference. The solid line represents the predicted curve based on the avoidance distance *a*, which is non-zero for these cases.

# 7 Conclusions and Future Work

We have presented a rigorous methodology for designing control policies that distribute a swarm of robots among a set of boundaries. The control policies rely only on local information obtained by the robots and have probabilistic guarantees on steady-state performance. We investigated the effect of robot interactions on the actual steady-state allocations that were achieved with the use of control policies An enzyme-inspired approach to stochastic allocation of swarms around boundaries

derived from a coarse-grained description of the swarm population. Our simulation results illustrate that the actual system behavior follows predictable and controllable trends when robot interactions at the boundaries and in the free space are introduced. In this way, we build a foundation for further work on encounter-based swarm robotic allocation that obviates an analytical characterization of encounter rates.

In future work, we will repeat these investigations for more arbitrary shapes. We also plan to gain a better understanding of the relationship between actual allocation ratio and the mean space between robots. Leveraging the several degrees of freedom in the enzymatic swarms (i.e., binding and unbinding probabilities, robot geometry, avoidance distances, and motion primitives that shape encounter-rate ratios), we will explore the design of optimal robot control policies to achieve convergence to desired allocations subject to other constraints, such as ensuring minimal allocation variance or settling within a specified time. We will extend our control approach to other scenarios in which robotic swarms must allocate among both static and dynamically moving regions, such as surveillance and target-tracking applications, and we will investigate how this approach can simplify other stochastic strategies such as swarm self assembly. Finally, we plan to experimentally validate our approach on a physical multi-robot testbed.

Acknowledgements This work was supported in part by the National Science Foundation (grant no. CCF-1012029).

# References

- Berman S, Halász Á, Hsieh MA, Kumar V (2009) Optimized stochastic policies for task allocation in swarms of robots. IEEE Trans Robot 25(4):927–937, doi: 10.1109/TRO.2009.2024997
- Berman S, Kumar V, Nagpal R (2011) Design of control policies for spatially inhomogeneous robot swarms with application to commercial pollination. In: Proceedings of the 2011 IEEE International Conference on Robotics and Automation, Shanghai, China
- Berman S, Nagpal R, Halász Á (2011) Optimization of stochastic strategies for spatially inhomogeneous robot swarms: a case study in commercial pollination. In: Proceedings of the 2011 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS), San Francisco, CA, USA, pp 3923–3930, doi: 10.1109/IROS.2011.6094771
- Bonani M, Longchamp V, Magnenat S, Rétornaz P, Burnier D, Roulet G, Vaussard F, Bleuler H, Mondada F (2010) The marXbot, a miniature mobile robot opening new perspectives for the collective-robotic research. In: Proceedings of the 2010 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS), Taipei, pp 4187–4193, doi: 10.1109/IROS.2010.5649153
- Brambilla M, Ferrante E, Birattari M, Dorigo M (2013) Swarm robotics: a review from the swarm engineering perspective. Swarm Intell 7(1):1–41, doi: 10.1007/s11721-012-0075-2
- Correll N (2008) Parameter estimation and optimal control of swarm-robotic systems: a case study in distributed task allocation. In: Proceedings of the 2008 IEEE International Conference on Robotics and Automation, Pasadena, CA, USA, pp 3302–3307, doi: 10.1109/ROBOT.2008.4543714

- Correll N, Martinoli A (2004) Modeling and optimization of a swarm-intelligent inspection system. In: Proceedings of the Seventh International Symposium on Distributed Autonomous Robotics Systems (DARS 2004), Toulouse, France, pp 369–378, doi: 10.1007/978-4-431-35873-2\_36
- Dantu K, Berman S, Kate B, Nagpal R (2012) A comparison of deterministic and stochastic approaches for allocating spatially dependent tasks in micro-aerial vehicle collectives. In: Proceedings of the 2012 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS), Vilamoura, Portugal, pp 793–800, doi: 10.1109/IROS.2012.6386233
- 9. Diller E, Sitti M (2013) Micro-scale mobile robotics. Foundations and Trends in Robotics 2:143–259
- Evans WC, Mermoud G, Martinoli A (2010) Comparing and modeling distributed control strategies for miniature self-assembling robots. In: Proceedings of the 2010 IEEE International Conference on Robotics and Automation, Anchorage, AK, USA, pp 1438–1445
- 11. Gurarie E (2008) Models and analysis of animal movements: From individual tracks to mass dispersal. PhD thesis, University of Washington
- Hamann H, Wörn H (2008) A framework of space-time continuous models for algorithm design in swarm robotics. Swarm Intell 2(2–4):209–239, doi: 10.1007/s11721-008-0015-3
- Hutchinson JMC, Waser PM (2007) Use, misuse and extensions of "ideal gas" models of animal encounter. Biol Rev 82(3):335–359, doi: 10.1111/j.1469-185X.2007.00014.x
- Klavins E, Burden S, Napp N (2006) Optimal rules for programmed stochastic self-assembly. In: Proceedings of Robotics: Science and Systems II, Philadelphia, PA, USA
- 15. Kushleyev A, Kumar V, Mellinger D (2012) Towards a swarm of agile micro quadrotors. In: Proceedings of Robotics: Science and Systems VIII, Sydney, NSW, Australia
- Liu W, Winfield AFT (2010) Modeling and optimization of adaptive foraging in swarm robotic systems. Int J Robot Res 29(14):1743–1760, doi: 10.1177/0278364910375139
- Martinoli A, Easton K, Agassounon W (2004) Modeling swarm robotic systems: a case study in collaborative distributed manipulation. Int J Robot Res 23(4–5):415–436, doi: 10.1177/0278364904042197
- Mather TW, Hsieh MA (2011) Distributed robot ensemble control for deployment to multiple sites. In: Proceedings of Robotics: Science and Systems VII, Los Angeles, CA, USA
- Matthey L, Berman S, Kumar V (2009) Stochastic strategies for a swarm robotic assembly system. In: Proceedings of the 2009 IEEE International Conference on Robotics and Automation, Kobe, Japan, pp 1953–1958
- McLurkin J, Rykowski J, John M, Kaseman Q, Lynch AJ (2013) Using multi-robot systems for engineering education: teaching and outreach with large numbers of an advanced, low-cost robot. IEEE Trans Educ 56(1):24–33, doi: 10.1109/TE.2012.2222646
- Napp N, Klavins E (2011) A compositional framework for programming stochastically interacting robots. Int J Robot Res [Special Issue Stochasticity in Robot Bio-Systems Part 2] 30(6):713–729, doi: 10.1177/0278364911403018
- Napp N, Burden S, Klavins E (2009) Setpoint regulation for stochastically interacting robots. In: Proceedings of Robotics: Science and Systems V, Seattle, WA, USA
- Odhner LU, Asada H (2010) Stochastic recruitment control of large ensemble systems with limited feedback. J Dyn Syst, Meas, Control 132(4):041008, doi: 10.1115/1.4001706
- Prorok A, Correll N, Martinoli A (2011) Multi-level spatial modeling for stochastic distributed robotic systems. Int J Robot Res 30(5):574–589, doi: 10.1177/0278364910399521
- Robotics Virtual Organization (Robotics VO) (2013) A Roadmap for U.S. Robotics: From Internet to Robotics, 2013 Edition. http://robotics-vo.us/sites/default/files/2013%20Robotics% 20Roadmap-rs.pdf
- Wilensky U (1999) NetLogo. Center for Connected Learning and Computer-Based Modeling, Northwestern University, Evanston, IL, USA, URL http://ccl.northwestern.edu/netlogo/
- Wood RJ, Finio B, Karpelson M, Ma K, Pérez-Arancibia NO, Sreetharan PS, Tanaka H, Whitney JP (2012) Progress on 'pico' air vehicles. Int J Robot Res 31(11):1292–1302, doi: 10.1177/0278364912455073

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