

# Dynamic Redistribution of a Swarm of Robots Among Multiple Sites

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**Abstract**—We present an approach for the dynamic assignment and reassignment of a large team of homogeneous robotic agents to multiple locations with applications to search and rescue, reconnaissance and exploration missions. Our work is inspired by experimental studies of ant house hunting and empirical models that predict the behavior of the colony that is faced with a choice between multiple candidate nests. We design stochastic control policies that enable the team of agents to distribute themselves between multiple candidate sites in a specified ratio. Additionally, we present an extension to our model to enable fast convergence via switching behaviors based on quorum sensing. The stability and convergence properties of these control policies are analyzed and simulation results are presented.

## I. INTRODUCTION

We are interested in deploying swarms of homogeneous robots to distinct locations for simultaneous execution of tasks at each locale. This is relevant in applications such as surveillance of multiple buildings, large scale environmental monitoring or providing aerial coverage for various ground units. In these applications, robots must have the ability to distribute themselves among many locations/sites and have the ability to autonomously redistribute to ensure task completion at each site which may be affected by robot failures or changes in the environment. In addition, there are situations when robots may not be able to easily communicate across sites, *e.g.* mining at multiple sites, and thus it makes sense to develop a strategy that can achieve (re)distribution of the team with little to no communication.

This work draws inspiration from the process through which an ant colony selects a new home from several sites using simple behaviors that arise from local sensing and physical contact [1]. Rather than choosing a “new home”, we propose a strategy for the deployment of a swarm of homogeneous robots such that the team collectively distributes itself to multiple sites in predefined proportions without the use of inter-agent communication. This is similar to the task/resource allocation problem where the objective is to determine the optimal assignment of robots to tasks. As such, the proposed strategy can be readily applied to the problem of dynamic (re)assignment of a large team of homogeneous robotic agents to multiple tasks.

Previous works that considered the assignment of multiple robots to one task include [2]–[4]. In particular, market-based approaches have gained much success in various multi-robot applications like robot soccer and treasure hunting [5]–

[7]. However, these existing methods do not address the controller synthesis problem and often suffer in terms of scalability with respect to the number of tasks and robots. As such performance guarantees are often sacrificed to reduce the computation and communication requirements [8], [9].

Other task allocation strategies include [10] where the problem is formulated as a Distributed Constraint Satisfaction Problem. This approach, however, requires the explicit modeling of tasks, their requirements, and robot capabilities making implementation to large populations difficult. In [11], large robot populations are modeled using a partial differential equation. Then a centralized optimal control strategy is used to maximize robot occupation at a desired position. In [12], an adaptive multi-foraging task with no explicit communication or global knowledge is modeled as a stochastic process. While the model was verified through simulations, the only way to control robot task reallocation is to modify the task distribution in the environment.

Similar to [12], [13], our proposed strategy employs a multi-level representation of swarm activity. Rather than the bottom-up analysis procedure, our methodology is based on a top-down design approach. We build on [14], [15] and extend our synthesis procedure to the problem of deployment to multiple sites and consider the allocation of hundreds and thousands of robots. Our proposed approach is computationally inexpensive and scalable, and can be easily modified to simultaneously solve the controller synthesis problem. Furthermore, we are able to control the final allocation among the various sites by translating the global specification into agent closed-loop control laws that produce convergence to the desired allocation. Lastly, our approach enables the system to respond to robot failures in a natural way thus ensuring graceful degradation.

This paper is organized as follows: Section II formulates the problem and outlines the system model. Section III shows the stability and convergence properties of our system. Section IV presents our simulation results. Section V provides an extension of our methodology to include a quorum sensing mechanism to speed up convergence. Finally, we conclude with a discussion of directions for future work in Section VI.

## II. PROBLEM FORMULATION

### A. Definitions

Consider  $N$  agents to be distributed among  $M$  sites. We denote the number of agents at site  $i \in \{1, \dots, M\}$  at time  $t$  by  $n_i(t)$  and the desired number of agents at site  $i$  by  $\bar{n}_i$ . We define the population fraction at each site at time  $t$  as  $x_i(t)$  where  $x_i(t) = n_i(t) / \sum_i \bar{n}_i$ . Then the system state vector is given by  $\mathbf{x} = [x_1, \dots, x_M]^T$ . For some initial distribution of

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the agents given by  $\{n_i\}^0, i = 1, \dots, M$ , we denote the target configuration as a set of population fractions to occupy each site given by

$$\bar{x}_i = \frac{\bar{n}_i}{\sum_{i=1}^M \bar{n}_i} \quad \forall i = 1, \dots, M. \quad (1)$$

A specification in terms of fractions rather than absolute agent numbers is practical for scaling as well as for potential applications where losses of agents to attrition and breakdown are common.

We model the interconnection topology of the  $M$  sites via a directed graph,  $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ , where the set of vertices,  $\mathcal{V}$ , represents sites  $\{1, \dots, M\}$  and the set of edges,  $\mathcal{E}$ , represents physical routes between sites. We say two sites  $i, j \in \{1, \dots, M\}$  are *adjacent*,  $i \sim j$ , if a route exists for agents to travel from  $i$  to  $j$ , and we represent this relation by the ordered pair,  $(i, j)$ , such that  $(i, j) \in \mathcal{V} \times \mathcal{V}$ , with the set  $\mathcal{E} = \{(i, j) \in \mathcal{V} \times \mathcal{V} | i \sim j\}$ . We assume the graph  $\mathcal{G}$  is a strongly connected graph, i.e. a path exists for any pair of vertices  $(u, v) \in \mathcal{V}$ . Furthermore, we assign every edge in  $\mathcal{E}$  constant *transition rates*,  $k_{ij} > 0$ , where  $k_{ij}$  defines the transition probability per unit time for one agent from site  $i$  to go to site  $j$ . Here  $k_{ij}$  is essentially a stochastic transition rule and in general  $k_{ij} \neq k_{ji}$ . In addition, we will define the flux from site  $i$  to site  $j$ , denoted by  $\phi_{ij}$ , as the fraction of agents per unit time going from  $i$  to  $j$  and denote the time required to travel from site  $i$  to site  $j$  as  $\tau_{ij}$ .

We assume every agent has complete knowledge of  $\mathcal{G}$  as well as all the transition rates  $k_{ij}$ . Furthermore, we assume that each edge in  $\mathcal{E}$  is associated with a maximum capacity, which limits the number of agents that can travel simultaneously along the edge. This limitation is enforced by a maximum rate (agents per unit time) that may initiate a transition from site  $i$  to  $j$  and vice versa.

### B. Problem Statement

Consider the system of  $N$  homogeneous agents and  $M$  sites. The task is to redeploy the swarm of  $N$  robots to the desired configuration given by the set  $\{\bar{x}_i\}_{i=1, \dots, M}$ , starting from the initial distribution  $\{x_i\}_{i=1, \dots, M}^0$  as fast as possible using no communication. Our proposed strategy achieves this by endowing each agent with a small set of instructions based on the transition rates  $k_{ij}$ . Furthermore, rather than model the system as a collection of individual agents, we use differential equations to model the swarm as a continuum. These deterministic models are used as *abstractions* to the stochastic formulation of the system, which has a more legitimate physical basis than the deterministic formulation [16]. We refer the interested reader to [17] for the theoretical justification for this abstraction.

In the limit of large  $N$ , the time evolution of the population fraction at site  $i$  is given by a linear law:

$$\frac{dx_i(t)}{dt} = \sum_{\forall j|(i,j) \in \mathcal{E}} k_{ji}x_j(t) - \sum_{\forall j|(i,j) \in \mathcal{E}} k_{ij}x_i(t). \quad (2)$$

Then the system of equations for the  $M$  sites is given by

$$\frac{d\mathbf{x}}{dt} = \mathbf{K}\mathbf{x} \quad (3)$$

with  $\mathbf{K}_{ij} = k_{ji}$  for  $i \neq j$  and  $\mathbf{K}_{ii} = -\sum_{j=1, \mathbf{A}_{ij} \neq 0}^M k_{ij}$ . We note that the columns of  $\mathbf{K}$  sum to 0 and since the number of agents is conserved, the system is subject to the following conservation constraint

$$\sum_{i=1}^M x_i(t) = 1. \quad (4)$$

In this model, agents will move between sites even at equilibrium, i.e. when the net flux for each site is zero. This is due to our model which forces a trade-off between maximizing the link capacities for fast equilibration and achieving long-term efficiency during equilibrium. Lastly, equation (2) can be equivalently written as

$$\frac{dx_i(t)}{dt} = \sum_{\forall j|(i,j) \in \mathcal{E}} \phi_{ji}(t) - \sum_{\forall j|(i,j) \in \mathcal{E}} \phi_{ij}(t).$$

The above model assumes agents instantaneously switch from one site to another and does not take into consideration the time needed to travel between sites. In reality, it takes finite time  $\tau_{ij}$  to travel from site  $i$  to site  $j$ . We note that the loss of agents at a site due to transfers to other sites is immediate, while the gain due to incoming agents from other sites is delayed. The effect of time travel can be included by converting (2) into a delay differential equation given by

$$\frac{dx_i(t)}{dt} = \sum_{\forall j|(i,j) \in \mathcal{E}} k_{ji}x_j(t - \tau_{ji}) - \sum_{\forall j|(i,j) \in \mathcal{E}} k_{ij}x_i(t). \quad (5)$$

Similar to (2), at equilibrium, agents will also move between sites in this model. However, because of the time delays, at equilibrium, the system will always result in a finite number of agents *en route* between sites. Furthermore, the fraction of agents en route versus the fraction at sites increases as the time delays increase. Thus, the conservation equation for this system is given by

$$\sum_{i=1}^M n_i(t) + \sum_{i=1}^M \sum_{\forall j|(i,j) \in \mathcal{E}} k_{ij}\tau_{ij}n_i(t) = N, \quad (6)$$

where the first term gives the number of agents at sites and the second term gives the number of agents on the roads. Thus, in this model  $\sum_{i=1}^M \bar{n}_i < N$ . Lastly, equation (5) can be equivalently written as

$$\frac{dx_i(t)}{dt} = \sum_{\forall j|(i,j) \in \mathcal{E}} \phi_{ji}(t - \tau_{ji}) - \sum_{\forall j|(i,j) \in \mathcal{E}} \phi_{ij}(t).$$

### III. ANALYSIS

In this section we consider the uniqueness and stability properties of the equilibrium points for the systems given by (3) subject to (4) and (5) subject to (6). We state our first theorem and a brief proof for the sake of completeness and refer the interested reader to [18]–[20] for more detailed exposition on the topic.

*Theorem 1:* For a strongly connected graph  $\mathcal{G}$ , the system (3) subject to (4) has a unique stable equilibrium point.

*Proof:* Since the columns of the matrix  $\mathbf{K}$  sum to 0, the rank of  $\mathbf{K}$  is  $(M - 1)$ . Furthermore, the vector  $\mathbf{1}$  exists in the

nullspace of  $\mathbf{K}^T$ . Thus, the system  $\mathbf{K}\mathbf{x} = \mathbf{0}$  subject to (4) has a unique equilibrium point. To show it is stable, we recall that a  $m_t \times m_t$  Markov matrix,  $\mathbf{T}$ , has the following properties:  $\mathbf{T}_{ij} > 0$  for  $1 \leq i, j \leq m_t$ ; and  $\sum_{j=1}^{m_t} \mathbf{T}_{ij} = 1$ .

Then, from the Perron-Frobenius theorem, one can show that every eigenvalue,  $\lambda$ , of a Markov matrix satisfies  $|\lambda| \leq 1$ . Consider the matrix given by  $\mathbf{S} = (1/s)(s\mathbf{I} + \mathbf{K}^T)$ , where  $s > 0$  and  $\mathbf{I}$  is the  $M \times M$  identity matrix. We note that for  $s$  large enough,  $\mathbf{S}$  is a Markov matrix with nonnegative entries and the rows sum to 1. Let  $\mathbf{J}$  denote the matrix of eigenvectors of  $\mathbf{K}^T$  such that  $\mathbf{K}^T = \mathbf{J}\mathbf{\Lambda}\mathbf{J}^{-1}$ , where  $\mathbf{\Lambda}$  is the diagonal matrix with the eigenvalues of  $\mathbf{K}^T$  on the diagonal. Then the following holds

$$\begin{aligned} \mathbf{S} &= \frac{1}{s}(s\mathbf{I} + \mathbf{J}\mathbf{\Lambda}\mathbf{J}^{-1}) \\ &= \mathbf{J}\left(\frac{1}{s}\right)(s\mathbf{I} + \mathbf{\Lambda})\mathbf{J}^{-1}. \end{aligned}$$

Thus, the eigenvalues of  $\mathbf{S}$  are given by  $1 + \lambda(\mathbf{K}^T)_i/s$  for  $i = 1, \dots, M$ . Since  $|\lambda(\mathbf{S})_i| \leq 1$  for all  $i$ , we can conclude that  $\lambda(\mathbf{K}^T)_i \leq 0$  for all  $i$ . Furthermore, since the eigenvalues of  $\mathbf{K}^T$  are equivalent to the eigenvalues of  $\mathbf{K}$ ,  $\mathbf{K}$  is negative semi-definite with at least one zero eigenvalue and so the system is stable. By design the system converges to the desired occupancy fractions given by (1). ■

Next, we consider the uniqueness and stability properties of the equilibrium point for the time-delayed system (5) subject to (6).

**Theorem 2:** Given a strongly connected graph  $\mathcal{G}$  and the set  $\{k_{ij}\}$ , the system with time delays  $\tau_{ij}$  for every  $(i, j) \in \mathcal{E}$  subject to (6) has a unique and stable equilibrium at  $\mathbf{x} = [\bar{x}_1, \dots, \bar{x}_M]^T$  given by the specification (1).

*Proof:* We first note the time-delayed model (5) is in fact an abstraction of a more realistic model in which the delays are random variables with a certain distribution. Thus, we can convert the time-delayed model into the linear model with no time delays by representing each edge with a finite sequence of "dummy sites". If we let  $y_{ij}(t)$  denote the fraction of the population en route between sites  $i$  and  $j$  and the set of dummy sites between  $i$  and  $j$  as  $\{y_{ij}^1, \dots, y_{ij}^p\}$ . For finite  $p$ , the transition rates between dummy sites  $y_{ij}^l$  and  $y_{ij}^{l+1}$  with  $l = 1, \dots, p$  are given by  $p/\tau_{ij}$ . Since  $\mathcal{G}$  is a strongly connected graph, the graph resulting from the addition of the dummy sites is also a strongly connected graph. Furthermore, the expanded system can also be expressed in the form of (2). Thus, by Theorem 1, this system is stable and has a unique equilibrium given by (1). ■

#### IV. SIMULATIONS

We consider  $N$  agents moving in the plane. While our focus is on the global design and properties of the swarm, our methodology takes into account the exact number of agents assigned to each site as well as the travel initiation and termination times for each individual traveler. When considering the continuous population model (2), transitions between sites occur spontaneously based on the probability

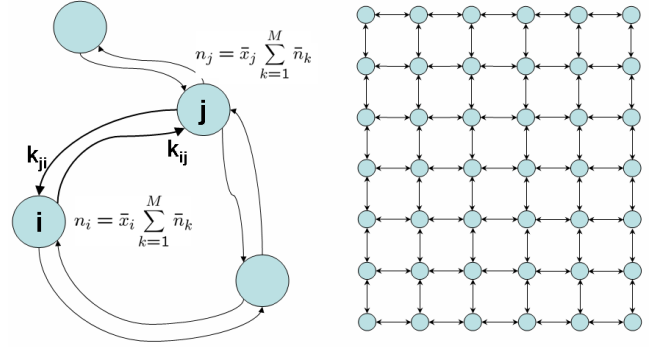


Fig. 1. Left: Schematic of the main features of the network of sites. All links are bi-directional, in that they allow the movement of agents in both directions. The rate of transitions in each direction along links is set by predetermined transition coefficients ( $k_{ij}$ ). Right: the network of 42 sites used in our simulations.

per unit time of transition to any adjacent site. For the model given by (5), agents transitioning between sites leave the origin site at the moment of the transition and spend the delay time associated with the respective transition traveling between sites. Thus, at any given time some of the agents will be *travelers*, assigned to the directed edge corresponding to their transition rather than to any site. Our simulation methodology is based on the abstraction procedure in [17] which is a centralized approach that can be converted to an equivalent decentralized approach. We refer the interested reader to [17] for further details.

#### A. Results

Our first simulation models the system using (5) with 10,000 agents and 42. Agents are initially scattered at sites configured to form the letter "T" and the task is to redistribute the team to another set of sites such that these sites form the letter "C". The interconnection topology of the sites is shown in Figure 1. Each edge represents bi-directional connections between sites. We distinguish between agents located at sites, shown in red and green, and traveling agents, shown in light blue. Agents at sites above the desired fraction are shown in red, agents at sites below the desired fraction are shown in green, and travelers are shown in light blue. Snapshots for this simulation are shown in Figure 2. The snapshots in the figure are in chronological order, from 10, 100, 200, 250, 500, 750, 1000, 3800 and 9750 minutes into the simulation.

In Figure 3, we provide a comparison between the continuous and stochastic simulations of the linear model (2). We plot the occupancy ratio (number of agents at a site over design occupancy) for a single site during a redistribution process for stochastic simulations of 1000 and 10000 agents and an ODE simulation. The site is originally empty and we see that the occupancy for the stochastic simulations fluctuates around the continuum value. Another interesting comparison is the difference between the desired and current configurations which can be seen in Figure 4. Here we show the time evolution of the fraction of misplaced agents, i.e. the sum of the absolute values of the differences between

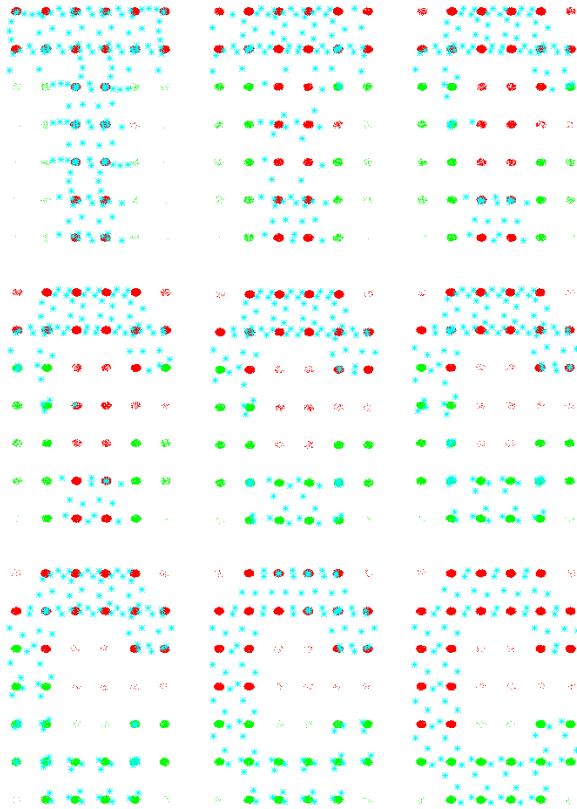


Fig. 2. Snapshots from a simulation using 10,000 agents in the 42-dot display network. The initial configuration forms the letter “T” with the design specification for a letter “C”. Agents at sites above design specs are shown in red, agents at sites below design specs are shown in green, and travelers in light blue. The system is driven by the model given in (5).

the actual and desired number of agents at each site over the total, for the same scenarios considered in Figure 3.

### B. Discussion

To accomplish the multi-site deployment task, we allow individual agents to transition between available sites at random times, with transition rates that are pre-determined and known by all agents. However, the inefficiency of this simple solution quickly becomes evident if the cost of time and energy involved is considered. The fast transition rates that ensure quick switching from a remote configuration may lead to many idle trips once the design configuration is achieved. Conversely, low transition rates appropriate to maintaining the configuration under mild perturbations imply long switching times.

This phenomenon is due to the fact that agents do not interact with one another in these models since each agent transitions independently of the others. If one puts a high premium on the lack of interactions, this may be the appropriate solution for the deployment task. However, the linear non-interacting model given by (2) cannot modulate transfer rates between sites. In situations when there is a high disturbance, or when the initial condition differs greatly from the design equilibrium, it makes sense to have the agents travel as close as possible to the maximum rate associated with each edge

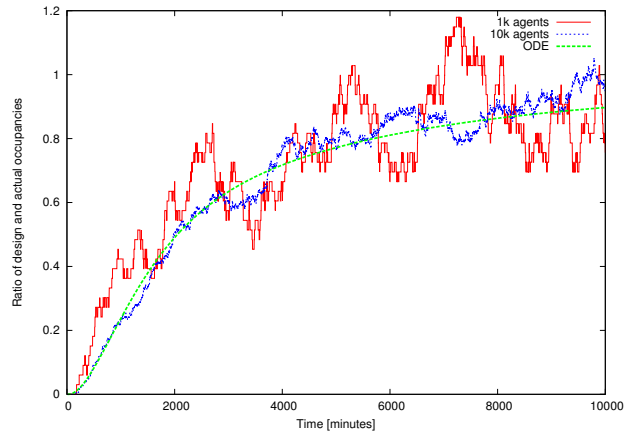


Fig. 3. Occupancy fractions at one site for different total agent numbers compared to the ODE model.

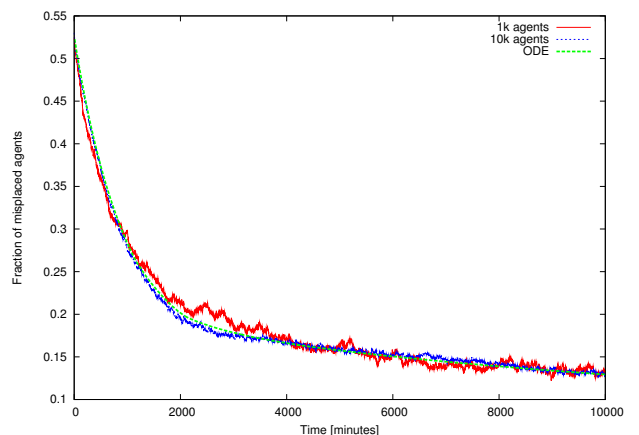


Fig. 4. Total number of misplaced agents for different  $N$  agent numbers compared to the ODE model.

to enable fast convergence to the desired state. However, in actual robotic systems, the extraneous traffic resulting from the movement between sites at equilibrium may come at a significant cost. The cost of breaking out of this dilemma is to provide the agents with a rough notion of the overall state of the system. In the following section we present an extension of our system model (2) that incorporates such elements in the form of switching behaviors based on quorum sensing.

## V. EXTENSIONS

In this section we propose an extension to (2) to enable fast convergence to the desired state using switching behaviors based on quorum sensing. The *quorum* is defined as a threshold occupancy. A site that is below quorum draws agents from adjacent sites that are above quorum at a fixed maximal rate. Similar to before, we will assume that agents have a map of the environment and know the interconnection graph,  $\mathcal{G}$ , as well as the transition rate matrix,  $\mathbf{K}$ . We will also assume quorum information is instantly available which can be achieved by providing sites with some mechanism to communicate with adjacent sites.

Each site  $i$  is then characterized by a quorum,  $q_i$ , a minimal number of agents, which we specify as a fraction of the design occupancy  $\bar{x}_i$ . If site  $i$  is above quorum, the transition rate from  $i$  to a site  $j$  that is below quorum is automatically set to the maximum transition rate,  $\phi_{ij}^{max}$ . We refer to such an edge as *activated*. This is maintained until the imbalance is resolved, either by the underpopulated site exceeding quorum or by the site above quorum becoming underpopulated. In this model, we will further require the following balancing condition:

$$\bar{x}_i k_{ij} = \bar{x}_j k_{ji} \quad (\forall) i \neq j. \quad (7)$$

For simplicity, we assume that  $\phi_{ij}^{max} = \phi_{max}$  for all  $i, j$ . The differential equation model with quorum is

$$\frac{dx_i(t)}{dt} = \sum_{\forall j, (i,j) \in \mathcal{E}} \phi_{ji}(t) - \sum_{\forall j, (i,j) \in \mathcal{E}} \phi_{ij}(t), \quad (8)$$

where the rates  $\phi_{ij}$  are given by:

$$\phi_{ji}(t) = k_{ji}x_j(t) + \sigma_{ji}(\phi_{max} - k_{ji}x_j(t)) \quad (9)$$

where  $\sigma_{ji} \in [0, 1]$  is the analytic switching function given

$$\sigma_{ji} = \left(1 + e^{-\gamma_1\left(\frac{x_i}{\bar{x}_i} - q_i\right)}\right)^{-1} \left(1 + e^{\gamma_2\left(\frac{x_j}{\bar{x}_j} - q_j\right)}\right)^{-1}. \quad (10)$$

We note  $\sigma_{ji} \rightarrow 1$  as  $x_i/\bar{x}_i \rightarrow 0$  and  $x_j/\bar{x}_j \rightarrow +\infty$  with  $\sigma_{ji} \rightarrow 0$  otherwise. Furthermore,  $\gamma_1, \gamma_2 > 0$  and chosen such that  $\sigma_{ji} = 1$  when  $x_i + \epsilon = q_i$ ,  $x_j = q_j + \epsilon$  where  $\epsilon > 0$  is small.

We can show that the system described by (8) assuming  $q_i = q$  for all  $i$  has a stable equilibrium point that satisfies the desired specifications (1). Consider the function given by

$$V = \sum_{i=1}^M \frac{x_i^2}{2\bar{x}_i}. \quad (11)$$

**Theorem 3:** The system defined by equations (8) for  $i = 1, \dots, M$  such that  $(i, j), (j, i) \in \mathcal{E}$  with condition (7) and conservation constraint (4), converges asymptotically to  $\mathbf{x} = [\bar{x}_1, \dots, \bar{x}_M]^T$ , defined by the specification (1).

*Proof:* We begin by showing the system is stable. First, note that  $V$  is a radially unbounded function of  $\|\mathbf{x}\|$ . We define the net influx from site  $i$  to site  $j$  as  $\Phi_{ij} = -\phi_{ij} + \phi_{ji}$ . We note that  $\Phi_{ij} = -\Phi_{ji}$  and by design  $\Phi_{ij} = -\phi_{max} + k_{ji}x_j < 0$  if and only if  $x_i/\bar{x}_i > x_j/\bar{x}_j$ . Consider the time derivative of the Lyapunov function given by (11)

$$\begin{aligned} \frac{dV}{dt} &= \sum_{i=1}^M \frac{x_i}{\bar{x}_i} \frac{dx_i}{dt} \\ &= \sum_{i=1}^M \frac{x_i}{\bar{x}_i} \left( \sum_{\forall j, (i,j) \in \mathcal{E}} \phi_{ji}(t) - \sum_{\forall j, (i,j) \in \mathcal{E}} \phi_{ij}(t) \right) \\ &= \sum_{\forall j, (i,j) \in \mathcal{E}} \left( \frac{x_i}{\bar{x}_i} - \frac{x_j}{\bar{x}_j} \right) \Phi_{ij} < 0 \end{aligned}$$

By design, if  $x_i/\bar{x}_i > x_j/\bar{x}_j$  then  $\Phi_{ij} < 0$  and if  $x_i/\bar{x}_i < x_j/\bar{x}_j$  then  $\Phi_{ij} > 0$ . Thus, the time derivative of the

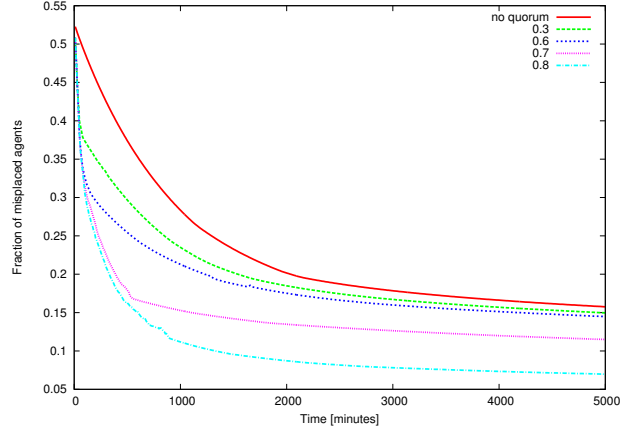


Fig. 5. Global 'distance' from the desired configuration as measured by the total number of misplaced agents for different quorum values in the ODE model, compared to the quorum-less model.

Lyapunov function is always negative and so system is stable. To show that the equilibrium point is given by (1), consider the set of equilibrium states  $\mathbf{x}_e$  satisfying (4), such that  $\frac{dV}{dt} = 0$ . The time derivative of the Lyapunov function evaluates to zero when all  $\Phi_{ij} = 0$  or when  $x_i/\bar{x}_i = x_j/\bar{x}_j$  for all  $(i, j) \in \mathcal{E}$ . By (7),  $\Phi_{ij} = 0$  if and only if  $x_i/\bar{x}_i = x_j/\bar{x}_j$ . Since  $\left(\frac{x_i}{\bar{x}_i} - \frac{x_j}{\bar{x}_j}\right) \Phi_{ij} < 0$  for all  $i, j$  where  $x_i/\bar{x}_i \neq x_j/\bar{x}_j$ , the only stable equilibrium is when (1) is satisfied. Thus, the system converges asymptotically to (1). ■

To show the effects of adding quorum dependent edge activation to our linear model (2), we performed simulations with different quorum values and compared them with the quorum-less linear model, i.e. quorum is zero. We show the number of misplaced agents over time in Figure 5. This figure suggests that increasing the quorum speeds up convergence. However, when combined with time delays, it may be possible for the system to get "stuck" for lengthy periods of time in states characterized by high transition rates (driven by below-quorum sites) with a very high fraction of travelling agents. In such cases, choosing a lower quorum, with all else being equal, may help avoid these configurations.

Lastly, we present simulation results of a redistribution of 200 agents among 3 sites, taking into account travel times between sites in an urban environment. Initially, the team is equally distributed between two sites in the urban environment shown in Figure 6. The task was to redeploy the team such that they distribute between the sites 1, 2, and 3 following the ratio 2 : 2 : 1. Initially sites 1 and 2 are above quorum. When the agents observe that site 3 is below quorum, the agents at sites 1 and 2 transfer out of these sites at a higher rate. Individual agent transitions are determined stochastically based on predetermined transition rates. Here, inter-site navigation is achieved via potential functions.

## VI. CONCLUSION

We have presented an approach to redistribute a swarm of robots among a set of available sites. Our methodology models the swarm as a continuum via a system of deterministic





Fig. 6. Snapshots from a simulation redeploying 200 agents, initially located at two sites, to three sites in an urban environment. Agents leaving sites are denoted by square markers and agents entering sites are denoted by diamond-shaped markers. Agents located at sites above quorum are shown in red while agents located at sites below quorum are shown in blue. The system is driven by the model given in ((8)) and taking travel times between sites into account.

linear ordinary differential equations in terms of population fractions that enables the synthesis of decentralized controllers with no communication. We then extended our linear models to include a quorum dependent redistribution mechanism to speed up convergence. This second approach required the communication of quorum status between adjacent sites. Uniqueness and stability of solutions were analyzed and simulation results were presented.

There are many directions for future work. We would like to extend our convergence results for the quorum model to more general interconnection topologies. Additionally, we would like to incorporate stochastically distributed travel times to our time delayed model and give agents the ability to determine whether a quorum exists based on their own observations. Lastly, we would like to implement nonlinear transition rules that may allow us to take inspiration from motifs in biomolecular networks and allow for interesting phenomena such as spontaneous switching as a result of an external input.

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