

A Stochastic Hybrid System Model of Collective Transport in the Desert Ant *Aphaenogaster cockerelli*

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ABSTRACT

Collective food transport in ant colonies is a striking, albeit poorly understood, example of coordinated group behavior in nature that can serve as a template for robust, decentralized multi-robot cooperative manipulation strategies. We investigate this behavior in *Aphaenogaster cockerelli* ants in order to derive a model of the ants' roles and behavioral transitions and the resulting dynamics of a transported load. In experimental trials, *A. cockerelli* are induced to transport a rigid artificial load to their nest. From video recordings of the trials, we obtain time series data on the load position and the population counts of ants in three roles. From our observations, we develop a stochastic hybrid system model that describes the time evolution of these variables and that can be used to derive the dynamics of their statistical moments. In our model, ants switch stochastically between roles at constant, unknown probability rates, and ants in one role pull on the load with a force that acts as a proportional controller on the load velocity with unknown gain and set point. We compute these unknown parameters by using standard numerical optimization techniques to fit the time evolution of the means of the load position and population counts to the averaged experimental time series. The close fit of our model to the averaged data and to data for individual trials demonstrates the accuracy of our proposed model in predicting the ant behavior.

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Keywords

stochastic hybrid system, collective transport, social insect behavior modeling, distributed robot systems, bio-inspired robotics, biomimicry

1. INTRODUCTION

Recent advances in technologies for swarm robotic systems, consisting of hundreds to thousands of autonomous, relatively expendable robots with limited capabilities, are facilitating the development of robotic teams to collectively manipulate and transport a variety of objects in their environment. These multi-robot transport teams can be used to amplify productivity in construction, manufacturing, and automated warehouse applications, as well as to aid in disaster scenarios and search-and-rescue missions. The problem of controlling swarm robotic transport teams in such applications presents certain challenges. The control approach should be scalable to arbitrary team sizes and should accommodate limitations on the robot platform's sensing, communication, and computation abilities. In addition, the control strategy should be robust to robot failures and not rely on detailed *a priori* information about the payload or environment so that it is generalizable to a wide range of scenarios.

As a step toward synthesizing a control approach for an adaptable, resource-constrained robotic transport team, we develop a model of collective transport by a group of agents

with minimal capabilities. Toward this end, we look to an analogous system in nature: the food retrieval teams formed in colonies of the desert ant *Aphaenogaster cockerelli*. Although most ant species are relatively unskilled at group transport, *A. cockerelli* has evolved the impressive coordination skills necessary for this task. This behavior is an example of a fully decentralized cooperative manipulation strategy that is scalable in the number of transporters and successful for a wide range of payloads in environments with uneven terrain and obstacles.

Berman et al. [1] and Czaczkes and Ratnieks [2] present overviews of the incidence, advantages, and organization of group retrieval teams in ants. Group transport requires individual ants to coordinate their movement to carry a bulky food item to the nest. How this is achieved remains very poorly understood [2, 3, 11]. Many have argued that coordination proceeds entirely by indirect interactions through the item itself, known as stigmergy [7], but more direct interactions and signaling among carriers may also play a role. A new approach to this problem is to describe the range of behavioral states occupied by transporters, the rates at which ants change states, and the contextual influences on these changes. Models of this type can link individual behavior to the dynamics of group transport, ultimately allowing the identification of behavioral rules crucial to successful coordination. Furthermore, these models can be adapted to describe other species and to explain why some species are better at collective transport than others. Thus, modeling group transport in ants not only provides a template for the engineering of multi-robot transport systems, but such models can also assist biologists in the analysis of natural transport behavior.

In previous work of S.M.B. and S.C.P. [1], qualitative observations of *A. cockerelli* transporting elastic vision-based force sensors were used to develop a model of ants collectively dragging a load with compliant attachment points. The model consisted of a behavioral component, comprised of a hybrid system with probabilistic transitions between two modes, and a dynamic component, described by a quasi-static planar manipulation model that incorporated friction on the load surface. In the current work, we quantitatively derive a model of collective retrieval that combines the stochastic ant behavioral transitions and the continuous load dynamics in a single framework, a *polynomial stochastic hybrid system (pSHS)* [4, 6], that is amenable to analysis and control. Using this framework, we can derive the time evolution of the model variables’ statistical moments, which can be fit to experimental data. The pSHS framework has been recently applied to problems of stochastic task allocation in multi-robot teams [8] and the control of self-assembly of stochastically interacting robots [10].

We conduct experiments on group transport in *A. cockerelli* using a rigid load, similar to one that would be encountered in nature, which the ants can lift as well as pull. This type of load allows us to develop a simpler dynamical model than our original one, and we expand the previous behavioral model of Berman et al. [1] with an additional mode to explicitly capture the directionality of the ants’ efforts. We propose that the ants switch stochastically between roles at constant probability rates and that their pulling force on the load acts as a proportional controller on the load velocity. The unknown parameters of our model, consisting of a set of behavioral transition rates, the load velocity set point, and

the gain on the load velocity regulator, are estimated from the experimental data using a weighted-least-squares procedure that fits theoretical to sampled means. The resulting model generates trajectories that not only closely match in the mean, but also accurately predict the load dynamics as a function of measured counts of ants in different roles in several individual trials.

2. EXPERIMENTAL TRIALS

We filmed colonies of *A. cockerelli* collectively retrieving a standardized artificial load. A total of 17 colonies were located in South Mountain Park in Phoenix, AZ. Experiments were carried out during the activity period of the colony in the early morning (0600–0830 hours) and in the late afternoon (1700–1900 hours) in May 2012.

A new colony was located each day. A Plexiglas® sheet with dimensions 61 cm × 46 cm × 0.5 cm was positioned such that one edge, Edge A, was 50 cm south of the main nest entrance, and the opposite edge, Edge B, was 111 cm south of the nest. The sheet was covered with white paper and leveled to avoid inclination in any direction. An artificial load was constructed by gluing a dime with mass $m_L = 2.30$ g and radius 0.90 cm to an ethylene vinyl acetate (EVA) foam disk (0.2 mm thickness, 1.0 cm radius), which was rubbed with fig paste to attract ants. We filmed the transport with a Canon G12 camera positioned above the sheet. The camera’s field of view was 1280 pixels × 720 pixels, centered on the sheet. Foragers were recruited to a whole fig placed at Edge B. Once 10 workers were feeding on the fig, we replaced the fruit with the artificial load. Ants were able to manipulate the load by gripping the excess 0.1 cm of foam around the perimeter of the dime. Ants carrying the load were filmed until they reached Edge A.

From the video recording of each experimental trial, we selected a segment of duration 145 s during which the ants were smoothly transporting the load; i.e., the load moved a nonzero distance during each consecutive 5-second interval of the segment. From this segment, we extracted the positions of the ants around the load and the position of the center of the load using ImageJ [12] and the Mtrack plugin [9]. This information was obtained from single frames at 5-second intervals.

We observed that during these segments, the ants moved the load along an approximately straight path (left column of Fig. 1), allowing us to model the load movement as one-dimensional. In addition, we found that the transport teams move the load at approximately the same speed across trials (right column of Fig. 1). Figure 2(a) shows an overhead snapshot of the ants and load during one trial. We observed that ants switched at random times between three behavioral states. The *Detached* state describes ants that are not attached to the load; two ants in Fig. 2(a) are in this state at the instant of the snapshot. To classify ants that were attached to the load, we divided the load in half by a line perpendicular to the direction of the load motion. Ants that gripped the half of the load in the direction of travel were labeled as being in state *Front*, and ants that gripped the other half were assigned state *Back*. In Fig. 2(a), three ants are in state *Front* and three are in state *Back*. From side-view videos of transport (Fig. 2(b)), we observed that ants on both sides lift the load off the ground in addition to exerting forces parallel to the surface that drive the load across the substrate.

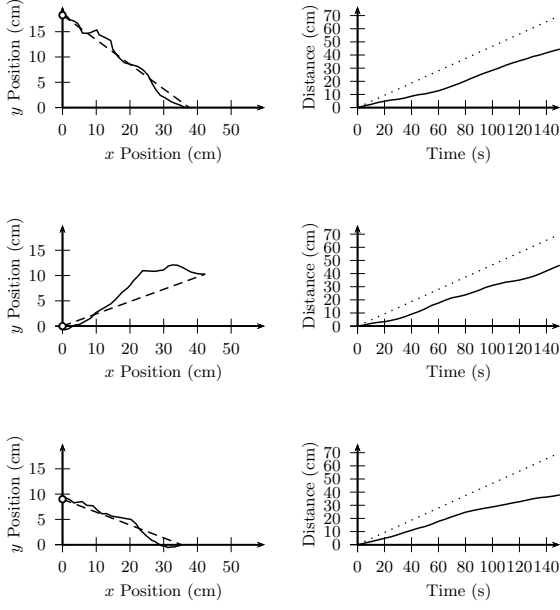


Figure 1: The x - y trajectory of the artificial load center (left column) and the distance traveled by the load over time (right column) for three experimental trials. The dashed lines in the left column show the ideal straight-line path. The dotted reference lines in the right column all have the same slope.

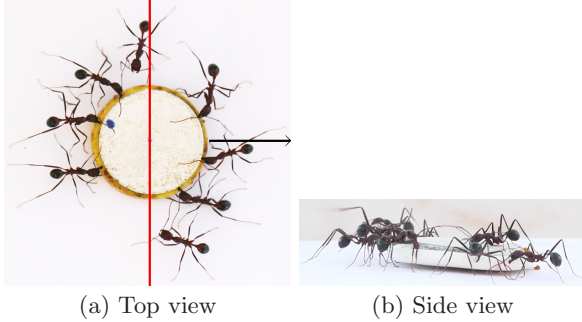


Figure 2: *Aphaenogaster cockerelli* ants transporting an artificial load: (a) top view, with the arrow indicating the direction of the load motion and the red line dividing the load into *front* (right) and *back* (left) halves; (b) side view. The views are from different trials.

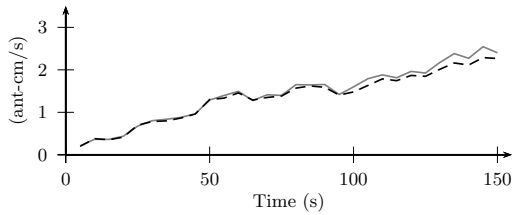


Figure 3: The solid line shows the sample mean of the product $N_F v_L$ from the data; the dashed line shows the product of the sample mean of N_F with the sample mean of v_L from the data.

The average pulling force of a single ant on an elastic load was previously measured by Berman et al. [1] to be 10.5 ± 5.0 mN, with 99.1% of the 10906 samples less than 30.0 mN. To further characterize the dynamics of the artificial load, we estimated the coefficient of kinetic friction μ of the load on the surface by measuring the angle θ_s at which the load started to slide down an inclined plane covered with the same paper used in the transport experiments. We measured $\theta_s = 30^\circ$, yielding the value $\mu = \tan(\theta_s) = 0.58$. Finally, the force F_l with which an individual ant lifts the load was estimated as $F_l = 2.653$ mN by averaging the peak lifting forces that several ants applied individually to a rigid plastic disk (0.5 mm thickness, 5 mm radius) glued to the pin of a 10 gram capacity load cell (Transducer Techniques GSO series).

3. COLLECTIVE TRANSPORT MODEL

In this section, we model collective transport as a stochastic hybrid system (SHS) [6]. This SHS is a cascade connection of a chemical reaction network representing stochastic ant behavioral transitions followed by the deterministic dynamics of a load transported along a surface with friction. In Section 4, we derive moment dynamics of the model that are used in Section 5 to fit model parameters to statistics from the experimental ant data.

Discrete Behavioral Modes.

We represent the stochastic switching of ants between behavioral states in the form of a set of chemical reactions. The species X_i denotes an ant in behavioral state $i \in \{F, B, D\}$, where i signifies the states *Front*, *Back*, and *Detached*, respectively. Each ant is assumed to switch from state i to state $j \neq i$ at a constant probability per unit time r_{ij} , which we call the *transition rate*. The six reactions representing these transitions take the form

$$X_i \xrightarrow{r_{ij}} X_j, \quad i, j \in \{F, B, D\}, \quad i \neq j. \quad (1)$$

We define $N_i(t)$ as the number of ants in state i at time t . The instantaneous probability rate of the reaction in Eq. (1) occurring within the group of ants is called the *transition intensity*, λ_{ij} [5]. Because all reactions are unimolecular, this quantity is given by $\lambda_{ij} = r_{ij} N_i$. Hence, although each transition rate is constant, the transition intensities vary with the number of ants in each state. Consequently, when the number of ants in a given state drops to zero, so does the probability per unit time of further reactions out of that state. We note that the total ant population, $N_F(t) + N_B(t) + N_D(t)$, is conserved at all times.

Load Dynamics.

As stated in Section 2, we model the load movement as one-dimensional. We specify that the load is initially located at the origin and then travels only in the positive direction along the x axis toward the nest. The load position and velocity at time t are denoted by $x_L(t)$ and $v_L(t)$, respectively. We assume that each ant in state *Front* walks backward in the positive x direction toward the nest and pulls on the load with force F_p . Because the ant teams in our experiments moved the load at an approximately constant velocity (see Fig. 1), we assume a proportional velocity regulation policy for F_p of the form

$$F_p = K(v_L^d - v_L(t)), \quad (2)$$

where the positive gain K and velocity set point v_L^d are parameters fit from the experimental data. Ants in state *Back* are assumed to only lift the load, and so the force applied by the ants to the load in the x direction at time t is $N_F(t)F_p$. We assume that ants in both states *Front* and *Back* lift the load with the upward force $F_l = 2.653 \text{ mN}$ from the measurements described in Section 2. The load is subject to a kinetic frictional force μF_n , where the normal force F_n on the load is determined by assuming that the load is in static equilibrium in the vertical direction. That is,

$$F_n = m_L g - (N_F(t) + N_B(t))F_l,$$

where g is the acceleration due to gravity. The net force F on the load in the x direction is then given by

$$F = N_F(t)F_p - \mu F_n.$$

Stochastic Hybrid System.

Our models of the ant behavioral dynamics and the load dynamics together constitute a polynomial stochastic hybrid system (pSHS) [4, 5]. The pSHS is characterized by the state vector $\mathbf{x} = [N_F \ N_B \ N_D \ x_L \ v_L]^\top$ with the continuous flow

$$\dot{\mathbf{x}} = [0 \ 0 \ 0 \ v_L \ F/m_L]^\top \quad (3)$$

and a set of discrete *reset maps* $\phi_{ij}(\mathbf{x})$, each representing the stochastic transition $(N_i, N_j) \mapsto (N_i - 1, N_j + 1)$ corresponding to the reaction in Eq. (1) that occurs with intensity λ_{ij} . The fact that the λ_{ij} , $\phi_{ij}(\mathbf{x})$, and components of $\dot{\mathbf{x}}$ are all finite polynomial functions of the continuous state variables establishes our SHS model as a pSHS [5]. The stochastic variation in $N_F(t)$, $N_B(t)$, and $N_D(t)$ over time causes $x_L(t)$ and $v_L(t)$ to vary randomly over time as well.

4. MOMENT DYNAMICS OF THE MODEL

The *extended generator* L of an SHS can be used to predict the time evolution of the statistical moments of the SHS continuous state [4, 5]. For any function $\psi(\mathbf{x}) : \mathbb{R}^n \rightarrow \mathbb{R}$ that is continuously differentiable, the dynamics of the expected value of ψ are given by $dE(\psi)/dt = E(L\psi)$. In our case, L is defined as

$$L\psi(\mathbf{x}) \triangleq \frac{\partial \psi}{\partial x_L} \dot{x}_L + \frac{\partial \psi}{\partial v_L} \dot{v}_L + \sum_{\substack{i,j \in \{F,B,D\} \\ i \neq j}} (\psi(\phi_{ij}(\mathbf{x})) - \psi(\mathbf{x})) r_{ij} N_i.$$

Hence, by setting $\psi = N_i$, we can derive the dynamics of the mean number of ants in state i , $E(N_i)$, as

$$\frac{d}{dt} E(N_i) = \sum_{\substack{j \in \{F,B,D\} \\ j \neq i}} (r_{ji} E(N_j) - r_{ij} E(N_i)). \quad (4a)$$

Likewise, using $\psi = x_L$, the mean load position is such that

$$\frac{d}{dt} E(x_L) = E(v_L), \quad (4b)$$

and, using $\psi = v_L$ and the approximation that the instantaneous N_F and v_L are uncorrelated (see Fig. 3), the mean load velocity is such that

$$\frac{d}{dt} E(v_L) = c_g + c_F E(N_F) + c_B E(N_B) + c_{Fv} E(N_F) E(v_L) \quad (4c)$$

where $c_g \triangleq -\mu g$, $c_F \triangleq (K v_L^d + \mu F_l)/m_L$, $c_B \triangleq \mu F_l/m_L$, and $c_{Fv} \triangleq K/m_L$.

5. PARAMETER ESTIMATION RESULTS

In this section, we discuss the fitting procedure used to estimate the pSHS model parameters: the transition rates r_{ij} of the reactions in Eq. (1) and the gain K and velocity set point v_L^d in Eq. (2). We compute parameters that best fit the sample means of the experimental data to the first-order moment dynamics derived in Section 4.

As described in Section 2, the experimental data consist of the load position and velocity and the counts of ants in each behavioral state sampled at 5 s intervals during 17 trials. The number of *Detached* ants at each sampling time was generated by subtracting the number of attached ants during that sampling period from the maximum number of attached ants across all periods. Thus, the total number of ants is constant within each trial. The load velocity was estimated numerically from the position data. For the purpose of fitting model parameters to the data, we computed the across-trial means of load position and velocity as well as the across-trial mean numbers of *Front*, *Back*, and *Detached* ants at each sampling time. Weighted-least-squares (WLS) optimization was then used to fit the theoretical moment dynamics Eq. (4) at these times to the corresponding empirical mean values. In particular, the numerical optimizer minimized the sum of weighted squared errors between each theoretical and empirical mean, where each weight was the sample variance of the corresponding data set. The numerical operation was performed in MATLAB using the `fmincon` tool for active-set optimization; the theoretical mean-field trajectories were integrated using `ode15s` with initial conditions chosen to match the initial means from the data.

The best-WLS-fit parameters were the transition rates

$$\begin{aligned} r_{DB} &= 0.0197 \text{ s}^{-1}, & r_{BD} &= 0.0205 \text{ s}^{-1}, \\ r_{DF} &= 0, & r_{FD} &= 0, \\ r_{BF} &= 0.0301 \text{ s}^{-1}, & r_{FB} &= 0.0184 \text{ s}^{-1}, \end{aligned}$$

and the gain and velocity set point parameters

$$K = 0.0035 \text{ N/(cm/s)}, \quad v_L^d = 0.3185 \text{ cm/s}.$$

Based on the rates that best fit the model to the data, each ant in the transport team appears much more likely to attach to the back of the moving load than the front (i.e., $r_{DB} > r_{DF} = 0$). This hypothesis can be tested in future experiments and in a more detailed analysis of the video data. If it is supported, we can speculate as to reasons for the trend. For example, during movement, the front of the load may have near-maximal occupancy, hindering new attachment to the front even by detached ants downstream of the load motion. Alternatively, it may be easier to attach to the back of the moving object because that end is vertically stationary and moving horizontally in the same direction as a forward-walking ant approaching it. When an *A. cockerelli* transports a small object individually, she lifts the seed off the ground and carries it forward toward her nest. Hence, it is possible that backward carrying in teams results from the inability to carry the load forward in the normal posture. Moreover, the model infers a relatively high *Back-to-Front* transition rate r_{BF} , which is supported by the observation that backward walking is a contingency when loads are difficult to move. Even if these speculations are not biologically accurate, they may assist in the design of grasping mechanisms and approach postures for robots performing collective transport.

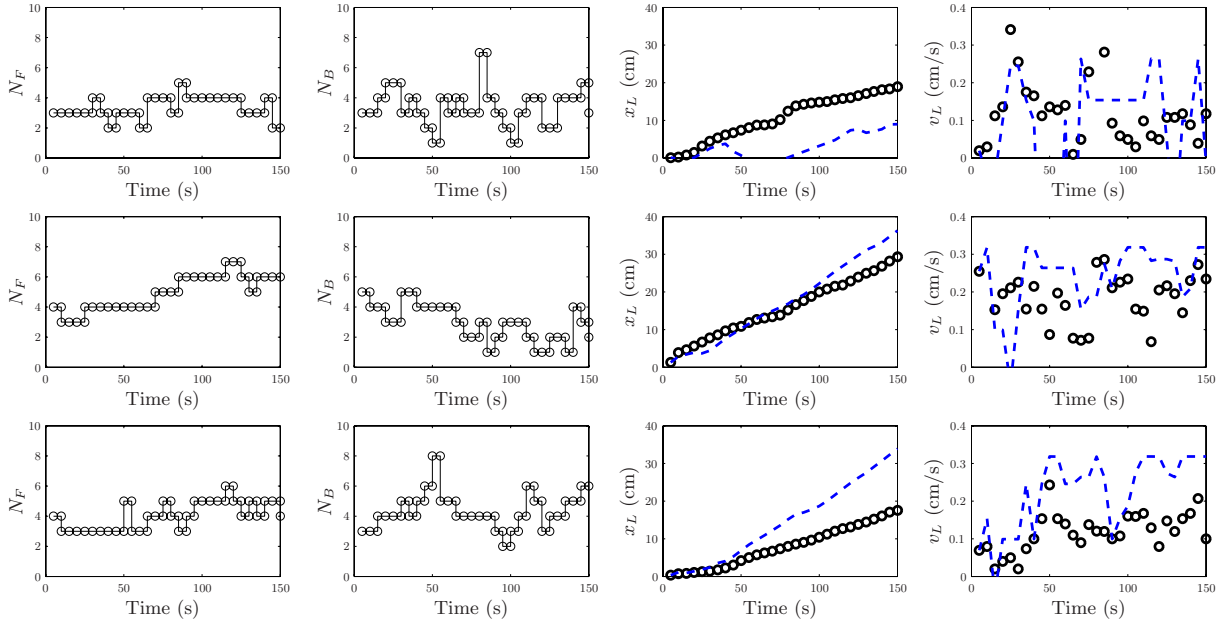


Figure 4: Observed numbers of ants in states *Front* (first column) and *Back* (second column) over time and observed (circles) and predicted (dashes) load position (third column) and velocity (fourth column) over time for three selected experimental trials (one per row).

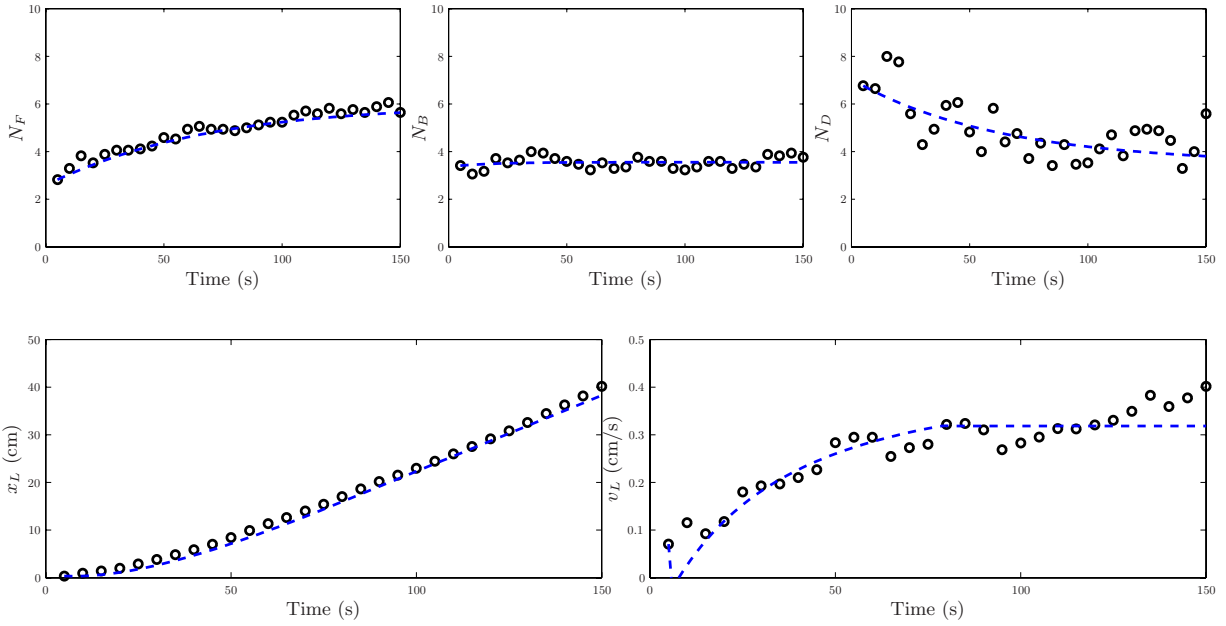


Figure 5: Observed (circles) and predicted (dashes) time evolution of the mean numbers of ants in each state (top row) and the mean load position and velocity (bottom row).

Figures 4 and 5 compare the experimental data with the model predictions. Figure 4 shows this comparison for three selected experimental trials: the first two columns show the measured numbers of *Front* and *Back* ants, and the third and fourth columns show the measured load position and velocity and their predictions based on the measured ant counts and the deterministic feedback policy from Eq. (2) instantiated with the WLS-fit K and v_L^d parameters in the load dynamics Eq. (4b), Eq. (4c). In Fig. 5, the first row displays the mean numbers of ants in each state measured across trials against the predicted mean-field trajectories from Eq. (4a) with the WLS-fit transition rates r_{ij} , and the second row shows the analogous plots for load position and velocity. The parameter v_L^d may be viewed as a desired traveling speed of the ant transport teams that can only be achieved in the absence of friction. That is, some amount of steady-state velocity error is required to balance the frictional force. Additional experiments can further validate the model for a higher load mass and coefficient of friction. The model should then predict a shift in the steady-state load velocity.

6. CONCLUSION

In this work, we conducted an experimental investigation of collective transport of a rigid artificial load by *Aphaenogaster cockerelli* ants. A stochastic hybrid system (SHS) model was developed to describe the dynamics of the load and the behavioral transitions of the ants during transport. The model was fit to the data to minimize the difference in mean behavior, and the resulting best-fit parameters are presented as reduced-order metrics of collective transport.

In future work, we plan to further validate our model by fitting both first-order and second-order moments to statistics from experimental data. We will also investigate how the best-fit parameters vary from optimized parameters that minimize criteria including path variance, load transport time, and transport team size.

We also plan to expand the model to incorporate other features of collective transport by ants. Additional behavioral states as well as heterogeneity and stochasticity within states will be included. The modeled behaviors will include teams of individuals pulling and lifting with different time-varying forces. An important future direction is to adjust the transition rates so that they depend on factors such as the load position, load velocity, ant force applied to the load, and the number of ants attached. Especially in the uncoordinated phase before smooth transport, it is likely that the probability of an ant attaching to the front or back of the load is strongly determined by the load's nascent motion as well as the number of ants gathered around it. State-dependent transition rates can capture this initial behavior and still allow for the smooth motion that is the focus of this paper. As the uncoordinated phase has less directionality than the smooth phase discussed here, it will require augmenting the model for two-dimensional load motion.

In general, we hope to catalyze bio-inspired research on multi-robot transport teams. SHS frameworks are utilized in robotics, and so they have potential to be substrates for trans-disciplinary knowledge transfer.

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